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String Cosmology
versus Standard and Inflationary Cosmology

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Abstract

This paper presents a review of the basic, model-independent differences between the pre-big bang scenario, arising naturally in a string cosmology context, and the standard inflationary scenario. We use an unconventional approach in which the introduction of technical details is avoided as much as possible, trying to focus the reader's attention on the main conceptual aspects of both scenarios. The aim of the paper is not to conclude in favour either of one or of the other scenario, but to raise questions that are left to the reader's meditation. Warnings: the paper does not contain equations, and is not intended as a complete review of all aspects of string cosmology.

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String cosmology versus standard and inflationary cosmology

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Abstract. This paper presents a review of the basic, model-independent differences between the pre-big bang scenario, arising naturally in a string cosmology context, and the standard inflationary scenario. We use an unconventional approach in which the introduction of technical details is avoided as much as possible, trying to focus the reader's attention on the main conceptual aspects of both scenarios. The aim of the paper is not to conclude in favour either of one or of the other scenario, but to raise questions that are left to the reader's meditation. Warnings: the paper does not contain equations, and is not intended as a complete review of all aspects of string cosmology.

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1. Introduction

The standard cosmological scenario [1, 2], rightfully one of the best celebrated conquests of the physics of the XX century, cannot be extrapolated to arbitrarily high energy and curvature scales without clashing with the singularity problem.

A singularity, on the other hand, often represents a signal that the physical laws we are applying have been extrapolated outside their domain of validity. As a well known example, we may quote here the case of the spectral energy distribution of radiation in thermal equilibrium. By applying the laws of classical physics one finds indeed the Rayleigh-Jeans spectrum, that diverges like ω^3 at high frequency. By taking into account instead the appropriate quantum corrections, this classical singularity is regularized by the bell-like Planck distribution, $\omega^4 \left(e^{\omega/T} - 1\right)^{-1}$, as illustrated in Fig. 1.

String theory suggests that the initial curvature singularity of the standard cosmological scenario could be similarly regularized by a bell-shaped curve, as qualitatively illustrated in Fig. 2. As we go back in time the curvature, instead of blowing up, could reach a maximum controlled by the string scale, and then decrease towards a state of very low curvature and weak coupling, approaching asymptotically the so-called string perturbative vacuum. This behaviour, indirectly suggested by duality and thermodynamical arguments [3], as well as, independently, by the motion of

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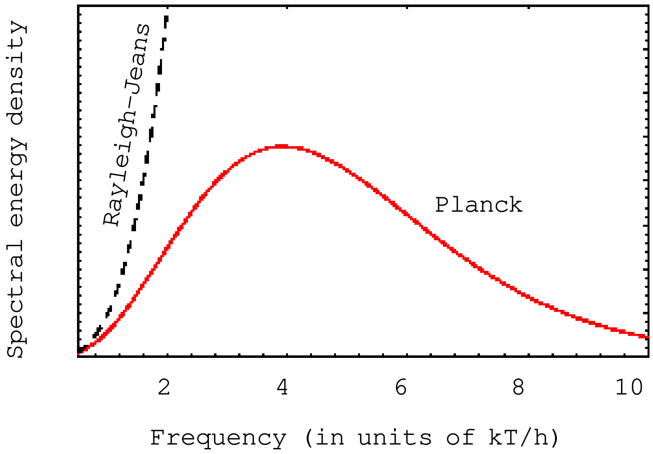

Figure 1. *The Planck distribution (full line) regularizes the classical Rayleigh-Jeans prediction (dashed line), for the spectral energy density of radiation in thermal equilibrium. The classical distribution is only valid at low enough frequency scales.*

strings in rolling backgrounds [4], is naturally grounded on the duality symmetry of the cosmological string effective action [5], and its possible implementation in the context of a realistic model of the early Universe has a lot of dynamical and phenomenological consequences [6].

In the string comology scenario of Fig. 2, the big bang singularity is replaced by a phase of high but finite curvature. It comes thus natural, in such a context, to call "pre-big bang" the initial phase in which the curvature is growing, in contrast to the subsequent "post-big bang" phase, with decreasing curvature and standard decelerated evolution.

The conventional inflationary scenario [2, 7], on the other hand, suggests a picture of the early Universe which is approximately intermediate between the standard and

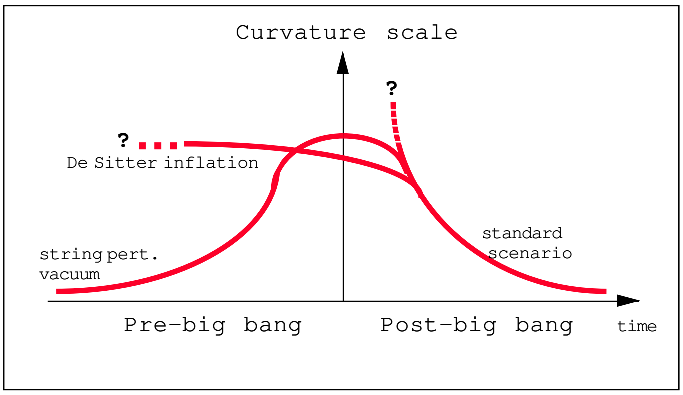

Figure 2. *Curvature scale versus time for standard, inflationary (de Sitter), and string cosmology models of the Universe. In the pre-big bang scenario the classical curvature singularity is regularized, and the standard cosmological evolution is only valid at late enough time scales.*

the string cosmology one. In that context the curvature, instead of blowing up, is expected to approach a nearly constant value, typical of a de Sitter-like geometry (see Fig. 2). However, a de Sitter phase with exponential expansion at constant curvature, implemented in the context of the conventional, potential-dominated inflation, cannot be extended back in time for ever, as first discussed in [8]. Indeed, quoting Alan Guth's recent survey of inflationary cosmology [9]:

"... Nevertheless, since inflation appears to be eternal only into the future, but not to the past, an important question remains open. How did all start? Although eternal inflation pushes this question far into the past, and well beyond the range of observational tests, the question does not disappear."

A possible answer to the above question, suggested by string cosmology and represented graphically in Fig. 2, is that all started from a state approaching, asymptotically, the flat, cold and empty string perturbative vacuum. Even if this starting point is infinitely far into the past, however, the initial state of our Universe might be non completely beyond the range of present observations, in contrast to the sentence quoted above, because the starting point may affect the dynamics of the subsequent inflationary evolution. The initial curvature scale, for instance, is constant or decreasing according to the conventional inflationary scenario, while it is growing according to the pre-big bang scenario, and this may lead to important phenomenological consequences.

In spite of the existence of a few particular examples [10], an unambiguous regularization of the curvature singularity (to all orders in the string effective action), toghether with a complete description of the transition from the pre- to the post-big bang regime, is not an easy achievement (see however [11] for recent encouraging results). Also, it is fair to say that the pre-big bang models, at present, are not free from other (more or less important) difficulties, that some aspects of the pre-big bang scenario are still unclear, and that further work is certainly needed for a final answer to all the difficulties.

Assuming that all the problems can (and will) be solved in a satisfactory way, string cosmology will provide eventually a model of the early Universe somewhat different, however, from the standard inflationary picture. The aim of this paper, therefore, is to present in a compact form a comparison (and a short discussion) of the way in which a phase of inflation could be implemented within a cosmology based on the string effective action, with respect to the standard cosmology based on the Einstein equations. Basically all the differences arise, as we will see, from the fact that in string cosmology the Universe starts evolving from an initial state at very low curvature and weak coupling, while in conventional inflation the initial state is assumed to approach the Planckian, quantum gravity regime.

The paper is organized as follows. We will discuss kinematical and dynamical differences in Sect. 2, quantum cosmology differences in Sect. 3, and phenomenological differences in Sect. 4. Sect. 5 is devoted to some concluding remarks. We will avoid as much as possible the introduction of technical details – all contained in the papers quoted in the bibliography – trying to emphasize the ideas and the main physical aspects of the different inflationary scenarios.

As already stressed in the Abstract, it seems appropriate to recall that this paper is *not* intended as a complete review of all aspects of string cosmology. Rather, the paper is narrowly focused on those aspects where there is an important overlapping of methods and objectives and, simultaneously, a strong contrast of basic assumptions, for the string cosmology and the standard scenario. For a more exhaustive and systematic approach, the interested reader is referred to the excellent review paper devoted to a presentation and a technical discussion of all presently existing (super)string cosmology models [12], as well as to two recent introductory lectures on the pre-big bang scenario [13].

2. Kinematical and dynamical differences

The idea of inflation hystorically was born [14] to solve the problem of monopoles that could be largely produced in the early Universe, at the energy scale of grand unified theories (GUT). More generally, inflation is now understood as a period of accelerated evolution that can explain why the present Universe is so flat and smooth over a so large scale of distances [2, 7].

For hystorical reasons, i.e. because inflation was first implemented as a period of supercooling of the Universe trapped in a "false" vacuum state, in the context of GUT phase transitions, inflation was first associated [14] to a phase of exponential expansion (in cosmic time) of the scale factor $a(t)$, corresponding to a de Sitter (or "quasi-de Sitter") geometrical state. But it was soon realized that any type of accelerated expansion ($\dot{a} > 0$, $\ddot{a} > 0$, where the dots denote differentiation with respect to cosmic time), can in principle solve the kinematic problems of the standard scenario [15].

Actually, even accelerated contraction ($\dot{a} < 0$, $\ddot{a} < 0$) is effective to this purpose [16], as expected from the fact that accelerated expansion can be transformed into accelerated contraction through an appropriate field redefinition, like the one connecting the String and the Einstein frame. Indeed, physical effects such as the dilution of inhomogeneities should be independent from the choice of the frame. So, inflation can be generally identified, in a frame-independent way, as a period of accelerated evolution of the scale factor (sign $\dot{a}$ = sign $\ddot{a}$).

There are, however, two classes of accelerated evolution very different from a dynamical point of view, and depending on the behaviour – growing or decreasing in time – of the curvature of the space-time manifold. A first important point to be stressed is thus the fact that the pre-big bang scenario corresponds to a phase of accelerated evolution characterized by growing (or non-decreasing) curvature [5, 6], while the phase that we shall call "standard inflation " [2, 7] is characterized by a curvature scale which tends to be constant – in the limiting case of a de Sitter metric – or slightly decreasing in time.

Before proceeding further, two remarks are in order. The first is that a phase of accelerated evolution and growing curvature, also called superinflation (or pole-inflation) [17], is not a peculiarity of string cosmology, but is possible even in general relativity: in higher-dimensional backgrounds, for instance, in the context of dynamical dimensional reduction. The important difference is that, in string cosmology, superinflation does not necessarily requires neither the shrinking of the internal dimensions, nor some exotic matter source [18] or symmetric breaking mechanism [19]: it can be simply driven, even in three spatial dimensions, by the kinetic energy of the rolling dilaton field ϕ [5, 6], parametrizing the growth of the string coupling $g = \exp(\phi/2)$ from zero (the string perturbative vacuum), to the strong coupling regime, $g \sim 1$.

The second remark is that the words "pre-big bang" should be referred to the complete cosmological evolution from the initial state approaching the string perturbative vacuum, up to the beginning of the hot, radiation-dominated phase of the standard scenario. Altough in most of this paper we shall restrict our discussion to the low-energy part of the pre-big bang phase, appropriately described in terms of the low-energy string effective action, the complete pre-big bang history necessarily includes a high-curvature "stringy" phase [20], which may also be of the inflationary type [21], and which has a curvature expected to be, on the average, non decreasing.

For a power-law, accelerated, conformally flat background, the time-behaviour of the curvature scale follows the behaviour of the absolute value of the Hubble parameter, $|H| = |\dot{a}/a|$. In such a background, on the other hand, the inverse of the Hubble parameter (i.e. the Hubble horizon $|H|^{-1}$) also controls the (finite) proper distance between the surface of the event horizon and the wordline of a comoving observer. Such a distance is shrinking for pre-big bang inflation (where $|H|$ is growing), while it is non-decreasing in standard inflation.

As a consequence of the fact that the horizon is shrinking, and the curvature is growing, it turns out that the initial state of the phase of inflation, in the pre-big bang scenario, is characterized by a curvature which is very small in Planck (or string) units, and by a Hubble horizon very large in the same units (for the sake of simplicity we may identify, at the end of inflation, the present value of the string length L_s and of the Planck lenght L_p; indeed, at tree-level, they are related by [22] $L_s = \langle g \rangle L_p = \langle \exp \phi/2 \rangle L_p$, with a present dilaton expectation value $\langle g \rangle \sim 0.3 - 0.03$). The initial state is also characterized by another very small dimensionless number, the initial string coupling $g \ll 1$, corresponding to a dilaton approaching the perturbative vacuum value, $\phi \to -\infty$.

By contrast, the standard inflationary scenario is characterized by a dilaton already settled to its present vacuum expectation value; the coupling is always strong (g is of order one), and the size of the initial horizon may be of order one in Planck units, if the initial curvature approaches the Planck scale. These kinematical and dynamical differences between standard and pre-big bang inflation are summarized in Table I.

As evident from the pre-big bang curve of Fig. 2, the smaller is the value of the initial curvature, the longer is the duration of the phase of pre-big bang inflation. The request that the inflation phase be long enough to solve the horizon and flatness

	Standard Inflation	**Pre-big bang Inflation**
Time evolution	*accelerated*	*accelerated*
Driving energy	*inflaton potential*	*dilaton kinetic energy*
Curvature	*constant or decreasing*	*growing*
Event horizon	*constant or growing*	*shrinking*
Initial curvature scale	*model-dependent*	*arbitrarily small*
Initial coupling	*strong, non-perturbative*	*arbitrarily weak, perturbative*

Table 1. Kinematical and dynamical differences between standard and pre-big bang inflation.

problems [14] thus imposes bounds on the dimensionless parameters of the initial state, controlled by the curvature and by the string coupling. Parameters such as the typical size of an initial homogeneous domain, in Planck units, must be by far greater than one for inflation to be successful. This aspect of string cosmology was pointed out already in the first papers on the pre-big bang scenario [5, 6] and, even before, also in the context of string-driven superinflation [4].

The fact that the initial curvature and coupling are small, and that the initial state is characterized by very large dimensionless numbers, may be interpreted however as a possible fine-tuning of the pre-big bang models [23], or even as a serious drawback, preventing the solution of the flatness and homogeneity problems, and supporting the conclusion that "the current version of the pre-big bang scenario cannot replace usual inflation" [24].

Consider, for instance, the horizon/homogeneity problem. At the onset of pre-big bang inflation, as usual in an inflationary context, the maximal allowed homogeneity scale is bounded by the size of the horizon. The point is that the initial horizon size H^{-1} is very large, instead of being of Planckian order. The basic question thus becomes: is an initial homogeneity scale of the order of the maximal scale H^{-1} necessarily unnatural if the initial curvature is small and, consequently, the initial horizon is large in Planck (or string) units? In other words, which basic length scale has to be used to measure the naturalness of the initial homogeneous domain, which subsequently inflates to reproduce the presently observed Universe? The Planck length or the radius of the causal horizon?

The choice of the Planck length, emphasized in [24], is certainly appropriate when the initial conditions are imposed in a state approaching the high-curvature, quantum gravity regime, like in models of chaotic inflation [25], for instance. In the pre-big bang scenario, on the contrary, the initial conditions are to be imposed when the Universe is deeply inside the low-curvature, weak coupling, classical regime. In that regime the Universe does not know about the Planck length, and the only available classical scale of distance, the horizon, should not be discarded "a priori" as unnatural [26].

This does not mean, of course, that the horizon should be always *assumed* as the natural scale of homogeneity. This suggests, however, that the naturalness of homogeneity over a large horizon scale should be discussed on the grounds of some quantitative and objective criterium, as attempted for instance in [27], taking into account also the effects of quantum fluctuations [28] that could destroy the initial, classical homogeneity (see also [29] for a discussion of "generic" initial conditions in a string cosmology context).

Another point concerns the flatness problem. In order to explain the precise fine-tuning of the present density to the critical one, the initial state of pre-big bang inflation must be characterized by large dimensionless parameters, thus reintroducing from the beginning the large numbers that one would like to explain.

This may seem to be quite unsatisfactory, as emphasized in [24]. It should be pointed out, however, that if one accepts the point of view that large numbers are always to be avoided at the onset of inflation, then should also accept the fact that natural initial conditions are only possible in the context of models in which inflation starts at the Planck scale, in order to have, for the curvature, an initial dimensionless ratio of order one. This rules out, as a satisfactory explanation of our present cosmological state, not only the pre-big bang scenario, but any model in which inflation starts at scales smaller than Planckian (unless we imagine a scenario with different stages of inflation, each of them responsible for solving different problems, and occurring at different scales: but, again, is this a natural cosmological configuration?)

Even for a single stage of inflation, occurring very near to the Planck scale, we are not free from problems, however, as we are led eventually to the following question: can we trust the naturalness of inflation models in which classical general relativity is applied to set up initial conditions at Planckian curvature scales, i.e. deeply inside the non-perturbative, quantum gravity regime? (in string cosmology, the Planckian regime directly affects the exit from the inflationary phase, and only indirectly set constraints on the initial conditions, through the finite duration of the low energy phase).

Assuming that the answer be positive, we are led to a situation that can be graphically summarized, in a qualitative way, as in Fig. 3.

- Case (a) represents a standard inflationary model of the Universe in which inflation starts at the Planck scale. The time arrow points from bottom to top, and the shaded area at the time t_0 represents a spatial section of our present homogeneous Universe, of size fixed by the present Hubble radius H_0^{-1}. As we go back in time, according to the solutions of the standard cosmological model, the horizon shrinks linearly in cosmic time, while the proper size of the present homogeneous region, controlled by the scale factor $a(t)$, shrinks slowly. When we reach the Planck scale, at the time t_f, the causal horizon is smaller than the homogeneous region, roughly by the factor 10^{-30}.

To solve this problem, the phase of standard evolution is preceeded by a phase of exponential de Sitter inflation, long enough in time from t_i to t_f, during which the curvature and the horizon stay frozen at the Planck scale, and our present portion of the Universe may "re-enter" inside the causal horizon.

It should be stressed that, in a realistic inflationary model, the horizon has to be slightly increasing from t_i to t_f, because the scale corresponding to our present Hubble radius has to cross the horizon, during inflation, at a curvature scale H_1 smaller than Planckian. We must require, in particular, that $H_1/M_p \lesssim 10^{-5}$ in order to avoid too much amplification of gravitational perturbations, that would contradict the present degree of homogeneity observed by COBE [30] at large angular scales. However, for the sake of graphical simplicity, we shall ignore this complication that is not essential for our present discussion.

- Case (b) represents a string cosmology model of the Universe, in which the inflationary pre-big bang phase, from t_i to t_f, is represented in terms of the contracting metric of the Einstein frame [16], in order to emphasize (graphically) the underlying duality and time-reversal symmetry of the scenario (there is no need, of course, that in the Einstein frame the pre-big bang scale factor exactly coincide with the time-reversal of the post-big bang solution).

  The main difference from case (a) is that in the pre-big bang epoch the curvature is growing, and the event horizon shrinks linearly in cosmic time, from t_i to t_f, instead of being constant. Since the scale factor shrinks at a slower rate, however, it is still possible for the initial homogeneous domain to be "pushed out" of the horizon, and for the Universe to emerge at the Planck scale, at the time t_f, in the same configuration as in case (a). The subsequent evolution from t_f to t_0 is the same as in the standard scenario.

- Case (c), finally, represents a string cosmology model in which the period of pre-big bang inflation corresponds in part to a phase of growing curvature, growing dilaton and shrinking horizon (from t_i to t_s), and in part to a phase in which the curvature, the horizon, and eventually the dilaton, are frozen at the Planck scale (from t_s to t_f). The initial horizon is still large in Planck units, but it is no longer reflection-symmetric to t_0, depending on the duration of the high curvature phase from t_s to t_f.

According to [24], the model (a) represents an acceptable solution to the horizon and flatness problem because inflation starts at the Planck scale, and all the dimensionless parameters characterizing the initial configuration are of order one. When the phase at constant curvature of the model (c) extends in time like in case (a) the two models pratically coincide for what concerns the naturalness of the initial conditions, as in both cases our Universe emerges at the Planck scale from a single domain of Planckian size, and we loose any observational tracks of what happened before.

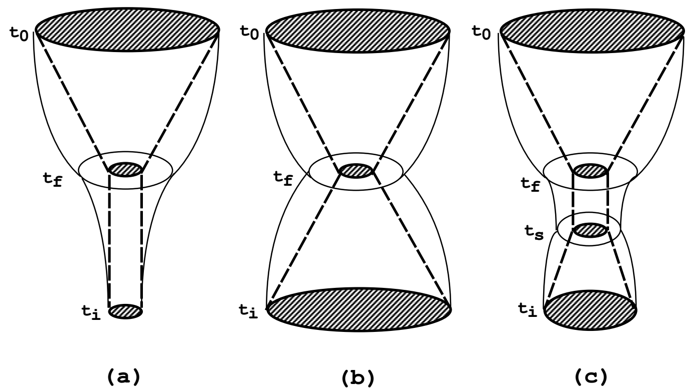

Figure 3. *Qualitative evolution of the horizon scale and of the proper size of a homogeneous region for (a) standard de Sitter inflation with constant Hubble horizon, (b) pre-big bang inflation (in the Einstein frame) with shrinking horizon, and (c) pre-big bang including a phase of high curvature inflation at the string scale. The time direction coincides with the vertical axis. The three horizontal spatial sections t_0, t_f and t_i corresponds, from top to bottom, to the present time, to the end, and to the beginning of inflation. The shaded area represents the horizon, and the dashed lines its time evolution. The full curves represent the time evolution of the proper size of the homogeneous region, controlled by the scale factor.*

The aim of the pre-big bang scenario, on the other hand, is to attempt a description of the possible cosmological evolution *before* the Planck epoch. The main difference between case (a) and (c) is that, if the duration of the phase of inflation at constant curvature is shorter than the minimal duration required for a solution of all the standard kinematic problems, what happened before the Planck scale may then become visible. In other words, there are phenomenological consequences that can be ascribed to the phase of pre-Planckian evolution, and that can be tested (at least in principle) even today (see Section 4). The model of case (b), in particular, is the limiting case in which the duration of the high curvature phase shrinks to a point, and the Universe emerges at the Planck curvature scale with a homogeneous domain large enough to fill our present Hubble radius through the subsequent standard evolution.

The above discussion refers to the curvature in Planck units, but the same arguments can also be applied to the initial value of the string coupling. If the coupling is of order one at the beginning of inflation then it is natural, in the sense of [24], if it is much smaller than one, then the inflationary growth of the coupling may have observable consequences.

To conclude this discussion it seems difficult, in our opinion, to discard a model of pre-Planckian evolution, like those illustrated in case (b), (c), only on the grounds of the large parameters characterizing its initial conditions. Such an argument could be applied in the impossibility of observational tests, namely in the absence of any phenomenological evidence about the cosmological evolution before the Planck era. But, as pointed out before, the pre-Planckian epoch becomes invisible only in the limiting case (a), in which the effective models reduces to standard inflation starting at the

Planck scale, with no unnatural initial conditions.

If, on the contrary, the high curvature phase is shorter than in case (a), then the model requires an initial horizon larger than Planckian, and a small initial coupling – which are possibly unnatural according to standard criteria [24]. In that case, however, such initial conditions are in principle accessible to present observations, so why do not try to test the scenario observationally, and try to analyze the naturalness in terms of a Bayesan approach, as attempted in [27]? In that case, the computation of "a posteriori" probabilities suggests that the observation of a large initial horizon and a small initial coupling may become "a posteriori" natural [27], because of the duality symmetry intrinsic to the pre-big bang scenario.

Finally, even if the initial state should require a certain degree of fine-tuning, this does not necessarily implies that the pre-big bang cosmology described by string models is to be discarded (after all, the description of our late-time Universe given by the standard cosmological model is rather satisfactory, in spite of the fine-tuning required in such a model if the initial state is extrapolated back in time until the Planck epoch).

Usually, the need for fine-tuning in the initial conditions means that the model is incomplete, and that a more general dynamical mechanism is required, to explain the particular initial conditions. Thus, it might well be that the pre-big bang picture provided by string cosmology does not represents the whole story of our Universe, and that only an earlier evolution can explain why, at a certain instant of time, the Universe is lead to a state so similar to the string perturbative vacuum.

3. Quantum cosmology differences

In the standard inflationary scenario the phase of exponential, de Sitter-like expansion at constant curvature cannot be infinitely extended towards the past, for a well known reason of geodesic completeness. A complete manifold requires an earlier contracting phase: in that case, however, it seems impossible, in models dominated by the inflaton potential, to stop the collapse and to bounce towards the expanding phase [8]. One has thus the problem of explaining how the Universe could emerge at the Planck scale in the initial state appropriate to exponential expansion.

At curvature scales of order one in Planck units we are in the full quantum gravity regime, and the use of the quantum cosmology approach seems to be appropriate. In this approach the Universe is represented by a wave function satisfying the Wheeler-De Witt (WDW) equation [31], and evolving in the so-called superspace, whose points represent all possible spatial geometric configurations. For pratical applications, however, the evolution of the WDW wave function is usually studied in a "minisuperspace" context, where only a finite numbers of coordinates is chosen to parametrize the different geometrical configurations.

With an appropriate choice of the boundary conditions it is then possible to obtain cosmological solutions of the WDW equation describing the "birth of the Universe" as an effect of quantum tunnelling [32, 33]. In that case, if the geometric state of the

Universe is characterized by a cosmological constant Λ (due, for instance, to the vacuum energy-density induced by a scalar field potential), the tunnelling probability is found to be proportional to $\exp(-\Lambda^{-1})$, where Λ is measured in Planck units. The Universe tends thus to emerge in a state of big vacuum energy, just appropriate to the onset of inflation. The quantum cosmology approach seems thus to provide a natural mechanism to explain the formation of "baby universes", emerging at the Planck scale, and ready to inflate according to the standard inflationary scenario [34].

The minisuperspace approach to quantum cosmology is known to be affected by various problems of technical nature: the probabilistic interpretation, the unambiguous determination of an appropriate time parameter, the semiclassical limit, the ordering of quantum operators, and so on. The most unsatisfactory aspect of this approach, in our opinion, is however the fact that the boundary conditions for the tunnelling process are to be chosen "ad-hoc". They are by no means compelling, and it is possible indeed to impose different boundary conditions, for instance according to the "no boundary" criterium [35], leading to a completely different result for the probability of creation of universes – results that are not always appropriate to inflationary initial conditions.

The source of this problem is the fact that, in a quantum description of the birth of the Universe, the final cosmological state (i.e. the Universe that we want to obtain) is well known, while the initial cosmological state (before the quantum transition) is completely unknown, at least in the context of the standard inflationary scenario. Indeed, the cosmological tunnelling is usually referred as a proces of tunnelling "from nothing" [32], just to stress the ignorance about the initial vacuum state. The classical theory of the standard cosmological scenario cannot help, because the initial state, in that context, is the big bang singularity, i.e. just what the quantum approach would like to avoid.

In a string cosmology context, the quantum approach based on minisuperspace can be implemented in a straightforward way, with the only difference that the differential WDW equation represents the Hamiltonian constraint following not from the Einstein action, but from the low-energy string effective action [36, 37, 38]. As a consequence, the "minimal" minisuperspace is at least two-dimensional, because the action always contains the dilaton, besides the metric. The formal problems related to the minisuperspace approach remains, with the possible exception of the operator-ordering problem, as the quantum ordering is unambiguosly fixed by the global, pseudo-orthogonal $O(d,d)$ symmetry of the low-energy string effective action [36, 38]. Another possible exception is the identification of the time-like coordinate in minisuperspace [39].

There is, however, a radical difference for what concerns boundary conditions. In the context of the pre-big bang scenario the initial, asymptotic state of the Universe is unambiguosly prescribed – the string perturbative vacuum – and cannot be chosen "ad-hoc". Such initial state is perfectly appropriate to a low-energy normalization of the WDW wave function, and the transition probability of string cosmology only depends on the dynamics, i.e. on the effective potential appearing in the WDW equation.

It is now interesting to observe that if we compute, in the context of the pre-big bang scenario, the transition probability from the perturbative vacuum to a final, post-big bang configuration characterized by a non-vanishing cosmological constant, we obtain [37] a probability distribution $P(\Lambda)$ very similar to the one of the conventional quantum cosmology, computed with tunneling boundary conditions. The reason is that, by imposing the perturbative vacuum as the boundary condition to the WDW equation, the WDW solutions contain only outgoing waves at the singular spacetime boundary, just like in the case of tunnelling boundary conditions [32]. In this sense, we can say that the "ad-hoc" prescription of tunnelling boundary conditions simulates, in a phenomenological way, the birth of the Universe from the string perturbative vacuum. This suggests that, instead of "tunnelling from nothing", we should speak of "tunnelling from the string perturbative vacuum" or, even better, of quantum instability and "decay" of the perturbative vacuum [40].

A further, important difference should be mentioned. The transition from the pre- to the post-big bang phase induced by the cosmological constant (or, more generally, by an appropriate dilaton potential), is represented, in the minisuperspace of string cosmology, not like a tunnelling effet, but like a quantum reflection of the WDW wave function, over an effective potential barrier. The correct description that we obtain in string cosmology for the birth of the Universe, therefore, is that of a "quantum scattering" effect [36, 37, 39].

The various differences between quantum inflationary cosmology and quantum string cosmology are summarized in Table II. Besides the formal aspects (such as tunnelling versus reflection), the basic difference is that in the standard inflationary scenario the Universe, because of quantum cosmology effects, is expected *to enter* in the inflationary regime, while in the pre-big bang scenario the Universe is expected *to exit* from the inflationary regime (or at least from the phase of growing curvature). So, in standard inflation, quantum effects at the Planck scale are expected to be responsible for inflationary initial conditions. In string cosmology, on the contrary, initial conditions are to be imposed in the opposite, low energy quantum regime, where quantum effects are negligible.

It seems appropriate, at this point, to comment on the fact that the initial state of pre-big bang inflation seems to be characterized by a large entropy S, if one assumes, as in [24], that the de Sitter relation between entropy and horizon area remains valid also when the horizon is not constant in time (or, in other words, if one assumes a saturation of the bound provided by the holographic principle [41], applied however to the Hubble horizon [42]). If S is large in Planck units, the probability that such a configuration be obtained through a process of quantum tunnelling, $\exp(-S)$, is exponentially small, as emphasized in [24]. However, as stressed above, in string cosmology quantum effects such as tunnelling or reflection are expected to be effective *at the end* of inflation, and not *at the beginning*, i.e. *not* to explain the origin of the initial state. A large entropy of the initial state, in the weakly-coupled, highly-classical regime, can only correspond to a large probability of such configuration, which is proportional to $\exp(+S)$, like for every classical and macroscopic configurations (not arising from quantum tunnelling).

	Quantum cosmology	**Quantum string cosmology**
Formal approach	*WDW, minisuperspace*	*WDW, minisuperspace*
Quantum ordering	*arbitrary*	*fixed by duality*
Boundary conditions	*arbitrary*	*string perturbative vacuum*
Outgoing waves	*tunnelling from nothing*	*reflection from pre-big bang*
Quantum transition	*beginning of inflation*	*exit from inflation*

Table 2. Quantum cosmology differences between standard and pre-big bang inflation.

Let us stress, finally, another important difference between conventional quantum cosmology and quantum string cosmology. In string cosmology quantum geometrical effects cannot be fully accounted for, as fas as we limit ourself to a WDW equation obtained from the low-energy string effective action. Indeed, when approaching the Planck scale, the string theory action acquires (even at small coupling, i.e. at tree-level in the quantum loop expansion) higher curvature correction [43], weighed by the inverse string tension α'. They are to be included into the Hamiltonian constraint, and lead in general to a higher-derivative WDW equation. This problem has been discussed in [44], and it has been shown in [45] that when the higher curvature corrections appear in the form of an Eulero density, then the WDW approach can only be applied to a dimensionally reduced version of the theory.

The quantum cosmology results reported in this section refer, in this sense, only to a model of “low-energy” quantum string cosmology [40]. In the full quantum gravity regime, in order to include all the higher-derivative contributions, the correct WDW equation should follow not from the effective action, but possibly from a conformal, sigma-model action [46], which automatically takes into account all orders in α'. We note, however, that duality transformations in toroidal moduli space have recently suggested [47] that the Lorenztian structure of the low-energy minisuperspace may have an exact meaning also in an M-theory context, even if the exact M-theory equations are expected to be in general different.

In fact, when the curvature is large in string units ($\alpha' H^2 > 1$), and also the string coupling is large ($g^2 = e^{\phi} > 1$), we necessarily enter the M-theory regime where new quantum effects are possible, such as a copious production of higher-dimensional D-branes [48]. If the curvature is small enough, the strong coupling regime of string cosmology is then expected to be described by 11-dimensional supergravity theory, and the dilaton to be interpreted as the radius (i.e., the modulus field) of the 11-th dimension

[49]. In this context string cosmology becomes U-duality covariant [12, 50, 51], and the presence in the action of Ramond-Ramond fields may be helpful to evade the problem of the curvature singularity [52, 53, 54]. The singularity, in addition, could also disappear as a result of the embedding of the low-energy solutions of string theory into a higher-dimensional ($d = 11$) manifold [51, 55].

These results seem to suggests that an appropriate quantum description of the birth of the Universe will be probably achieved only within a full M-theory approach to the strong coupling regime, in which the pre-big bang acceleration is damped, the curvature is regularized, and the Universe bounces back to the phase of standard evolution.

4. Phenomenological differences

One of the most important (and probably also most spectacular) phenomenological predictions of inflation is the parametric amplification of metric (and of other different types of) perturbations [56], and the corresponding generation of primordial inhomogeneity spectra, directly from the quantum fluctuations of the background fields in their vacuum state (see [57] for a review). Such fluctuations, when decomposed in Fourier modes, satisfy a canonical Schrodinger-like equation, whose effective potential is determined by the so-called" pump field", which depends in its turn on the background geometry.

It is then evident that different backgrounds lead to different pump fields, to a different evolution of perturbations, and thus to different spectra. In string cosmology, in particular, there are two main properties of the background that can affect the final form of the perturbation spectra. They are:

- (A) the growth of the curvature scale;
- (B) the scalar-tensor (i.e. gravi-dilaton) nature of the background.

Property (A) has two important consequences. The first, that we will call (A1), is that the pre-big bang scenario leads to metric perturbation spectra growing with frequency [58] (instead of being flat, or decreasing, like in standard inflation), because the spectral distribution of metric perturbations tends to follow the behaviour of the curvature scale at the time of the first horizon crossing. The second, that we will call (A2), is that the growth of the curvature can also force the comoving amplitude of perturbations to grow (instead of being frozen) outside the horizon. This effect, implicitly contained in the earlier, pioneer studies [59], was first explicitly pointed out in [60], and only later independently re-discovered in a string cosmology context [61]. As a further consequence of property (A) we should mention, finally, the fact that perturbations are amplified in a final "squeezed vacuum" state, and not in a "squeezed thermal vacuum" [62].

Let us first discuss the second effect (A2). This effect, on one hand, is interesting because it may lead to an amplification of perturbations more efficient than in the standard inflationary scenario. On the other hand, however, it is dangerous, because the perturbation amplitude could grow too much, during the pre-big bang phase, so as to prevent the application of the standard linearized approach, which neglects effects of back-reaction [57].

Such an "anomalous" growth of perturbations cannot be eliminated by a change of frame, because the associated physical (i.e. observable) energy density spectrum is obviously frame-independent. However, the breakdown of the linear approximation *is*, in general, *gauge-dependent*. For the particular case of scalar metric perturbations, in three isotropic dimensions, the linear approximation breaks down in the standard longitudinal gauge, but is restored in a more appropriate "off-diagonal" gauge [63], also called "uniform-curvature" gauge [64]. Moreover, as a consequence of a particolar form of duality invariance that appears explicitly in the Hamiltonian approach to perturbation theory [65], the final energy-density spectrum can always be correctly estimated by neglecting the growing mode, provided one includes in the full Hamiltonian both the contribution of the amplitude and of its conjugate momentum.

There are backgrounds, however, in which the growth of perturbations remains too strong even after the elimination of all unphysical gauge effects, and we have to limit ourselves to a restricted portion of parameter space for the linear approximation to be valid. Even in this case, however, the effect (A1) has two interesting consequences.

The first is that a growing spectrum leads to the formation of relic backgrounds whose amplitude is higher at higher frequency, where in general the backgrounds are also more easily detectable. A typical example is the formation of a relic background of cosmic gravitons which, in the frequency range of present resonant-mass and interferometric detectors ($10^2 - 10^3$ Hz), could be up to eight orders of magnitude stronger than expected in the context of standard inflation [58, 66]. Thus in principle detectable, in a not-so-far future, by the (planned) advanced version of the interferometric gravitational antennas, or by spherical resonant detectors.

The second consequence is that the normalization of the peak of the spectrum, at high frequency, is automatically controlled by the string scale [67]. The peak amplitude may be high enough to support a picture in which all the radiation, that becomes dominant at the beginning of the standard era, is produced through a process of parametric amplification, directly from the quantum fluctuations of the vacuum during the pre-big bang epoch [67, 68, 69]. Indeed, for the background fields that interact more strongly than gravitationally, the amplified fluctuations are expected to thermalize, and their energy-density is expected to grow in time with respect to the dilaton kinetic energy that was driving the background during the phase of pre-big bang inflation. This possibility is absent in the standard inflationary scenario, where the spectrum of perturbations is decreasing, and the normalization of the spectrum is determined at low frequency by the observation of the large scale CMB anisotropy [30]: the resulting energy-density of the fluctuations, in that case, is by far too low to dominate, eventually, the post-inflationary background.

Up to now we have reported some phenomenological consequences of the property (A). For what concerns the property (B), i.e. the gravi-dilaton nature of the background, we will quote here a peculiar string cosmology effect, the amplification of the quantum fluctuations of the electromagnetic field due to their direct coupling to the dilaton, according to the effective Lagrangian density $\sqrt{-g}e^{-\phi}F_{\mu\nu}F^{\mu\nu}$.

In general relativity the dilaton is absent, the Lagrangian is invariant under a conformal rescaling of the metric, and the coupling of the electromagnetic field to a conformally flat metric, typical of inflation, can always be eliminated (unless the coupling is non-minimal and/or violates $U(1)$ gauge invariance [70]). As a consequence, the inflationary evolution of the metric background is unable to amplify the electromagnetic fluctuations. In string cosmology, on the contrary, such fluctuations are amplified by the accelerated growth of the dilaton (acting as the pump field) during the pre-big bang phase. If the high-curvature string phase is long enough, it is then possible to produce in this way the "seeds", required for instance by the galactic dynamo, for the generation of cosmic magnetic fields on a large scale [71]. String cosmology thus provides a possible solution to a longstanding astrophysical "puzzle", i.e. the generation of the primordial seed fields, through a mechanism which is uneffective in the standard inflationary scenario.

This certainly represents an advantage with respect to the standard scenario. The different amplification of perturbations, however, is also asssociated to possible drawbacks. In particular, the fact that the metric perturbation spectrum, in string cosmology, grows with a very steep slope, and it is rigidly normalized at high frequency, makes problematic the matching to the anisotropy observed at the present horizon scale [30]. The generation of the observed CMB anisotropy, with the right spectrum, is instead one of the most celebrated results of standard inflation [2, 7, 57].

A possible solution to this problem, in string cosmology, comes from the amplification of the fluctuations of the Kalb-Ramond axion, which is one of the fundamental fields appearing in the string effective action, already at low energy. Indeed, unlike metric perturbations, the axion perturbations can be amplified with a rather flat spectrum [72] and, through the (integrated) Sachs-Wolfe effect [73], they can induce the temperature anisotropy observed in the CMB radiation on a large scale, both in case of massless [74] and massive [75] axion fluctuations.

It is important to observe, in that case, that the slope of the spectrum is no longer arbitrary, but rigidly determined by the COBE normalization (imposed at low frequency), and by the string normalization (imposed at the opposite, high-frequency end of the spectrum). The resulting slope turns out to be slightly increasing [74, 75], but still in agreement with the observational limits at the horizon scale [76]. At higher frequency scales, however, important differences from standard inflation may appear in the peak structure of the spectrum [77]. The possible axionic origin of the fluctuations of the CMB temperature is thus expected to be confirmed (or disproved) in a very near future, by the planned satellite observations.

The main differences between standard and string cosmology inflation, for what concerns cosmological perturbations, are summarized in Table III. The last entry of the Table refers to the amplification of the fluctuations of the dilaton background [78], another peculiar effect of string cosmology, because the dilaton is absent in the standard

	Standard Inflation	**Pre-big bang Inflation**
Pump field	*metric*	*metric and dilaton*
Spectrum (vs frequency)	*flat or decreasing*	*increasing*
Amplitude outside horizon	*frozen*	*increasing*
Normalization	*low frequency (COBE)*	*high frequency (string scale)*
Electromagnetic fields	*unaffected*	*amplified by the dilaton*
CMB anisotropy	*metric fluctuations*	*axion fluctations*
Dilaton fluctuations	*absent*	*dilaton productions*

Table 3. Amplification of vacuum fluctuations in the inflationary and pre-big bang scenario.

scenario. This effect, analogous to the amplification of tensor metric perturbations, can be interpreted (in a second quantization approach) as a process of dilaton production, which leads to the formation of a relic background of cosmic dilatons. The background is subject to various constraints, depending on the slope of the spectrum and on the mass of the dilaton, but if dilatons are light enough [79] (namely $m \lesssim 10$ KeV), they are not yet decayed and could represent today a significant fraction of the dark matter, that seems required to match various astrophysical observations.

Detecting such a background, through the gravity-like interactions of the dilatons at low-energy scales, is however a challenge that seems beyond the possibilities of present technology [80], unless the dilaton couples universally to macroscopic bodies, represents a significant fraction of dark matter, and its mass lies within the sensitivity band of gravitational detectors [81].

5. Conclusion

The pre-big bang scenario provides a model, suggested and supported by string theory, of the possible cosmological evolution before our Universe emerged at the Planck scale.

If the subsequent post-Planckian evolution follows the standard inflationary scenario, then initial conditions can be imposed at the Planck scale, and are probably natural in the sense of [24], but any track of what happened before disappears from our observational range. If, on the contrary, inflation at the Planck scale is not too long, and inside our present Hubble radius there are comoving length scales that crossed the horizon during the low-curvature pre-big bang phase, then pre-Plackian initial conditions are in principle accessible to present observations, and their naturaleness can be discussed in terms of a Bayesan analysis [27], based on "a posteriori" probabilities. Quantum cosmology methods can also be applied, taking into account however that quantum effects are possibly important at the end, and not at the beginning, of inflation.

We believe that the possibility of looking back in the past before the Planck era is the most distinctive aspect of string cosmology, with respect to the standard inflationary cosmology. Concerning the possible tracks of the pre-Planckian Universe, we have emphasized , in particular, three effects, referring to observations to be performed 1) in a not-so-far future, 2) in a near future, and 3) to observations already (in part) performed. These effects are, respectively: 1) the production of a cosmic gravity wave background, 2) the axion-induced anisotropy of the CMB radiation, and 3) the production of seeds for the cosmic magnetic fields.

These effects are not necessarily compatible among them (at least in the "minimal" version of the pre-big bang models), and it seems thus possible to test, and eventually exclude (or confirm) the pre-big bang scenario on the grounds of its phenomenological consequences. Even if, as recently stressed [82], the idea itself of inflation "cannot be falsified", the particular models can (and must) be tested, and the pre-big bang scenario, after all, can be regarded as a particular, unconventional model of primordial inflation.

It seems thus appropriate to stress, in conclusion, that the pre-big bang scenario is not alternative to the idea of inflation, but only alternative to a more conventional realization of inflation which, for hystorical reason, is still deeply anchored to the standard big bang picture, where the initial state must necessarily represent a very small, curve and dense Universe. The effort of this paper aims at stimulating the reader's meditation on the fact that this standard picture is a possibility, not a necessity, and that quite different initial conditions are possible, and not necessarily unlikely.

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