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SCALE FACTOR DUALITY FOR CLASSICAL AND QUANTUM STRINGS

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Abstract

Duality under inversion of the cosmological scale factor is discussed both for the classical motion of strings in cosmological backgrounds and for genus zero, low energy effective actions. The string-modified, Einstein-Friedmann equations are then shown to possess physically-inequivalent, duality-related solutions which generically describe the "decay" of an initial, perturbative, flat $D = 10$ superstring vacuum towards a more interesting strong-coupling state through an appealing pre-big-bang cosmological scenario.

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1 INTRODUCTION

Undoubtedly, one of the deepest quantum symmetries of string theory is (target space) duality. In its simplest form [1], duality says that a (closed) string moving on a circle of radius R is equivalent to one which moves on a circle of radius λ_s^2/R where $\lambda_s^2 = 2\alpha'\hbar$ is the fundamental length parameter (Planck constant) of string theory. Duality has been extended to more complicated situations [2, 3, 4] and is more generally termed modular invariance (in target space). It is believed to be an exact symmetry [5], at least order by order in the string-loop expansion.

Duality appears to have far-reaching consequences, such as introducing a minimal compactification scale [6], restricting the possible form of scalar (or super) potentials [7] and determining some characteristics of non-perturbative supersymmetry breaking [8]. It could also have lots to do with the notion of a minimal observable scale $O(\lambda_s)$ in string collisions [9] and with extended forms [10] of the Uncertainty and Equivalence principles, although these concepts appear to retain their validity irrespectively of compactification.

Under duality the roles of X' and $P \sim \dot{X}$ (winding number and momentum for zero modes) are interchanged. Indeed, a somewhat simplified (see below) derivation of duality [3] consists of performing a canonical transformation on the string's position and momentum variables which are integrated over in the (Hamiltonian) path integral defining the partition function Z. If no anomaly gets in the way, this immediately leads to a symmetry of Z under certain discrete changes of the metric and torsion background fields, which is one of the possible definitions of duality [3].

So far duality has been fully discussed and implemented for a variety of <u>constant</u> background fields $G_{\mu\nu}, B_{\mu\nu}, ...$and there has been some controversy as to the possibility of extending it beyond such static situations[11].

In this paper I wish to present some evidence, both at the classical and at the quantum level, that duality is a very useful concept even for time-dependent backgrounds. Being a symmetry of the effective action (including classical stringy sources), at least to lowest order in derivatives and in the string loop expansion, duality will relate, in general, physically inequivalent cosmological solutions of the string-modified Einstein-Friedmann equations (which include a non-trivial dilaton).

Unlike the usual R-duality, this symmetry does not rest on compactification and connects (physically) expanding to (physically) contracting Universes. To distinguish it R-duality, I shall refer to it as scale-factor-duality (SFD).

In Sect.2 I shall present some heuristic arguments for SFD starting from the classical motion of strings in cosmological backgrounds. In Sect.3 SDF will be substantiated by the analysis of the low-energy, tree-level string effective action. The final result will be a system of modified, SFD-invariant Einstein equations coupled to the SFD-invariant classical sources of Sect.2. In Sect.4 I shall present some explicit solutions and physical considerations, while Sect.5 will contain some conclusions and a rather speculative outlook.

In this paper I shall only present the general ideas and some results. A more detailed account, as well as extensions to more general backgrounds, will be given elsewhere [12, 13].

2 SCALE FACTOR DUALITY AT THE CLASSICAL LEVEL

The possibility of extending R-duality to time-dependent scale factors comes naturally from the study [14, 15, 16] of classical string propagation in homogeneous, isotropic, cosmological backgrounds:

$$g_{\mu\nu} = diag(-1, a^2(t)) \; ; \; \mu, \nu = 0, 1, ... (D-1)$$

In the orthonormal gauge, the corresponding string equations of motion and constraints read (i = 1,2, ..., D-1; $\mathrm{X}^0 \equiv \mathrm{t}$)

$$\ddot{\mathrm{X}}^0 - \mathrm{X}''^0 = \mathrm{a}\frac{\mathrm{da}}{\mathrm{dt}} \sum_{\mathrm{i}} \left[\left(\mathrm{X}'^{\mathrm{i}}\right)^2 - \left(\dot{\mathrm{X}}^{\mathrm{i}}\right)^2 \right]$$

$$\ddot{\mathrm{X}}^{\mathrm{i}} - \mathrm{X}''^{\mathrm{i}} = \frac{2}{\mathrm{a}}\frac{\mathrm{da}}{\mathrm{dt}} (\mathrm{X}'^0 \mathrm{X}'^{\mathrm{i}} - \dot{\mathrm{X}}^0 \dot{\mathrm{X}}^{\mathrm{i}}),$$

$$\left(\dot{\mathrm{X}}^0\right)^2 + \left(\mathrm{X}'^0\right) = \mathrm{a}^2 \sum_{\mathrm{i}} \left[\left(\mathrm{X}'^{\mathrm{i}}\right)^2 + \left(\dot{\mathrm{X}}^{\mathrm{i}}\right)^2 \right]$$

$$\mathrm{X}'^0 \dot{\mathrm{X}}^0 = \mathrm{a}^2 \sum_{\mathrm{i}} \dot{\mathrm{X}}^{\mathrm{i}} \mathrm{X}'^{\mathrm{i}}. \tag{2.1}$$

Asymptotic solutions to the system (2.1) were discussed in [16] (see also [14, 15]) for the case $a(t) \to \infty$ in the form of a systematic large-$a(t)$ expansion. Of particular interest was the superinflationary case ($\dot{H} \equiv \frac{d}{dt}\left(\frac{\dot{a}}{a}\right) > 0$) of which a typical representative is

$$\mathrm{a(t)} = (-\mathrm{t})^{-\gamma} \quad (\gamma > 0) \tag{2.2}$$

For such backgrounds, a regime of "high instability" was found to develop inevitably at late times. It is characterized asymptotically by [16]:

1. Proportionality of world-sheet time τ and of conformal time η where, as usual,

$$\mathrm{a}d\eta \equiv \mathrm{dt} \tag{2.3}$$

2. The stretching ("freezing") of spatial string coordinates:

$$\mathrm{X}'^{\mathrm{i}} >> \dot{\mathrm{X}}^{\mathrm{i}} \quad (\mathrm{i} = 1, 2 ... \mathrm{D} - 1) \tag{2.4}$$

3. A negative pressure which, in the ideal gas approximation, takes the asymptotic value:

$$\mathrm{p} = -\frac{\rho}{(\mathrm{D} - 1)} \ ; \ \rho = \text{energy density} \tag{2.5}$$

In ref. [17] the case of a rapidly contracting Universe, e.g.

$$\tilde{\mathrm{a}}(\mathrm{t}) = (-\mathrm{t})^{\gamma} = \mathrm{a(t)}^{-1} \quad (\gamma > 0) \tag{2.6}$$

was also considered and solved at "late" times ($t \to 0, \tilde{a} \to 0$). In this case, the asymptotic solution was instead characterized by:

$\tilde{1}$. Proportionality between τ and $\tilde{\zeta}$ where:

$$\mathrm{d}\tilde{\zeta} \equiv \tilde{\mathrm{a}}\mathrm{d}\tilde{\mathrm{t}} \tag{2.7}$$

$\tilde{2}$. Very fast shrinkage of the string:

$$\tilde{\mathrm{X}}'^{\mathrm{i}} << \dot{\tilde{\mathrm{X}}}^{\mathrm{i}} \tag{2.8}$$

$\tilde{3}$. An equation of state typical of an ultrarelativistic gas:

$$\tilde{\mathrm{p}} = \frac{\tilde{\rho}}{(\mathrm{D} - 1)} \tag{2.9}$$

Comparison of properties 1,2,3 and $\tilde{1}, \tilde{2}, \tilde{3}$ suggests [17] some relation between these two regimes and the corresponding solutions. Since $\tilde{a} = a^{-1}$, the result $\eta = \tilde{\zeta} = \tau$ implies, up to a constant,

$$\mathrm{t} = \tilde{\mathrm{t}} \tag{2.10}$$

A detailed study of the equations of motion and constraints shows that, indeed, one can transform any solution of the system of equations for the background metric (2.1) into a solution for the "dual" metric by the replacements

$$\begin{aligned}
\mathrm{t} &\rightarrow \tilde{\mathrm{t}} = \mathrm{t} \\
\mathrm{X}'^{\mathrm{i}} &\rightarrow \tilde{\mathrm{X}}'^{\mathrm{i}} = \mathrm{a}^2 \dot{\mathrm{X}}^{\mathrm{i}} \equiv \mathrm{P}^{\mathrm{i}} \\
\mathrm{P}^{\mathrm{i}} &\rightarrow \tilde{\mathrm{P}}^{\mathrm{i}} \equiv \mathrm{a}^{-2} \dot{\tilde{\mathrm{X}}}^{\mathrm{i}} = \mathrm{X}'^{\mathrm{i}}
\end{aligned} \tag{2.11}$$

where the latter two equations are consistent with each other thanks to the equations of motion.

¿From (2.11) and from the definition of energy and pressure density for an ideal, isotropic gas of strings:

$$\mathrm{T}_0^0 = \rho \; ; \; \mathrm{T}_{\mathrm{i}}^{\mathrm{j}} = -\delta_{\mathrm{i}}^{\mathrm{j}} \, \mathrm{p}$$

$$\overset{\mu\nu}{\mathrm{T}}(\mathrm{x}) = \frac{1}{\sqrt{-g}\pi\alpha'} \sum_n \int d\sigma d\tau (\dot{X}_n^\mu \dot{X}_n^\nu - X_n'^\mu X_n'^\nu) \delta\,(X - x)\,. \tag{2.12}$$

one finds, after use of (2.11),

$$\sqrt{-g}\rho = \sqrt{-\tilde{g}}\tilde{\rho} \;\; ; \;\; \sqrt{-g}p = -\sqrt{-\tilde{g}}\tilde{p} \tag{2.13}$$

in agreement with (2.5) and (2.9).

A "surprise" comes when one writes [17] the usual Einstein-Friedmann equations in the presence of these classical stringy sources: SFD gets badly broken! Indeed, the left- and right-hand sides of the equations transform differently under $a \rightarrow \tilde{a} = a^{-1}$. For instance, one combination of Einstein's equations reads:

$$\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G_D}{(D-2)}\,(\rho + p)\,. \tag{2.14}$$

The l.h.s. of (2.14) is clearly odd under $a \rightarrow \tilde{a} = a^{-1}$, while the r.h.s., owing to eq. (2.13), has no definite symmetry. As we shall discuss in the next section, the solution of this puzzle lies in the string's modification of Einstein's equations. Since these equations reflect the absence of conformal anomalies in the quantum theory, we can also say that it is the consistent quantization of strings that restores SFD.

3 SCALE FACTOR DUALITY FOR THE EFFECTIVE ACTION

The way duality is implemented at the quantum level is a little subtle even for usual time-independent R-duality. It involves a non-trivial transformation [18] of both the metric $G_{\mu\nu}$ and of the Fradkin-Tseytlin dilaton [19] ϕ which appear in the Euclidean 2D action as:

$$\mathrm{S} = 1/2 \int \mathrm{d^2z}\sqrt{-\gamma}(\gamma^{\alpha\beta}\partial_\alpha \mathrm{X}^\mu \partial_\beta \mathrm{X}^\nu \mathrm{G}_{\mu\nu} + (\mathrm{R}/4\pi)\phi) \tag{3.1}$$

With this definition of the background fields (and up to a numerical factor), the tree-level string effective action takes, the form [20]:

$$\Gamma_{\mathrm{eff}} = \int \mathrm{d^Dx}\sqrt{-\mathrm{G}}\mathrm{e}^{-\phi}[\mathrm{R} + \partial_\mu\phi\partial^\mu\phi - \mathrm{V} + \text{higher deriv.}]. \tag{3.2}$$

where $V = \frac{(D-10)}{3}$ and R is the scalar curvature constructed out of $G_{\mu\nu}$.

It is clear from (3.2) that even the cosmological term (proportional to $(D-10)$) is <u>not</u> invariant under $G \rightarrow G^{-1}$ unless, at the same time, ϕ is also changed:

$$G \rightarrow G^{-1};\ \phi \rightarrow \phi - trlnG. \tag{3.3}$$

A possible way to understand why ϕ has to transform non-trivially under duality, in spite of the fact it does not couple to $\dot{X}$ or X' in (3.1), is to recall that the fields appearing in Γ_{eff} are "renormalized" fields while those appearing in S are "unrenormalized". The path-integral "proof" of duality

[3] implies that the bare dilaton is left invariant. However, the relation between bare and renormalized ϕ involves, at one loop order, precisely a term proportional to $trlnG$ (see e.g. [21]) This is a generalization of an observation by de Alvis [22] that, at the linearized level (around a flat G), the dilaton mixes with the trace of G.

Thus, if the bare dilaton is invariant under duality, the renormalized dilaton will transform as in (3.3). We have actually checked that the coefficient in front of the $trlnG$ term is the correct one if dimensional regularization is used. More generally, since we expect the renormalized and bare dilaton to be related by a local transformation, some generalization of (3.3) should also work at higher orders with just a more complicated, but local, transformation law for ϕ, as recently found to be the case at the next order [23].

Let us now proceed to the case of time-dependent scale factors. Take for instance

$$\lambda_s^2 G_{\mu\nu} = diag(-1, {a_i}^2\,(t))\ ;\ B_{\mu\nu} = 0\ ;\ \phi = \phi\,(t) \tag{3.4}$$

.

We may ask if the symmetry under (3.3) survives in this case. Surprisingly perhaps, the answer is in the affirmative at least up to second order in the space-time derivatives (slowly varying fields). Furthermore, for backgrounds of the type (3.4), the symmetry group appears to be extendible from a single Z_2 to ${Z_2}^d$ defined for each $i = 1, 2, ..d$ by:

$$\mathrm{a_i(t) \to a_i^{-1}(t)\ ;\ \phi \to \phi - 2lna_i} \tag{3.5}$$

This symmetry can be further extended, as shown in [12].

If one looks at the effective, low energy string action as being a generic Brans-Dicke-type [24] modification of Einstein's gravity, one concludes that the ω parameter of Brans-Dicke is fixed in string theory, by duality, to take the value -1. Such a value is unacceptably small, phenomenologically, unless the dilaton picks up a mass [25], presumably through the same mechanism that breaks supersymmetry.

We may ask at this point whether SFD is really distinguished from R-duality even in the case of compact dimensions. In order to see that this is so, consider the case of a single circle of fixed radius R and a time-dependent scale factor $a(t)$. A naive extension [26] of R-duality would connect this situation to the one in which

$$aR/\lambda_s \to (aR/\lambda_s)^{-1} \tag{3.6}$$

However, these R-duality-related situations both describe a contracting or expanding circle according to whether aR/λ_s approaches the fixed point value 1 or moves away from it. Instead, by fixing $a(0) = \tilde{a}(0) = 1$ and by letting the two evolve according to (3.5), we are connecting, through SFD, a physically expanding ($aR/\lambda_s \to \infty$) to a physically contracting ($aR/\lambda_s \to 1$) Universe.

It follows from the previous discussion that the correct interpretation of SFD is not that of a true symmetry, but rather of a group acting on the vacuum manifold and transforming solutions of the field equations into other (generally inequivalent) solutions. SFD thus appears to be a generalization of Narain's construction [27] to the case of possibly non-compact, time-dependent backgrounds. This analogy will be made much more complete in ref. [12].

The fact that SFD is a symmetry of the action can be verified immediately at the level of the equations of motion that follow from (3.2). In the general case these read:

$$\begin{aligned} \mathrm{R} - \partial_\mu\phi\partial^\mu\phi + 2\mathrm{D}_\mu\mathrm{D}^\mu\phi - \mathrm{V} = 0 \\ \mathrm{R}_{\mu\nu} + \mathrm{D}_\mu\mathrm{D}_\nu\phi = 0. \end{aligned} \tag{3.7}$$

where D_μ denotes the usual covariant derivative of General Relativity.

For our ansatz (3.4) eqs. (3.7) can be reduced to the following set of independent equations:

$$\begin{aligned} \sum \mathrm{H}_\mathrm{i}^2 - (\dot{\phi} - \sum \mathrm{H}_\mathrm{j})^2 + \mathrm{T}^{-2} = 0 \\ \dot{H}_i - H_i(\dot{\phi} - \sum H_j) = 0 \\ H_i \equiv \dot{a}_i/a_i \ , \ T \equiv \lambda_s V^{-1/2} = \lambda_s \Big(\frac{D-10}{3}\Big)^{-1/2} \end{aligned} \tag{3.8}$$

which are clearly invariant (respectively even and odd) under SFD.

We note, incidentally, that eqs.(3.8) admit, for $D > 10$, the particular solution [28]:

$$\phi = \mathrm{Qt} \ ; \ \mathrm{Q}^2 = \mathrm{T}^{-2} = (\mathrm{D} - 10)/3\lambda_\mathrm{s}^{-2} \tag{3.9}$$

which is well known to provide an exactly conformal invariant theory to all orders in λ_s. However, as discussed in [15] (see also [29]), these solutions do not seem to describe a physical expansion of the Universe. In this paper we shall stick to the claim [15] that the role of Einstein's metric is played in string theory by $G_{\mu\nu}\lambda_s^2$.

Before giving solutions to the above equations, let us combine the results obtained so far by coupling the classical string sources described in sect.2 to the string-modified Einstein equations of this section. This straightforward excercise yields the following modification of eqs.(3.8)

$$\begin{aligned} \sum \mathrm{H_i^2} - (\dot{\phi} - \sum \mathrm{H_j})^2 + \mathrm{T}^{-2} &= -\kappa \mathrm{e}^{\phi} \rho \\ \dot{H}_i - H_i(\dot{\phi} - \sum H_j) &= 1/2 \kappa e^{\phi} p_i \\ \kappa e^{\phi} &= \alpha' e^{\phi} \lambda_s^{D-4} \sim 8\pi G_N \\ \dot{\rho} + \sum H_i(\rho + p_i) &= 0. \end{aligned} \tag{3.10}$$

where we notice that, in this case, one of the field equations, instead of being redundant, yields the energy conservation equation.

It is now clear, after use of (2.13), that, unlike for the case of Einstein's equations, the left- and right-hand sides of (3.10) transform in the same way under SFD.

4 SOLUTIONS AND PHYSICAL CONSIDERATIONS

In this paper I shall consider in some detail the case in which the effect of classical string matter can be neglected, commenting briefly on more general situations. A detailed study of the general case is being made and will appear elsewhere [13]. The general solution of eqs. (3.7) can be written explicitly and reads ($D > 10$)

$$\begin{aligned} \mathrm{a_i(t)} &= \mathrm{a_i}(-\infty)(\tanh(\pm\tau/2))^{\alpha_\mathrm{i}} \\ \bar{\phi}(\mathrm{t}) \equiv \phi(\mathrm{t}) - \sum \ln \mathrm{a_i(t)} &= -\ln\sinh(\pm\tau) + \bar{\phi}_0 \\ \tau = t/T \ , \ \sum \alpha_i^2 &= 1 \end{aligned} \tag{4.1}$$

Two remarks are in order:

i) in general the solutions are defined on a half-line in t. Equivalently, there is in general a singularity at some finite value of t here taken conventionally to be $t = 0$. If, for the moment, we consider solutions defined for $t < 0$ the minus sign has to be chosen in eq. (4.1);

ii) the known, all-order solution (3.8) can be recovered formally by taking the limit $t \to -\infty$.

The solution (4.1) is general since it depends on $2D - 1$ integration constants (this includes the time at which the singularity occurs) and, indeed, the solution is completely determined once the initial values of a_i, H_i and ϕ are given. It describes a cosmological evolution from $t = -\infty$ to $t = 0$ whereby the Universe is initially very flat (and isotropic):

$$\mathrm{H_i} = -\alpha_\mathrm{i}/\mathrm{T}\sinh^{-1}(-\tau) \to 0 \tag{4.2}$$

and the D-dimensional coupling

$$\lambda_\mathrm{D} = \exp(\phi) \to \mathrm{const.}\exp(\tau) \to 0 \tag{4.3}$$

is very weak. Hence, the initial state is perturbative from the point of view of both the σ-model and the string-loop expansion.

As long as $\tau \ll -1$ ($t \ll -T$), things do not change much. However, from $\tau = O(-1)$ onward, scale factors begin to vary faster and faster. Depending upon the signs of the α_i's, some dimensions undergo (super)inflation or very fast contraction, i.e. precisely the kind of behaviours found in [17] from solving Einstein's equations in the presence of highly unstable strings in a self-consistent way. This certainly gives a hint that the addition of classical string sources will not modify qualitatively the solutions of the pure gravity-plus-dilaton system. Notice here that it is SFD that allows, for any solution with an expanding dimension, one with a reciprocally contracting dimension: anisotropic cosmologies are natural alternatives to isotropic ones in SFD-invariant theories.

As $t \to 0^-$, H_i blows up like $\frac{\alpha_i}{t}$ while the behaviour of λ_D depends on the actual value of $\sum \alpha_i$ via

$$\lambda_D \sim (a)^{\sum \alpha_i - 1}(1 - a^2)\ ,\ a \equiv \tanh(-\tau/2) \to 0 \tag{4.4}$$

Thus λ_D goes to zero, to a finite constant or to infinity for $\sum \alpha_i > 1$, $\sum \alpha_i = 1$ and $\sum \alpha_i < 1$, respectively. However, the relevant, effective coupling in the expanding dimensions after they have become much larger than all the contracting ones is related to λ_D by

$$\lambda_{eff}^{-1} = \lambda_D^{-1} \prod_{contr.\ dim's} a_i \tag{4.5}$$

and is thus easily seen to be always growing as one approaches the singularity. In conclusion, our solutions represent an evolution from flat D-dimensional space-time and weak coupling to a regime of high curvatures and large coupling through a period of super-inflation and dimensional reduction. The duration of inflation and the corresponding "e-folding" factor

$$\mathrm{N} \equiv \ln(\frac{\mathrm{a_{final}}}{\mathrm{a_{initial}}}) \tag{4.6}$$

are determined by requiring that, at the end of inflation, our approximations are still valid, implying, at least:

$$-\mathrm{t_{final}} \geq \lambda_\mathrm{s}\ ;\ |\mathrm{H_{final}}| < \mathrm{O}(\lambda_\mathrm{s}^{-1})\ ;\ \lambda_\mathrm{D} < \mathrm{O}(1). \tag{4.7}$$

Consequently one finds:

$$\mathrm{N} \leq \mathrm{O}(\lambda_\mathrm{s}^{-1}\mathrm{T}) = \mathrm{O}((\mathrm{D}-10)^{-1/2}) \tag{4.8}$$

We thus see that there is no hope of getting a large amount of inflation in the case of a tree-level cosmological constant. Paradoxically perhaps, the amount of inflation increases for decreasing Λ_0, the reason being that the smaller Λ_0, the longer inflation will last.

The above considerations suggest considering the case of critical dimensions, $D = 10$. In this case, because of non-renormalization theorems, no potential is generated at any finite order in the string loop expansion. Neglecting non-perturbative effects, eqs.(4.1) still give the general solution by letting $\tau \to 0$. One finds:

$$\mathrm{a_i(t)} = \mathrm{a_i}(-\mathrm{t_0})(-\mathrm{t}/\mathrm{t_0})^{\alpha_\mathrm{i}}$$

$$\bar{\phi}(\mathrm{t}) = -\ln(\mathrm{t}/\mathrm{t_0})$$

$$\sum \alpha_i^2 = 1\ \ (D = 10) \tag{4.9}$$

The amount of inflation is only limited now by the initial values of the Hubble constants H_i and of the dilaton. A simple calculation yields:

$$N < min(-\phi_{initial}, -1/2ln(\lambda_s^2 \sum H_i^2)) \tag{4.10}$$

If we imagine to start, at some initial time, with a slight perturbation of the $D = 10$ flat, weakly coupled, supersymmetric vacuum, we can easily achieve large enough values of N for solving the standard cosmological problems.

On the other hand, non-perturbative, supersymmetry breaking effects are expected to produce a non-vanishing V of order [8]

$$\mathrm{V} \sim \exp(-\mathrm{c}\,\exp(-\phi)) \ll 1\ ,\ (\mathrm{c} > 0\ ,\ \phi \ll -1) \tag{4.11}$$

Furthermore, the potential will depend, in general, on the a_i through the combination $r_i(t) \equiv a_i R/\lambda_s$ if the ith. direction is a circle of radius R with a dependence of V on the r_i, itself restricted by modular invariance [7, 8].

In this case eqs. (3.10) can be shown to become:

$$\sum \mathrm{H_i^2} - (\dot{\phi} - \sum \mathrm{H_j})^2 + \mathrm{V}\lambda_\mathrm{s}^{-2} = -\kappa \mathrm{e}^\phi \rho$$

$$\dot{H}_i - H_i(\dot{\phi} - \sum H_j) + 1/2(\partial V/\partial\phi) + 1/2(\partial V/\partial ln a_i) = 1/2\kappa e^\phi p_i$$

$$\dot{\rho} + \sum H_i(\rho + p_i) = 0. \tag{4.12}$$

The above equations appear to break SFD. Only ordinary R duality is strictly preserved in the compactified dimensions thanks to the symmetry properties of V. Eqs. (4.12) can no longer be solved in closed form for a generic V and their analysis will be investigated elsewhere [13]. Nonetheless, we can already anticipate that the possibility of a long inflation and of a large e-folding factor appears to be preserved even in this case.

5 CONCLUSIONS AND SOME ”PHILOSOPHY”

Obviously, any realistic situation will have to be much more complicated than the one described by the system of eqs. (3.10). Nonetheless, we may hope some general features of the real world to be shared by the solutions of the simpler system.

Consider, for instance, eqs. (4.12) with some generic V satisfying (4.11) and depending on the "radii" r_i in a modular-invariant way. Let us start evolving from a "very classical" situation, i.e.

$$H_i \ll \lambda_s^{-1} \ , \ \phi \ll -1 \tag{5.1}$$

Under these conditions, eqs. (4.12) imply, at early times,

$$(\dot{\phi} - \sum \mathrm{H_j}) \sim \pm\sqrt{\sum \mathrm{H_i^2}} \ , \ \frac{\mathrm{d}}{\mathrm{dt}}(\sum \mathrm{H_i^2})^{-1/2} \sim \mp 1 \tag{5.2}$$

and thus a two-fold ambiguity.

We thus see that the choice of sign depends, very generally, on whether one wants to describe a late- or an early-time solution. Since, evidently, we do not want to describe today's Universe in terms of a Kaluza-Klein cosmology with a time-dependent gravitational and gauge coupling, we are forced, by physics, to choose the early-time solution (upper signs in eq. (5.2)). We thus obtain an interesting cosmological scenario whereby a classical, weak coupling, small curvature regime evolves naturally into a quantum era, with large curvatures and/or coupling. This is naturally identified with the "big-bang", i.e. with the beginning of our epoch. Of course the (semi)classical description exhibits a singularity and, as such, cannot be continued across $t = 0$. Hopefully, this just reflects the inadequacy of the semiclassical picture, while quantum string theory has a way to go through the singularity on to $t > 0$.

What will happen during the fully quantum era is clearly mere speculation since we do not know, at present, how to tackle such a complicated non-perturbative regime. As already pointed out, SFD is expected to be broken as soon as higher loops and/or compactification effects will be felt. Hopefully, this will select, out of all SFD-related solutions, those evolutions where six dimensions contract while the other three expand. During the quantum era, at least two nice "miracles" should take place: the freezing of the internal dimensions to scales $O(\lambda_s)$ and the freezing of the dilaton at a value $O(1)$ with generation of a dilaton mass. Only under these circumstances the way could be paved for starting a more conventional kind of cosmology at $t > O(\lambda_s)$.

After completion of this work I became aware of ref. [30] where the solution (4.1) to the β -function equations already appears.

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