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Peak and End Point of the Relic Graviton Background in String Cosmology

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Abstract

Using general arguments we determine the allowed region for the end point frequency and the peak energy density of the stochastic background of gravity waves expected in string cosmology. We provide an accurate estimate of the minimal experimental sensitivity required to detect a signal in the Hz to GHz range.

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In a recent paper [1] we computed, in collaboration with M. Giovannini, the spectrum of relic gravity waves produced in the context of the so-called "pre-big-bang" scenario of string cosmology [2, 3]. We showed that the spectral energy density of the produced gravitons grows with frequency following a Rayleigh-Jeans-type behaviour at low frequencies, is followed by a flatter intermediate region, and then reaches a peak value $\Omega_G(\omega_1) \sim 10^{-5}$ (in critical units) at $\omega_1 \sim 10^2$ GHz. The stochastic background of relic gravity waves is thus expected to be much stronger, at high frequency, than in the context of the standard inflationary scenario, which predicts, in the most favourable case, a flat spectrum at a level [4] of $\Omega_G \sim 10^{-14}$ in its higher frequency range. Such an enhanced production of high-frequency gravitons represents a typical signature of the pre-big-bang scenario, as previously stressed in a number of papers [3, 5, 6].

The explicit computation of the spectrum performed in [1] made use of a two-parameter model of the metric–dilaton background and of the equation for tensor perturbations obtained from the low-energy string effective action. Such an equation may be questioned when applied to the truly "stringy" high-curvature regime in which all higher orders in the string tension have to be taken into account. In view of this, the present paper aims at confirming the main findings of [1] by determining, within some inherent uncertainty, the position and height of the peak signal from the expected graviton background, without using either the perturbation equation or an explicit parametrization of the shape of the spectrum. We also discuss to what extent the position and height of the peak are affected by late entropy production, associated with some additional reheating process occurring well below the string scale.

We shall work in the context of a scenario [1, 2, 3], in which the Universe evolves from the string perturbative vacuum, through a dilaton-driven phase and a high-curvature stringy phase, towards the final radiation-dominated epoch. For a detailed discussion of the initial, pre-big-bang epoch we refer the reader to more specific papers on the general picture [3, 7], on the underlying symmetries [2, 8], on the perturbation spectra [5, 9] and on the difficulties of a classical matching to the standard radiation era [10]. The main aspect of the scenario that we shall use here is the fact that the time evolution of the classical background amplifies, with similar efficiency, both metric perturbations (gravity waves) and the vacuum fluctuations of the electromagnetic [11] and of other gauge fields, as a consequence of their coupling to a dynamical dilaton.

Thus, unlike ordinary inflation, string cosmology naturally leads to a democratic production of all sorts of ultra-relativistic particles [12], most of which subsequently thermalize and start dominating the energy density. Only gravitationally coupled particles, such as gravitons and dilatons, drop out of thermal equilibrium soon after the string phase. Of course, such a thermal background may possibly represent only a small fraction of the Cosmic Microwave Background (CMB) that we now observe, if later, efficient sources of thermal entropy existed. Nevertheless, because of their common origin at the same (string) scale, the energy density of the produced gravitons remains linked to the energy density of this primordial thermal radiation [13], and this link allows us to relate the peak of the graviton spectrum to the present CMB temperature, $T_0 = 2.7$ K, and to the amount of late entropy production. The common, primordial production of all relativistic species, typical of our scenario, is an important ingredient for obtaining such a relation.

We start by recalling that, in our scenario, metric fluctuations are amplified with a spectrum that grows with frequency. However, without knowing explicitly the time evolution of the model during the string phase, we cannot compute exactly the maximal amplified proper frequency ω_1. We thus *define* ω_1 as the frequency corresponding to the production of one graviton per polarization and per unit phase-space volume. It is known that, for larger frequencies, the production has to be exponentially suppressed [14]. With this definition, the "end point" of the spectrum in the plane $(\omega, \rho_G(\omega))$, where $\rho_G(\omega) = d\rho_G/d\ln\omega$ is the spectral energy density, has coordinates ω_1 and $\rho_G(\omega_1) = \omega_1^4/\pi^2$.

We shall now relate these coordinates to the present temperature T_0, and to the temperature scale T_r marking the beginning of the phase dominated by thermal radiation, soon after the string era. Such a scale is defined by the Einstein equations as

$$H_r^2 = \frac{8\pi}{3M_p^2}\frac{\pi^2 N_r}{30}T_r^4 \tag{1}$$

where M_p is the Planck mass, H_r the Hubble factor at $t = t_r$, and N_r is the total effective number of massless degrees of freedom in thermal equilibrium [15] at $t = t_r$ (as $N_r \gg 1$, the graviton contribution to this equation is negligible). Let us also define the fraction δs of the present thermal entropy density, generated at some intermediate scale between t_r and the present time t_0, as $\delta s = (s_0 - s_r)/s_0$, where [15]

$$s_0 \equiv \frac{2\pi^2}{45} n_0 (a_0 T_0)^3 = \frac{2\pi^2}{45} n_r (a_r T_r)^3 + s_0 \delta s \equiv s_r + s_0 \delta s \ . \tag{2}$$

Here n_0, n_r are the number of species contributing (each with its own weight) to the thermal entropy at t_0 and t_r, respectively, and a_0, a_r are the corresponding scale factors. By expressing $\omega_1(t_0)$ as $\omega_1(t_r) a_r/a_0$, and using eqs. (1) and (2), the present coordinates of the end point of the spectrum can be written in the form

$$\omega_1(t_0) = T_0 \left[\frac{M_s(t_r)}{M_p}\right]^{1/2} \left(\frac{8\pi^3 N_r}{90}\right)^{1/4} \left[\frac{n_0}{n_r}(1-\delta s)\right]^{1/3} \frac{\omega_1(t_r)}{\sqrt{H_r M_s(t_r)}} \tag{3}$$

$$\rho_G(\omega_1, t_0) = \frac{\omega_1^4(t_0)}{\pi^2} = \rho_\gamma(t_0) \left[\frac{M_s(t_r)}{M_p}\right]^2 \frac{8\pi N_r}{3N_0} \left[\frac{n_0}{n_r}(1-\delta s)\right]^{4/3} \left[\frac{\omega_1(t_r)}{\sqrt{H_r M_s(t_r)}}\right]^4 \ . \tag{4}$$

We have multiplied and divided by the value of the string mass M_s at the time $t = t_r$ (defined by [1, 12] $M_s = e^{\phi/2} M_p$, where ϕ is the dilaton), and we have introduced the present photon CMB energy density, $\rho_\gamma(t_0) = (\pi^2 N_0/30) T_0^4$, where $N_0 = 2$ is the number of photon degrees of freedom. Note that eqs. (3), (4) are exact, and that the time-dependence of M_s/M_p accounts for possible residual variations of the dilaton field for $t > t_r$ (this time-dependence is attributed to M_s or to M_p, depending on the frame in which one is working [7]). Note also that n_0, N_0 are known numbers of order unity, while n_r, N_r are numbers of order 10^2–10^3, whose precise value depends on the superstring model unifying gravity and gauge interactions.

We shall now discuss the uncertainty with which we can fix the position of the peak of the spectrum in the plane $(\omega, \rho_G(\omega))$, by using the two previous equations at fixed δs. We shall treat δs as a parameter that accounts for all subsequent non-adiabatic processes, which are not expected to be significant in our context, but which can in principle dilute, to a certain extent, the primordial graviton production (we assumed $\delta s \ll 1$ in [1]). We distinguish two possibilities, which we shall discuss separately.

The first possibility, which seems to be favoured in our context, is the one in which $H_r \simeq M_s(t_r) \simeq \omega_1(t_r)$. In this case, the total energy density ρ_{qf} produced by the amplification of the vacuum fluctuations, which becomes critical at $t = t_r$, must satisfy

$$\frac{\rho_{qf}(t_r)}{M_s^4(t_r)} = \frac{\pi^2 N_r}{30} \frac{T_r^4}{M_s^4(t_r)} = \frac{3}{8\pi} \frac{M_p^2}{M_s^2(t_r)} \ . \tag{5}$$

According to the above equation ρ_{qf} cannot be much larger than $N_r M_s^4 \simeq N_r \omega_1^4$, otherwise T_r would exceed M_s, which does not make sense in a string theory context. This implies that the integrated spectra are dominated by the end point values at $\omega_1(t_r) \simeq M_s(t_r)$. On the other hand, if $\rho_{qf} \simeq N_r M_s^4$, the value of M_p/M_s at $t = t_r$ is predicted from eq. (5) to be of order $N_r^{1/2}$, i.e. quite close to its present value [16]. Therefore, for $H_r \simeq M_s(t_r) \simeq \omega_1(t_r)$, the end point must coincide with the peak of the spectrum, and the present position of the peak follows directly from eqs. (3) and (4) with $M_s(t_r)$ fixed by a dilaton expectation value already in its present range (this is the case for which we computed an explicit spectrum [1]).

By inserting known numbers, and noting that $N_r \simeq n_r$, we obtain in this case that for fixed δs the peak position is controlled by the fundamental ratio (M_s/M_p), whose present value is expected [16] to lie in the range $10^{-2} \lesssim (M_s/M_p) \lesssim 10^{-1}$. By using this range to define our uncertainty on the peak position, we get

$$0.7 \times 10^{11}\mathrm{Hz}\ (1-\delta s)^{1/3} \left(\frac{10^3}{n_r}\right)^{1/12} < \ \omega_1(t_0) \ < 2 \times 10^{11}\mathrm{Hz}\ (1-\delta s)^{1/3} \left(\frac{10^3}{n_r}\right)^{1/12} . \quad (6)$$

This translates into an uncertainty for the height of the peak, which can be written in units of critical energy density as

$$0.7 \times 10^{-8} h_{100}^{-2}\ (1-\delta s)^{4/3} \left(\frac{10^3}{n_r}\right)^{1/3} < \ \Omega_G(\omega_1, t_0) \ < 0.7 \times 10^{-6} h_{100}^{-2}\ (1-\delta s)^{4/3} \left(\frac{10^3}{n_r}\right)^{1/3} \quad (7)$$

(for the present CMB energy density, in critical units, we have used the value [15] $\Omega_\gamma(t_0) = 2.6 \times 10^{-5} h_{100}^{-2}$, where $h_{100} = H_0(100 \text{ km sec}^{-1} \text{ Mpc}^{-1})^{-1}$). These two equations generalize to the case $\delta s \neq 0$ our previous numerical estimates [1] for the end point and the peak of the spectrum. In the case $h_{100} = 0.5$ and $M_s \simeq 0.3 M_p$ we recover the maximal values quoted at the beginning of this paper.

The corresponding allowed region for the peak of the spectrum is represented in **Fig. 1** by two boxes, which are obtained from eqs. (6) and (7) with $n_r = 10^3$, for the two cases $\delta s = 0$ and $\delta s = 0.99$. Note that even if 99% of the present entropy was produced during the latest stages of evolution, the graviton signal stays well above the standard inflationary prediction, which, in **Fig. 1**, is represented by the flat spectrum $\Omega_G = 10^{-10}\Omega_\gamma$. We also note that the theoretical estimate for the maximal allowed energy density, obtained from eq. (7), is consistent with the bound obtained from nucleosynthesis, which implies, roughly, that the total energy density in gravitons cannot exceed that of one massless degree of freedom in thermal equilibrium. According to standard nucleosynthesis analysis [15, 17] we get in fact the bound [18] $\int \rho_G(\omega, t_N) d\ln\omega \lesssim 0.1\rho_R(t_N)$, where ρ_R is the total radiation energy density at the freeze out of the neutron-to-proton ratio, $t = t_N$ (see however [19] for recent critical discussions of the standard nucleosynthesis analysis). When referred to the present CMB energy density, the above bound implies

$$h_{100}^2 \int \Omega_G(\omega, t_0) d\ln\omega \; < \; 0.2 \; \Omega_\gamma(t_0) h_{100}^2 = 0.5 \times 10^{-5}. \tag{8}$$

Unless ω_1 exactly coincides with the maximal allowed value of eq. (6), the spectrum may even be flat, from the end point down to a minimal frequency much smaller than one Hertz, without violating such a bound. This situation is described by the dashed lines [20] of **Fig. 1**, which define the allowed region for the maximal value of the spectral energy density, for the two cases $\delta s = 0$ and $\delta s = 99\%$. As clearly illustrated in the figure, the expected energy density can thus be as high as the peak value ($\Omega_G \sim 10^{-6}$) also in the typical frequency band of present gravity wave detectors, around and below the kHz range. This corresponds to an enhancement of *nine* orders of magnitude with respect to the most optimistic expectations of the standard inflationary models.

Fig. 1

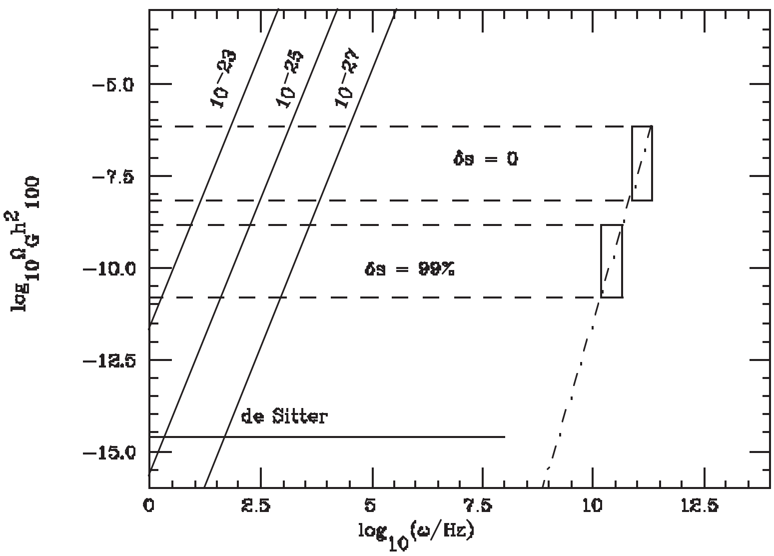

Fig. 1. *The area within the dashed lines defines the allowed region for the maximal value of the spectral energy density, for the two cases $\delta s = 0$ and $\delta s = 0.99$ (the plot is done using* $\mathrm{n_r} = 10^3$*). The two boxes on the right border define the position of the end point of the spectrum if the end of the string era occurs in the strong coupling regime. For comparison, the flat graviton spectrum of the de Sitter inflationary scenario is plotted for an inflation scale high enough to account for the observed large scale anisotropy. Also plotted are three*

lines of constant spectral amplitude $S_h^{1/2} = 10^{-23}, 10^{-25}$ *and* $10^{-27}\mathrm{Hz}^{-1/2}$*, as well as the (dash-dotted) "one-graviton" line, along which the end point is shifted as a function of late entropy production.*

Let us now consider, for completeness, a scenario in which the curvature starts decreasing from the maximal scale $H_1 \simeq M_s(t_1) \simeq \omega_1(t_1)$, while the string coupling e^ϕ (ϕ is the dilaton) is still very small. In this scenario the transition to the regime of decelerated expansion is induced by higher derivative corrections rather than by the back-reaction of the produced quanta. The radiation-dominated epoch is now reached at a scale $H_r << H_1$, and is preceded by a decelerated, dilaton-driven epoch [7]. Inserting the explicit background solutions we find $\omega_1(t_r)/\sqrt{H_r M_s(t_r)} << 1$ implying, from eq. (3), that the end point of the spectrum is shifted to much lower values (unless $M_s(t_r)/M_p$ is very large; this seems to be excluded, however, since it would correspond to the dilaton having gone very far into the non-perturbative region at $t = t_r$). A shifted value of the end point ω_1 implies a smaller total energy density Ω_G, unless the spectrum has a peak at some arbitrary frequency ω_P lower than ω_1, with such a height that the integrated graviton energy is still of the same order as that of a thermal degree of freedom, at $t = t_r$. In that case the peak would again be localized, for any given δs, within the dashed lines of **Fig. 1**. The allowed region of **Fig. 1** thus refers not only to a flat spectrum but also, in principle, to a spectrum with a peak energy density higher than the end point value. We note, however, that for $\omega_P << \omega_1$, and $\Omega_G(\omega_P) >> \Omega_G(\omega_1)$, present calculations based on the low-energy effective theory appear to preclude the possibility of having enough quantum fluctuations to make them dominant at $t = t_r$ (at least for a monotonic time evolution of the dilaton and of the metric scale factor).

In order to compare our prediction with the sensitivities of gravity waves detectors, it is convenient to express the spectral energy density in terms of the spectral amplitude $S_h^{1/2}(\nu)$, $\nu = \omega/2\pi$, defined by

$$\langle h(\nu)h^*(-\nu')\rangle = \frac{1}{2}\delta(\nu+\nu')S_h(\nu), \qquad h(\nu) = \int dt\ h(t)e^{-2\pi i\nu t} \tag{9}$$

where $h(x,t)$ is either one of the two polarized, dimensionless gravity wave amplitudes, and $\langle ...\rangle$ denotes time or ensemble average. The average energy density ρ_G, summing over polarizations, satisfies [15] $8\pi\rho_G = M_p^2\langle\dot{h}^2\rangle$. The corresponding spectral density, in critical units, is thus related to S_h by

$$\Omega_G(\nu) = \frac{8\pi\rho_G(\nu)}{3M_p^2 H_0^2} = \frac{4\pi^2\nu^3 S_h(\nu)}{3H_0^2} = 1.25 \times 10^{36} h_{100}^{-2}\ \nu^3 S_h(\nu)\ \mathrm{Hz}^{-2}. \tag{10}$$

In **Fig. 1** we have plotted three lines of constant sensitivity, corresponding to $S_h^{1/2} = 10^{-23}$, 10^{-25} and 10^{-27} $\mathrm{Hz}^{-1/2}$. Entering the region where we expect a signal, $\Omega_G h_{100}^2 \lesssim 10^{-6}$, would require a minimal sensitivity (from eq. (10))

$$S_h^{1/2}(\nu) \lesssim 3 \times 10^{-26} \left(\frac{\mathrm{kHz}}{\nu}\right)^{3/2}\ \mathrm{Hz}^{-1/2}. \tag{11}$$

Very recent, direct measurements with cryogenic resonant detectors provide an upper limit [21] on the existence of a relic graviton background, $S_h^{1/2} < 6 \times 10^{-22}$ $\mathrm{Hz}^{-1/2}$, at $\nu = 907$ Hz and $\nu = 923$ Hz. This limit is still too high to be significant for our background. However, much better sensitivities can be reached through the cross-correlation of existing resonant detectors [21] such as EXPLORER, NAUTILUS and AURIGA [22], as well as from interferometric detectors that will start operating in the near future, such as GEO [23], LIGO [24] and VIRGO [25]. Finally, spherical detectors [26] also appear promising, because of their high cross section at several frequencies for both tensor and scalar metric fluctuations.

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