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Birth of the Universe in String Cosmology

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Abstract

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BIRTH OF THE UNIVERSE IN STRING COSMOLOGY

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1. Introduction

In the context of the pre-big bang cosmological scenario[1], based on the string effective action, the process of "birth of the Universe" corresponds to the transition from the string perturbative vacuum to the standard radiation-dominated regime, passing through a high-curvature and strong coupling phase, in which quantum and "stringy" effects may become important. Such a process may be qualitatively illustrated as in Fig. 1, by plotting the time evolution of the curvature scale from the initial vacuum down to the present cosmological state. The aim of this paper is to introduce, and briefly discuss, the classical and quantum approach to the transition from the pre-big bang to the post-big bang regime.

Before starting the discussion, let me stress that the physical system that we call Universe evolves in time, like all physical systems. For our convenience, its evolution can be divided into various phases: now we are in the matter-dominated phase, but in the past there was certainly a radiation-dominated phase, and an explosive phase of very high curvature and density, that we may call "big bang". I cannot find any convincing reason, however, to believe that before the big bang there was "nothing".

To make an analogy with another physical system, consider for instance the β-decay of a neutron. The initial neutron is transformed into a proton, an electron and a neutrino. For these three particles, the decay process is a sort of "big bang" which marks the beginning of their existence. This does not means, however, that these three particles spring out of nothing: initially, the system was in a different quantum state, representing a neutron.

In the same way, coming back to cosmology, the high curvature big bang phase certainly marks the beginning of the present state of the Universe, i.e. of the standard Friedmann phase that we can observe today. It seems quite reasonable, however, to wonder about the state of the Universe preceeding the big bang explosion. The answer suggested by string theory is illustrated in Fig. 1: our present cosmological phase, decelerated, with decreasing curvature, might have been complemented by a dual phase, accelerated, with growing curvature, which seems natural to call "pre-big bang". In this context, the very early cosmological evolution from the perturbative vacuum, i.e. from a configuration with asymptotically flat metric and vanishing coupling constant, can be consistently described by the lowest-order string effective action, as will be discussed in the following Section.

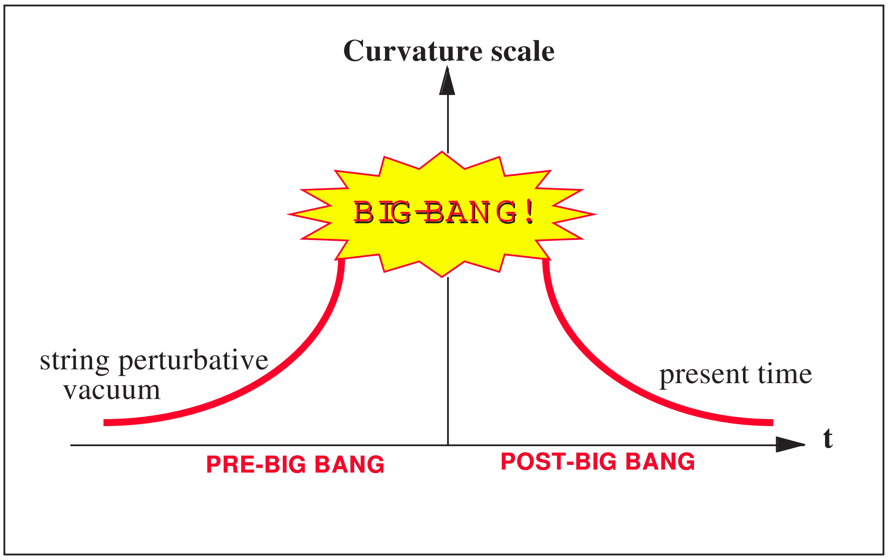

Figure 1: *Curvature scale versus time in the pre-big bang scenario.*

2. Low-energy pre-big bang evolution

In the context of the pre-big bang scenario, the global evolution of the universe can be conveniently represented in the phase space spanned by the Hubble factor and by the dilaton kinetic energy.

Consider in fact the lowest order gravi-dilaton effective action[2], which in d critical spatial dimensions, and in the string frame, takes the form

$$S = -\frac{1}{2\,\lambda_s^{d-1}} \int d^{d+1}x\,\sqrt{|g|}\,e^{-\phi}\,\left(R + \partial_\mu\phi\partial^\mu\phi\right). \tag{2.1}$$

($\lambda_s \equiv (\alpha')^{1/2}$ is the basic string length parameter). For an isotropic and spatially flat background (but we can also consider more general initial conditions, spatially curved[3] and inhomogeneous[4]), the asymptotic solutions approaching the singularity can be written as[5]:

$$a(t) = (\mp t)^{\mp 1/\sqrt{d}}, \qquad\qquad \overline{\phi}(t) = -\ln(\mp t), \tag{2.2}$$

where a is the scale factor, and $\overline{\phi}$ is the so-called "shifted" dilaton:

$$\overline{\phi} = \phi - d\ln a - \ln \int \frac{d^dx}{\lambda_s^d} \tag{2.3}$$

(we are assuming spatial sections of finite volume). Such solutions are characterized by four branches, depending on the range of time, and on the power of the scale factor. As illustrated in Fig. 2, these solutions are represented by the bisecting lines in the plane spanned by $\sqrt{d}H$ and $\dot{\overline{\phi}}$ ($H = \dot{a}/a$, and a dot denotes differentiation with respect to the cosmic time t).

The string perturbative vacuum is characterized by $H = 0 = \dot{\overline{\phi}}$, and corresponds to the origin of that plane. In the limits $|H| \to \infty$, $|\dot{\overline{\phi}}| \to \infty$, the background approaches a curvature singularity. The four branches of the solution describe expansion or contraction depending on the sign of H, and represent a pre-big bang configuration (evolving towards the curvature singularity) or a post-big bang configuration (evolving from the singularity towards an asymptotically flat spacetime), depending on the sign of $\dot{\overline{\phi}}$. Notice that, in the isotropic case, only the branch called "expanding pre-big bang" in Fig. 2 describes a true evolution from the perturbative vacuum (i.e. from a state of zero string coupling), because in the contracting branch the true dilaton ϕ is decreasing[1], and the string coupling e^{ϕ} is also decreasing.

The four branches of the solution are connected by T-duality transformations[5]:

$$a \to \tilde{a} = a^{-1}, \qquad\qquad \overline{\phi} \to \overline{\phi}, \tag{2.4}$$

and t-reversal transformations:

$$a(t) \to a(-t), \qquad\qquad \overline{\phi}(t) \to \overline{\phi}(-t). \tag{2.5}$$

A monotonic (expanding or contracting) transition from pre- to post-big bang thus requires a combination of both T and t transformations, as illustrated in Fig. 2. Smooth self-dual solutions, characterized by $a(t) = a^{-1}(-t)$, and defined over the whole time range $-\infty \le t \le +\infty$, would automatically connect the pre- and post-big bang regime, avoiding the curvature singularity. Such solutions are possible,

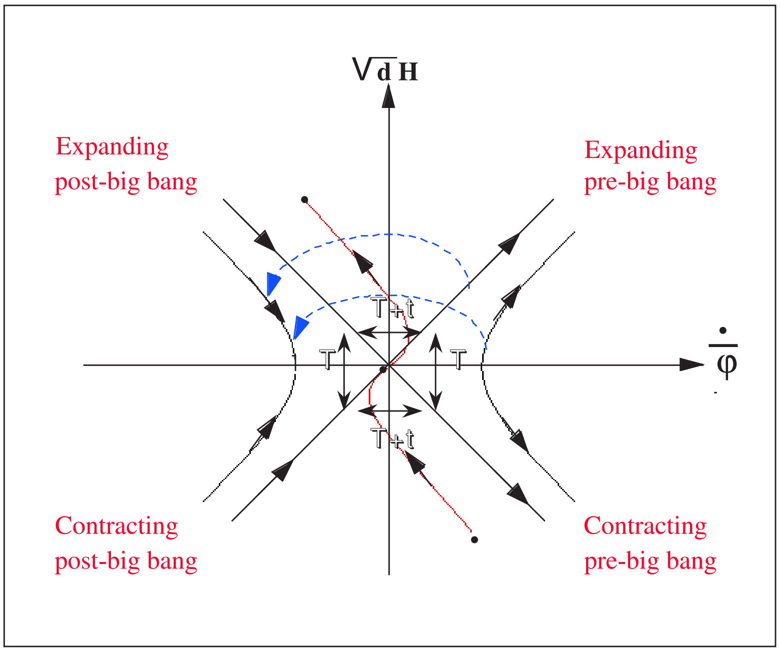

Figure 2: *Global view of the possible cosmological evolution from and towards the string perturbative vacuum.*

but only at the price of adding to the action an "ad-hoc", non-local potential for the shifted dilaton[1,6].

The simplest potential is the one induced by a cosmological constant, $V(\overline{\phi}) = \Lambda$. In that case the solutions are characterized by the condition[5]

$$\dot{\overline{\phi}}^{\,2} - (\sqrt{d}H)^2 = \Lambda, \tag{2.6}$$

representing an hyperbola in the plane of Fig. 2, and the initial vacuum is shifted to a state of flat metric and linearly evolving dilaton. The solution is still characterized by four branches disconnected by a curvature singularity, so that a transition from the pre- to the post-big bang phase remains classically forbidden. It may be allowed, however, at a quantum level.

If we compute, with the Wheeler-De Witt (WDW) equation[7], the probability of transition to a post-big bang phase with $\Lambda \neq 0$, we find indeed that such a probability is finite and non-vanishing, in spite of the presence of a curvature singularity disconnecting, classically, the two regimes[6,8] (such transitions are represented by the dashed curves of Fig. 1).

The transition from a state of positive $\dot{\overline{\phi}}$ to a state of negative $\dot{\overline{\phi}}$ is allowed even classically, and even in the absence of a dilaton potential, provided we add to the lowest-order action (2.1) the higher-derivative corrections, arising from an expansion of the string effective action in powers of the curvature (the so-called α' expansion[9]). Already to first order in α', we can find indeed exact solutions which smoothly interpolate between the pre- and post-big bang regime[10], connecting the perturbative vacuum to a final state of constant curvature and linearly evolving dilaton, as illustrated by the solid curve of Fig. 2.

Fig. 2 illustrates the two possibilities (classical, solid line, and quantum, dashed lines) of transition from pre- to post-big bang, and thus provides a schematic summary of my subsequent discussion.

3. Quantum string cosmology

The quantum approach to the transition is based on the WDW equation, obtained from the low-energy string effective action. All the standard quantum cosmology problems (time parameter, probability interpretation, semiclassical limit) remain, except perhaps the problem of quantum ordering, since the ordering is fixed by the duality symmetry[6]. The aim of this approach is to compare, and possibly contrast, the quantum results obtained in the context of the standard cosmological scenario, with the results obtained, with the same method and the same assumptions, in the context of the pre-big bang scenario.

The main difference between the two scenarios is qualitatively illustrated in Fig. 3, in which we compare classical cosmology, quantum cosmology with tunnelling boundary conditions[11,12], and quantum string cosmology for the pre-big bang scenario. While in classical cosmology the initial singularity is unavoidable, in quantum cosmology the Universe may avoid the singularity, emerging from the quantum regime through a tunnelling process. The natural suggestion of string theory is that tunnelling in not "from nothing"[11], but from the preceeding pre-big bang phase. Also, tunnelling in not through a curvature singularity, but through a string phase of high (but finite) curvature.

3.1. *Quantum transition from pre- to post big bang*

For a sort, but self-contained, illustration of how this program could be implemented, we can start with the simplest gravi-dilaton action (2.1), supplemented by

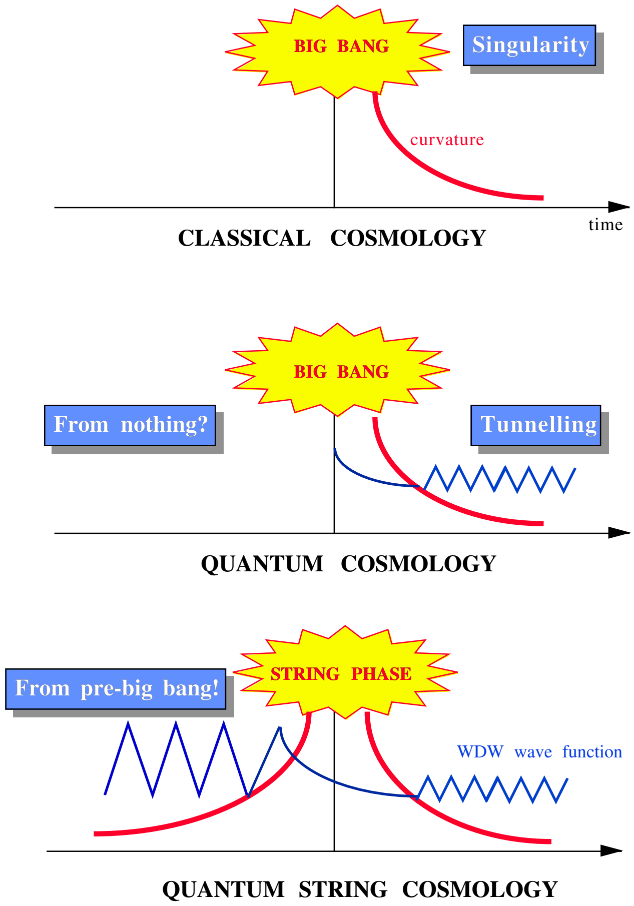

Figure 3: *A qualitative comparison of classical cosmology, quantum cosmology with tunnelling boundary conditions, and quantum string cosmology for the pre-big bang scenario.*

a dilaton potential $V(\phi)$, in $d=3$ isotropic dimensions:

$$S = \frac{\lambda_s}{2} \int dt \, \frac{e^{-\overline{\phi}}}{N} \left(\dot{\beta}^2 - \dot{\overline{\phi}}^2 - N^2 \, V \right) . \tag{3.1}$$

N is the usual lapse function, and $\beta = \sqrt{3} \ln a$. The variation of the lapse, in the cosmic time gauge $N=1$, gives the Hamiltonian constraint

$$\Pi_\beta^2 - \Pi_{\overline{\phi}}^2 + \lambda_s^2 \, V(\beta, \overline{\phi}) \, e^{-2\,\overline{\phi}} = 0 \ , \tag{3.2}$$

and the WDW equation

$$\left[\partial_{\overline{\phi}}^2 - \partial_\beta^2 + \lambda_s^2 \, V(\beta, \overline{\phi}) \, e^{-2\,\overline{\phi}} \right] \psi = 0, \tag{3.3}$$

where we have introduced the canonical momenta:

$$\Pi_\beta = \frac{\delta S}{\delta \dot{\beta}} = \lambda_s \, \dot{\beta} \, e^{-\overline{\phi}}, \qquad \Pi_{\overline{\phi}} = \frac{\delta S}{\delta \dot{\overline{\phi}}} = -\lambda_s \, \dot{\overline{\phi}} \, e^{-\overline{\phi}}. \tag{3.4}$$

In the absence of a dilaton potential the WDW equation (3.3) reduces to the massless Klein-Gordon equation, and we recover a plane wave representation of the four branches of the classical solution, $\psi_\beta^{(\pm)} \psi_{\overline{\phi}}^{(\pm)} \sim e^{\pm ik\beta \pm ik\overline{\phi}}$, where

$$\Pi_\beta \, \psi_\beta^{(\pm)} = \pm k \, \psi_\beta^{(\pm)}, \qquad \Pi_{\overline{\phi}} \, \psi_{\overline{\phi}}^{(\pm)} = \pm k \, \psi_{\overline{\phi}}^{(\pm)}. \tag{3.5}$$

They corresponds to expansion ($\Pi_\beta > 0$), contraction ($\Pi_\beta < 0$), growing dilaton ($\Pi_{\overline{\phi}} < 0$), decreasing dilaton ($\Pi_{\overline{\phi}} > 0$), according to eq. (3.4). A transition from a state of pre-big bang expansion to a state of post-big bang expansion thus corresponds, in this representation, to a transition from a state of positive momentum Π_β and negative momentum $\Pi_{\overline{\phi}}$, to a state with positive Π_β and $\Pi_{\overline{\phi}}$. In other words, the transition corresponds to a monotonic evolution along the β direction, and to a reflection along the $\overline{\phi}$ direction.

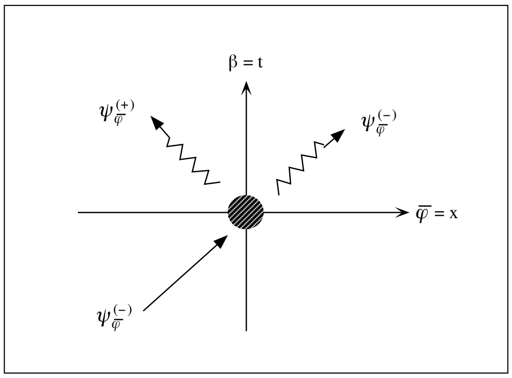

Figure 4: *Transition from pre- to post-big bang represented as a spacelike reflection of the wave function, in the minisuperspace spanned by β and $\overline{\phi}$.*

This suggests to look at the transition as at a scattering process[6,8] in the minisuperspace spanned by β and $\overline{\phi}$, in which β plays the role of the time-like coordinate, and $\overline{\phi}$ plays the role of the space-like coordinate, as illustrated in Fig. 4. The initial pre-big bang state, $\psi \sim e^{+ik\overline{\phi}-ik\beta}$, is partially transmitted towards the singularity, and partially reflected back to a post-big bang configuration, $\psi \sim e^{-ik\overline{\phi}-ik\beta}$. The transition probability is controlled by the reflection coefficient, $R = |\psi_{\overline{\phi}}^{(+)}|^2/|\psi_{\overline{\phi}}^{(-)}|^2$ (see Ref. [13] for a rigorous definition of scalar products in the appropriate Hilbert space).

The boundary conditions, in this context, are unambiguously fixed by the choice of the perturbative vacuum as the initial state of the Universe. It may be noted that, with this choice, there are only outgoing waves at the singular spacetime boundary ($\overline{\phi} \rightarrow +\infty$), just like in the case of tunnelling boundary conditions[11]. We may thus look at the process as at a sort of "tunnelling from the string perturbative vacuum", even if, in this minisuperspace representation, the WDW wave function is actually reflected.

When the reflection is induced by a cosmological constant, $V = \Lambda$, we have checked[8] that the transition probability is indeed very similar to the tunnelling probability. Both are exponentially suppressed, unless the cosmological constant is very large and the proper size of the transition volume is very small in string units. The only basic difference is that, in a string cosmology context, the coupling constant is controlled by the dilaton and it is thus running in Planckian units. As a consequence, the Universe tends to emerge from the pre-big bang phase in the strong coupling regime, since the transition probability has a typical instanton-like behaviour $\sim \exp(-g_s^{-2})$, where $g_s = \exp(\phi/2)$ is the string coupling parameter.

It is important to stress, finally, that the WDW eq. (3.3) is free from problems of operator ordering. Indeed, thanks to the duality symmetry of the effective action, the WDW Hamiltonian is associated to a globally flat minisuperspace.

For a more general discussion of this point, we may add to the effective action (3.1) a non-trivial antisymmetric tensor (also called torsion) background, $B_{\mu\nu} \neq 0$. The kinetic part of the action may then be written in compact form as

$$S = -\frac{\lambda_s}{2} \int dt e^{-\overline{\phi}} \left[(\dot{\overline{\phi}})^2 + \frac{1}{8} \mathrm{Tr}\ \dot{M} (M^{-1})^{\cdot} \right], \tag{3.6}$$

where M is a symmetric $2d \times 2d$ matrix, including the spatial part of the metric, $G \equiv g_{ij}$, and of the torsion, $B \equiv B_{ij}$:

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}. \tag{3.7}$$

This action is invariant under global $O(d,d)$ transformations[14] which leave invariant the shifted dilaton,

$$\overline{\phi} \rightarrow \overline{\phi}, \qquad M \rightarrow \Omega^T M \Omega, \tag{3.8}$$

where

$$\Omega^T \eta \Omega = \eta, \qquad \eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{3.9}$$

The Hamiltonian associated to torsion-graviton background,

$$H \sim \mathrm{Tr}\ (M\ \Pi_M M\ \Pi_M), \qquad \Pi_M = \delta S/\delta M, \tag{3.10}$$

would seem to have ordering problems, because $[M, \Pi_M] \neq 0$. However, thanks to the $O(d,d)$ properties of M,

$$M\eta M = \eta, \tag{3.11}$$

we can always rewrite the kinetic part of the action in terms of the flat $O(d,d)$ metric η:

$$\mathrm{Tr}\ \dot{M}(M^{-1})^{\cdot} = \mathrm{Tr}\ (\dot{M}\eta)^2. \tag{3.12}$$

The corresponding Hamiltonian

$$H \sim \mathrm{Tr}\ (\eta\ \Pi_M\ \eta\ \Pi_M) \tag{3.13}$$

has a flat metric in momentum space, with no ordering problems for the corresponding WDW equation.

It may be noted that, for a general curvilinear parametrization of superspace, the ordering fixed by the duality symmetry is equivalent to the ordering fixed by the requirement of reparametrization invariance[6]. It is crucial, for this equivalence, the fact that there are no contributions to the ordered Hamiltonian from the scalar curvature of minisuperspace, because minisuperspace is globally flat.

3.2. *Quantum transition from expansion to contraction*

The transition discussed in the previous Section was characterized by a monotonic evolution of the scale factor. In the context of the pre-big bang scenario, however, it is also important to consider the possibility of transitions from expansion to contraction. Indeed, if we start with a higher-dimensional and anisotropic perturbative vacuum, the initial growth of the dilaton requires a large enough number of expanding dimensions[1], in general larger than 3. On the other hand, a model of dynamical dimensional reduction requires only three expanding dimensions, with all the other dimensions contracting down to a final compactification scale. The late-time transition of an appropriate subsection of the spatial manifold, from the expanding pre-big bang phase to the contracting pre-big bang phase, may thus be useful to implement a realistic cosmological scenario.

If the dilaton potential depends only on $\overline{\phi}$, the action (3.1) is duality-invariant, the momentum along β is conserved, $[\Pi_\beta, H] = 0$, the evolution of the scale factor is monotonic, and the transition from expansion to contraction is classically forbidden. Such a transition is allowed at the quantum level, however, where it can be represented (in a third quantization formalism) as a process of pair production (pairs of universes, in this case), out of the string perturbative vacuum[15]. This process is described by the same action, same Hamiltonian, same minisuperspace as before, but with different boundary conditions, and with a 90 degree rotation of the time axis (see Ref. [16] for a classification of the different types of scattering of the perturbative vacuum in the $(\beta, \overline{\phi})$ minisuperspace).

Consider in fact the configuration illustrated in Fig. 5, where we have one incoming wave in the weak coupling regime $\overline{\phi} \to -\infty$, and two waves, one incoming, the other outgoing, in the strong coupling regime $\overline{\phi} \to +\infty$. All the asymptotic

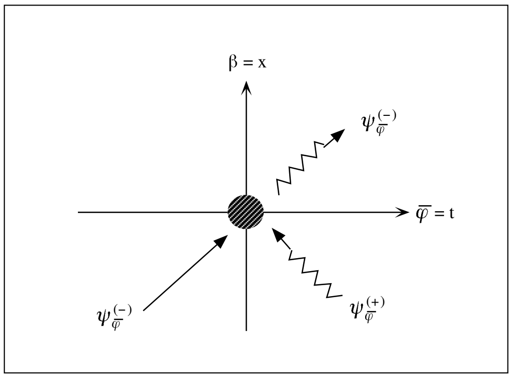

Figure 5: *Production from the vacuum of a universe–anti-universe pair, one expanding, the other contracting.*

states appearing in Fig. 5 are eigenstates of Π_β, with positive momentum $k > 0$. However, if $\overline{\phi}$ is the time-like coordinate, the final state is a mixture of positive and negative frequency modes, $\psi_{\overline{\phi}}^{(\pm)}$. In a quantum field theory context, this configuration describes a Bogoliubov process of pair creation out of the vacuum, in which the negative frequency mode is re-interpreted as an anti-particle of positive frequency (i.e. positive energy) and opposite spatial momentum.

In our case the spatial momentum is Π_β, and a negative value of Π_β corresponds to a contraction. The final state of the process illustrated in Fig. 5 thus describes, in a third quantization formalism, the production from the vacuum of a universe–anti-universe pair, one expanding, the other contracting.

With such a boundary conditions, the process cannot be interpreted as a tunnelling. It can be interpreted, instead, as an "anti-tunnelling from the string perturbative vacuum", i.e as a process in which the WDW wave function is parametrically amplified in superspace[15]. The probability of the process is indeed controlled by the Bogoliubov coefficient which weights the anti-universe content of the final state, and which becomes, in the parametric amplification regime, the inverse of the quantum mechanical transmission coefficient associated to the scattering process.

4. Classical evolution from pre- to post-big bang

A classical description of the transition, in terms of an exact solution connecting in a smooth way the pre- and post-big bang regime, is known to be excluded in the context of the lowest-order string effective action, for any local (and realistic) dilaton potential[17]. It may be allowed, however, when higher order corrections to the action are taken into account.

In string theory there are two types of higher order corrections: higher-derivative terms, appearing in the so-called α'-expansion[9], and and higher loops in the string coupling parameter $g_s = e^{\phi/2}$. The first is a truly "stringy" effect, due to the presence of the minimal, fundamental length $\lambda_s = (\alpha')^{1/2}$; the second is a quantum effect, controlled by the expectation value of the dilaton field.

Both types of corrections are probably required for a complete and successful description of the transition. For pedagogical reasons, however, it is better to discuss their effects separately, starting with the α' corrections that are probably the first to come into play when the background evolves from the perturbative vacuum[10].

4.1. *Higher-derivative corrections*

To first order in α', i.e. including all terms required by the gravi-dilaton sector of string theory up to four derivatives in the tree-level action, the action can be written as[9]:

$$S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left[R + (\nabla\phi)^2 - \frac{\alpha'}{4} \left(R^2_{GB} - (\nabla\phi)^4 \right) \right], \tag{4.1}$$

where $R^2_{GB} \equiv R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is the usual Gauss-Bonnet invariant. Note that there are no free parameters, except a possible number of order unity in front of the α' corrections, depending on the particular (super)string theory (bosonic, heterotic, ...) adopted. We have chosen here a convenient field-redefinition that eliminates higher-than-second derivatives from the equations of motion, but at the price of introducing dilaton-dependent α' corrections.

A numerical integration of the equations of motion (in any number of dimensions) shows that, already to this order in α', the effect of the higher-derivative terms is to contrast the growth of the curvature and of the dilaton, leading the background to a final regime in which the curvature is constant, and the dilaton is linearly growing (in cosmic time). This is clearly illustrated in Fig. 6, which shows the result of a numerical integration of the equations of motion, with the perturbative vacuum ($H = 0 = \dot\phi$) as initial conditions at $t \to -\infty$. The plot refers to the case $d = 3$, but the result is qualitatively the same[10] for any d.

It is important to stress that such a state of frozen curvature and linear dilaton may represent a solution to all orders in α' of the tree-level action. Indeed, the complete set of the string σ-model β-function equations, for a Bianchi-type I gravi-dilaton background, has been shown to reduce, to all orders, to an algebraic set of $d+1$ equations in $d+1$ unknowns, representing the (anisotropic) linear rate of growth of the log of the metric and of the string coupling[10]. The existence of real

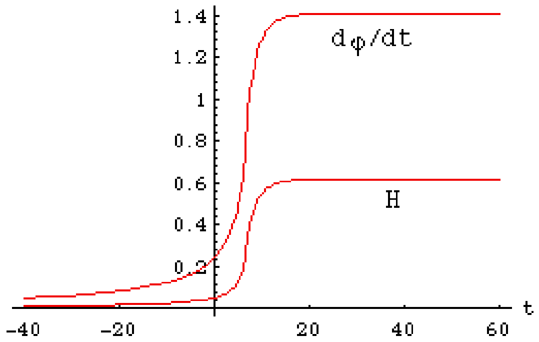

Figure 6: *Numerical integration of the equations of motion for the action (4.1), in $d = 3$, and with pre-big bang initial conditions.*

solutions for this system is a sufficient condition for the existence of an exact (in the sense of conformal field theory, i.e. to all orders in α') solution with constant curvature and linearly evolving dilaton.

It may be noted that a solution with $H = \text{const}$, $\dot{\phi} = \text{const}$, is in general allowed in many higher-derivative models of gravity. Such a fixed point of the cosmological equations, however, is in general disconnected from the trivial fixed point $H = 0 = \dot{\phi}$ (the perturbative vacuum, in this case) by a singularity, or by an unphysical region in which the curvature becomes imaginary. For the action (4.1), on the contrary, the constant fixed point is a late time time attractor for all isotropic backgrounds emerging from the string perturbative vacuum[10].

This attraction property, unfortunately, is not invariant under field redefinition, as we have checked, until the action is truncated at a given finite order in α'. Also, and most important, the growth of the curvature is stopped, but the transition is not completely performed. Indeed, the final fixed point is in the post-big bang regime $\dot{\overline{\phi}} < 0$, as illustrated in Fig. 2, but the transition cannot proceed further towards the lower curvature regime, without including additional corrections or an appropriate dilaton potential in the effective action.

The reason for this incompleteness is that the action (4.1) is invariant under time reflections, but is not duality-invariant. As a consequence, what is regularized is the expanding pre-big bang branch of the lowest order solution, and its time-symmetric counterpart, the *contracting post-big bang* branch. This is illustrated

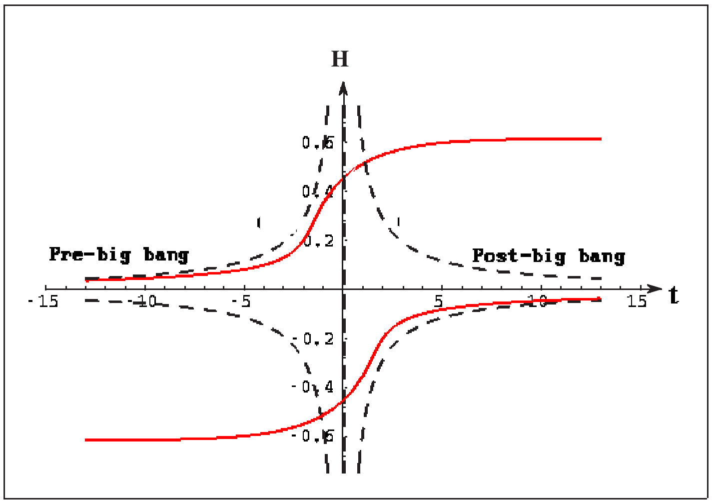

Figure 7: *Time-reversal symmetry of the regularized solution (the dashed curves represent the four branches of the lowest order, singular solution).*

in Fig. 7, where we plot the evolution in time of the curvature scale, according to a numerical integration of the equations of motion. The dashed curves are the four branches of the corresponding lowest-order, singular solution. The expanding post-big bang branch remains singular, and cannot be smoothly connected to the regularized pre-big bang branch. More corrections are to be added to the effective action.

4.2. *Loop corrections*

One-loop corrections to the string cosmology equations have been discussed by various authors, in $D = 2$ and $D = 4$ spacetime dimensions. In two dimensions[18], the one-loop corrections to the so-called CGHS model[19] lead to the action

$$S = S_{\rm tree} + \frac{k}{2} \int d^2x \sqrt{-g} \left(R \Box^{-2} R + \epsilon \phi R \right) , \tag{4.2}$$

where $S_{\rm tree}$ is the tree-level action, and k is a dimensionless parameter that depends on the number of conformal scalar field present in the model. Notice that the trace-anomaly term $R\Box^{-2}R$ has been supplemented by the local covariant counterterm $\epsilon\phi R$, that one is free to add to preserve classical symmetries[20].

In four dimensions, the dimensionally reduced one-loop action can be written as[21]

$$S = S_{\rm tree} - \alpha'\delta \int d^4x\sqrt{-g}\xi(\sigma)F\left[R^2, (\partial_\mu\phi)^4, R(\partial_\mu\phi)^2, ...\right] \qquad (4.3)$$

where δ is a model-dependent number of the order of unity, $\xi(\sigma) \approx -(2\pi/3)\cosh\sigma$ is a function of the modulus field σ parametrizing the size of the internal compactified manifold, and F is a complicated four-derivative function of the metric and of the dilaton, that in the Einstein frame factorizes into the Gauss-Bonnet form times a dilaton-dependent function. The tree-level action, in eq. (4.3), includes the modulus field and the first order α' corrections.

Both the analytical and numerical solutions of the actions (4.2) and (4.3) show that the effect of loop corrections is to induce a bounce in the curvature[18,21]: the curvature grows, reaches a maximum and then decreases, with a typical "bell-like" shape which mimics the effects of a transition from pre- to post-big bang.

The dilaton, however, remains growing from $-\infty$ to $+\infty$. It is true that a monotonic evolution of the dilaton from weak to strong coupling is equivalent, via S-duality transformations, to a smooth interpolation between two different weak coupling regimes. The final value of the string coupling should go to a finite constant, however, and not to zero. To this aim the dilaton has to be stopped, and this probably requires the addition of a realistic, non-perturbative potential.

The problem with the cosmological solutions of the one-loop corrected action, at least for the cases that we have analyzed, is that the final background configuration is still characterized by $\dot{\overline{\phi}} > 0$. This means, in other words, that there is no branch changing: the background is still in the pre-big bang regime (in spite of the fact that the curvature is decreasing), and the dilaton is not ready to be attracted to any stable minimum of the potential[17].

We may thus conclude, to the best of our present understanding of string cosmology, that α' and loop corrections can separately implement different aspects of the transition from pre- to post-big bang. They are both required, however, together with an appropriate non-perturbative potential, for the description of a complete transition.

5. Conclusion

String theory can provide (at least in principle) a consistent description of the Universe at all curvature scale. The duality symmetries of string theory, in particular, suggest a cosmological scenario in which the Universe evolves from the string perturbative vacuum, through a pre-big bang phase characterized by an accelerated growth of the curvature and of the string coupling (controlled by the dilaton).

I do not conceal that what I have presented here is no more than an approximate sketch of a complete and possibly realistic cosmological scenario. A lot of work is still needed to clarify all the aspects of this scenario, and in particular the dynamical details of the transition to the standard cosmological regime.

It seems to me, however, that such a work is worth to be done, because the study of this pre-big bang scenario, and of the associated non-standard phenomenology[1,22], may provide an efficient way to test string theory as well as alternative models of Planck scale physics.

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