Flavour symmetries in the symmetric limit (and the neutrino normal hierarchy)

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The content of this talk

Q1: can a flavour symmetry constraining light neutrino Majorana masses provide an approximate description of lepton flavour in the symmetric limit?

A1: yes, but only if neutrinos are inverted hierarchical or unconstrained (anarchical)

If NH is confirmed, symmetry breaking must play a leading role in the understanding of lepton flavour

Light neutrino majorana masses may originate from high-scale physics

Q2: is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

A2: not necessarily

Necessary and sufficient conditions for the equivalence in the case of see-saw I

Q3: can a flavour symmetry constraining a see-saw lagrangian provide an approximate description of lepton flavour in the symmetric limit?

A3: yes, and neutrinos can be normally hierarchical if the high-scale and low-scale actions of the flavour symmetry are not equivalent

Introduction: flavour symmetries

The flavour puzzle in the SM

- 3 families $\leftrightarrow$ $U(3)^5$ symmetry of the gauge lagrangian

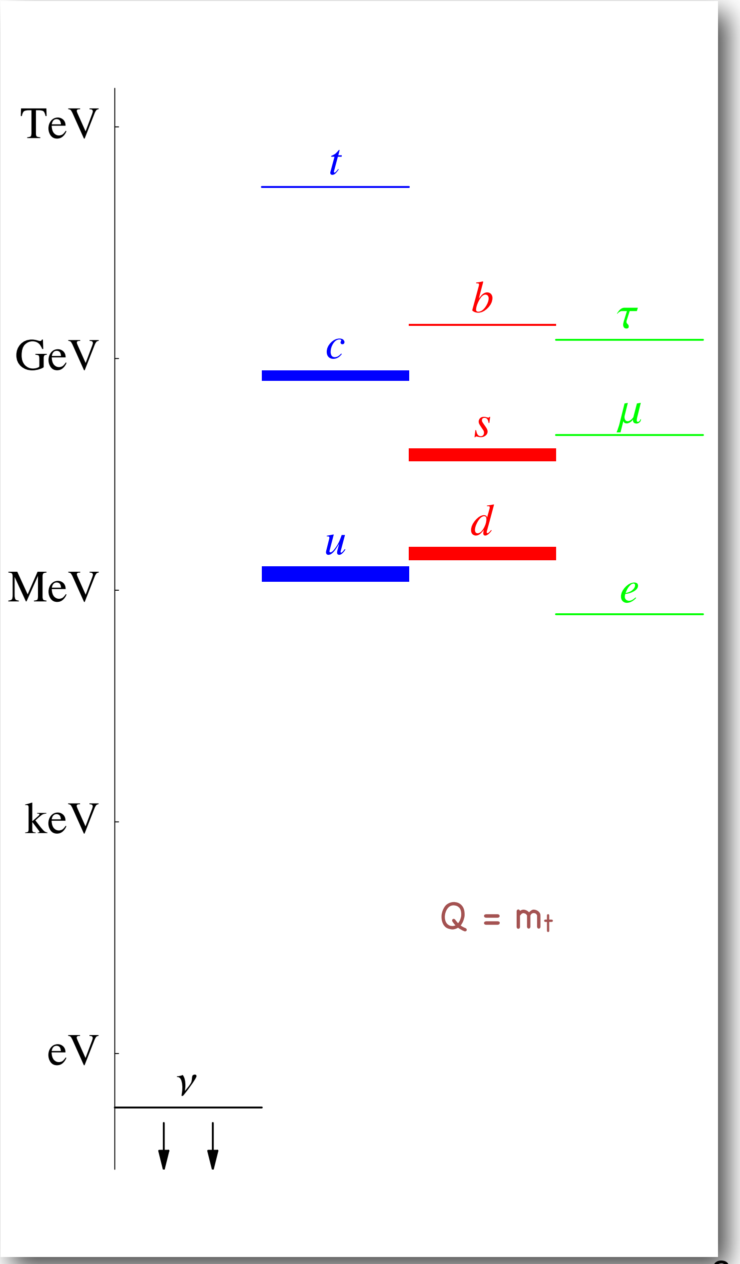
	1	2	3
l	l_1	l_2	l_3
e^c	$(e^c)_1$	$(e^c)_2$	$(e^c)_3$
q	q_1	q_2	q_3
u^c	$(u^c)_1$	$(u^c)_2$	$(u^c)_3$
d^c	$(d^c)_1$	$(d^c)_2$	$(d^c)_3$

family number (horizontal) not understood

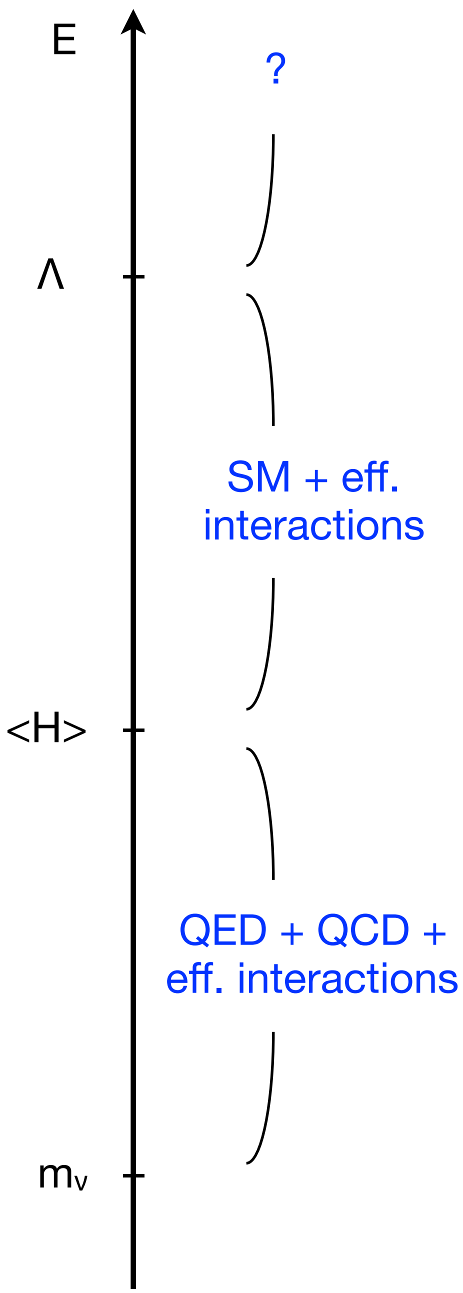
gauge irreps (vertical) understood?

The flavour puzzle in the SM

- 3 families $\leftrightarrow$ $U(3)^5$ symmetry of the gauge lagrangian
- Charged fermions: $m_1 \ll m_2 \ll m_3$
  Quarks: $V_{CKM} \sim \mathbf{1}$
- Neutrinos: lighter, milder hierarchy,
  $U_{PMNS} \neq \mathbf{1}$



Smallness of neutrino masses and high scales



$$\mathcal{L}^{\mathrm{eff}}_{E \ll \Lambda} = \mathcal{L}^{\mathrm{ren}}_{\mathrm{SM}} + \frac{h_{ij}}{2\Lambda}(l_i h)(l_j h) + \mathrm{h.c.}$$

$$m_{u,d,e} = \lambda_{u,d,e}\, v \qquad m_\nu = h\, v \times \frac{v}{\Lambda}$$

$$\Lambda \sim (0.5 \cdot 10^{15}\,\mathrm{GeV})\, h \left(\frac{0.05\,\mathrm{eV}}{m_\nu} \right)$$

This is the framework we consider
The results can be extended to other set-ups

Flavour symmetries

- G_f flavour group acting on "i", $\mathscr{L}$ invariant

- $$\mathcal{L}_f = \lambda_{ij}\psi_i\psi_j h \quad + \frac{\lambda_{ijk}}{\Lambda_f}\phi_k\psi_i\psi_j h + \dots$$

ϕ_k scalar "flavon"
(SM invariant)
spontaneously breaks G_f

$$\downarrow \qquad\qquad \downarrow$$

$$m^0_{ij} = \lambda_{ij} v \quad m^1_{ij} = \lambda_{ijk}\frac{\langle\phi_k\rangle}{\Lambda}v + \dots$$

$$\boxed{m_{ij} = m^0_{ij}\boxed{+\, m^1_{ij}}}$$

vanishes for $\phi_k = 0$
"symmetric limit"

Q1: can a flavour symmetry constraining light neutrino Majorana masses provide an approximate description of lepton flavour in the symmetric limit?

Flavour group

- G_f flavour group
  - any: discrete/continuous, abelian/non-abelian, global/gauge, etc
  - includes all "hidden" factors
  - unitary representation, commuting with Poincaré and G_{SM}

Flavour representation

$$g \in G_f : \quad \begin{cases} l_i & \to \quad U_l(g)_{ij} \, l_j \\ e_i^c & \to \quad U_{e^c}(g)_{ij} \, e_j^c \end{cases} \qquad e^c \leftrightarrow \overline{e_R}$$

Invariant lagrangian, <ϕ> = 0 (low-scale)

$$\mathcal{L}^{(0)}_{\text{low-scale}} = \lambda^E_{ij} e^c_i l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h$$

$$\to \boxed{m^{0E}_{ij} e^c_i e_j} + \boxed{\frac{m^{0\nu}_{ij}}{2} \nu_i \nu_j}$$

Symmetric limit

$$\begin{aligned} U_{e^c}(g)^T m_E^0 \; U_l(g) &= m_E^0 \\ U_l(g)^T \; m_\nu^0 \; U_l(g) &= m_\nu^0 \end{aligned}$$

(from the invariance of the lagrangian)

Symmetry breaking

$$\begin{aligned} m_E &= \boxed{m_E^0} + \boxed{m_E^1} \\ m_\nu &= m_\nu^0 + m_\nu^1 \end{aligned}$$

invariant under G_f

not invariant under G_f generated by ϕ

The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

$$m_E = \boxed{m_E^0} + \boxed{m_E^1}$$
$$m_\nu = \boxed{m_\nu^0} + \boxed{m_\nu^1}$$

approximate description of lepton observables

moderate correction necessary for an accurate description

$$\text{e.g.} \quad m_D^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_b \end{pmatrix} \qquad m_U^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_t \end{pmatrix}$$

$$m_E^0 = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_\tau \end{pmatrix} \qquad \begin{array}{l} V_{\mathrm{CKM}}^0 = \mathbf{1} \\ (\theta_C \ \text{undetermined}) \end{array}$$

The LO pattern of lepton flavour is determined by symmetry breaking

- e.g. if $m_E = 0$ or $m_\nu = 0$

$$m_E = \boxed{m_E^0} + \boxed{m_E^1}$$
$$m_\nu = \boxed{m_\nu^0} + \boxed{m_\nu^1}$$

e.g. $m_E = 0$ or $m_\nu = 0$ in the symmetric limit

fully determine the PMNS matrix

e.g. $\quad G = A_4 \qquad m_\nu^0 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \quad m_E^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

m_ν^1: H_1 invariant $\qquad m_E^1$: H_2 invariant

The symmetric limit provides an approximate description of lepton flavour

- $m_E \neq 0$ and $m_\nu \neq 0$

$$\begin{aligned} m_E &= \boxed{m_E^0} + m_E^1 \\ m_\nu &= \boxed{m_\nu^0} + m_\nu^1 \end{aligned}$$

approximate description of lepton observables

Neutrino masses

NH/IH	(a 0 0)	(0 a a)
NH or IH	(a a a)	(a b 0)
	(a b b)	(a b c)

Charged lepton masses

	(A 0 0)
	(A B 0)
	(A B C)

PMNS matrix

$$\begin{pmatrix} X & X & 0 \\ X & X & X \\ X & X & X \end{pmatrix} \text{ or } \begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix}$$

(X ≠ 0 generic)

G_f U_l U_e leading, in the symmetric limit, to lepton masses and mixings in the above form

- A complete and concise classification is possible, as the predictions in the symmetric limit only depend on the structure of the decomposition of the representations in irreducible components (irreps) and in particular on their
  - **Type** (real, pseudoreal, complex)
  - **Dimension**
  - **Equivalence**
- Notation
  - "**n**": dimension n **complex** or **pseudoreal** irrep
  - "n": dimension n **real** irrep
  - "n, n', n''": dimension n **inequivalent** irreps

G_f U_l U_e leading, in the symmetric limit, to lepton masses and mixings in the above form

U_l, U_{e^c} irreps	masses	ν hierarchy	PMNS zeros
$1 \quad 1 \quad 1$ $1 \quad r \not\supset 1$	$(A00)$ (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \overline{\mathbf{1}}$ $\overline{\mathbf{1}} \quad r \not\supset \mathbf{1}, \overline{\mathbf{1}}$	$(A00)$ $(0aa)$	IH	none $(\mathbf{13})$
$1 \quad 1 \quad 1$ $1 \quad 1 \quad r \neq 1$	$(AB0)$ (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \overline{\mathbf{1}}$ $\overline{\mathbf{1}} \quad \overline{\mathbf{1}} \quad r \neq \mathbf{1}$	$(AB0)$ $(0aa)$	IH	$\mathbf{13}$
$1 \quad 1 \quad 1$ $1 \quad 1 \quad 1$	(ABC) (abc)	NH or IH	none
$\mathbf{1} \quad \mathbf{1} \quad \overline{\mathbf{1}}$ $\overline{\mathbf{1}} \quad \overline{\mathbf{1}} \quad \mathbf{1}$	(ABC) $(0aa)$	IH	$\mathbf{13}, 23, 33$

- Only 6 cases
- Only d = 1 (abelian) irreps and no pseudoreal irreps (except possibly if $m_{e,\mu} = 0$)
- Neutrinos are either unconstrained (anarchical) or inverted hierarchical
- If NH confirmed, lepton flavour at low-scale can only be accounted for by SB

1+1+1

- "1" = real one-dimensional: $f \to \pm f$
- 1+1+1: $U(g)_{ij} = \pm \mathbf{1}_{ij}$
- any m_ν is trivially invariant
- neutrino masses and mixing completely unconstrained
- (anarchy)

SU(5) and SO(10)

- SU(5): assume $U_{\bar{5}} = U_l$ and $U_{10} = U_{ec}$, require $(V_{CKM})_0 = \mathbf{1}$ or V_{12}
    - only unconstrained (anarchical) neutrinos are allowed
- SO(10): assume $U_l = U_{ec} = U_{16}$
    - no solutions

Features of the proof

- in 2 steps: masses first, then mixings
- no need to write down any mass matrix, texture: the flavour pattern is directly determined by the irrep decomposition
- in particular, the form of the PMNS matrix is given by

$$U = H_E P_E V D^{-1} P_\nu^{-1} H_\nu^{-1}$$

  - V commutes with U_l
  - D maximal rotation, if U_l contains conjugated complex irreps (Dirac substructure)
  - P permutations possibly needed to bring mass eigenvalues in standard ordering
  - H ambiguity in case of degeneracies (fixed by symmetry breaking effects)

Q2: is studying the flavour symmetry at low-scale equivalent to studying it at high-scale?

[to appear]

Origin of lepton masses (high-scale)

$$\mathcal{L}_{\text{low-scale}} = \lambda^E_{ij} e^c_i l_j h^* + \frac{c_{ij}}{2\Lambda} l_i l_j h h \qquad \text{from}$$

Flavour representation

$$\text{high scale} \left\{ \begin{array}{lll} l_i & \to & U_l(g)_{ij}\, l_j \\ e^c_i & \to & U_{e^c}(g)_{ij}\, e^c_j \\ \nu^c_i & \to & U_{\nu^c}(g)_{ij}\, \nu^c_j \end{array} \right. \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{low scale version}$$

Equivalent, at least in the symmetric limit?

Equivalence of high and low-scale representations (in the symmetric limit)

- By definition, when for each invariant m_ν there exists invariant m_N and M such that $m_\nu = - m_N^T M^{-1} m_N$ (converse is always true)

- Given a low-scale representation does an equivalent high-scale version always exists? **YES**

- Is the low-scale version of a representation always equivalent to the high-scale version? **NO**

- Necessary and sufficient conditions for LS to be equivalent to HS:

1. U_{vc} vectorlike — real, or pairs of complex conjugated, or pairs of equivalent pseudoreal

2. The vectorlike part of U_l is contained in U_{vc}

1. U_{ν^c} is not vectorlike

- U_{ν^c} not vectorlike ⇔ M forced to be singular in the symmetric limit: the see-saw formula does not apply
- Example:

	low-scale	**high-scale**
$U_l = \mathbf{1}+\mathbf{1}+1$	$m_{ei} = (A\ 0\ 0)$	$m_{ei} = (A\ 0\ 0)$
$U_{e^c} = \bar{\mathbf{1}}+\mathbf{1}+\mathbf{1}$	$m_{\nu i} = (a\ 0\ 0)$	$m_{\nu i} = (a\ 0\ 0)$
$U_{\nu^c} = \bar{\mathbf{1}}+$ real	$U = \begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$U = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

low-scale:

$$m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} X & & \\ & & \\ & & \end{pmatrix}}$$

high-scale:

$$m_N = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix} \qquad m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$$

$$M = \begin{pmatrix} X & X & \\ X & X & \\ & & \end{pmatrix} \qquad \boxed{m_\nu = \begin{pmatrix} & & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}}$$

single RH neutrino dominance

det = 0

2. $U_{\nu c}$ is vectorlike but the vectorlike part of U_l is not contained in $U_{\nu c}$

- Example: **low-scale** **high-scale**

	low-scale	high-scale
$U_l = \boxed{1+1}+\mathbf{1}$	m_{ei} = (A 0 0)	m_{ei} = (A 0 0)
$U_{e^c} = 1+\mathbf{1}+\mathbf{1}$	$m_{\nu i}$ = (a b 0)	$m_{\nu i}$ = (a 0 0)
$U_{\nu^c} = 1+\bar{\mathbf{1}}+\mathbf{1}$	$V = \begin{pmatrix} X & ? & ? \\ ? & X & X \\ 0 & X & X \end{pmatrix}$ or $\begin{pmatrix} ? & ? & X \\ X & X & ? \\ X & X & 0 \end{pmatrix}$	$V = \begin{pmatrix} X & X & ? \\ X & X & X \\ X & X & X \end{pmatrix}$

$$m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$$

$$\boxed{m_\nu = \begin{pmatrix} & & \\ & X & X \\ & X & X \end{pmatrix}}$$

$$m_N = \begin{pmatrix} & & \\ X & & \\ & X & X \end{pmatrix} \qquad m_E = \begin{pmatrix} & & \\ & & \\ & X & X \end{pmatrix}$$

$$M = \begin{pmatrix} & X & \\ X & & \\ & & X \end{pmatrix} \qquad \boxed{m_\nu = \begin{pmatrix} & & \\ & \boxed{\begin{matrix} X & X \\ X & X \end{matrix}} \end{pmatrix}}$$

det = 0

Q3: can a flavour symmetry constraining a see-saw lagrangian provide an approximate description of lepton flavour in the symmetric limit?

- If U_{vc} vectorlike and the vectorlike part of U_l is contained in U_{vc}: yes, at the same conditions as in the low-scale analysis

- If instead the low- and high-scale analyses are not equivalent, predictive (non-unconstrained) cases corresponding to NH can be found

- The complete list of solutions can be again found based only on the structure of the irrep decompositions

Conclusions

- The **complete** set of lepton flavour **predictions** of any flavour group and representation **in the symmetric limit** has been found, both at low scale (Weinberg operator) and high scale (see-saw). The predictions only depend on the type, dimension, and equivalence of the irrep components.
- In the **low-scale** case: the symmetric limit is close to what observed only if neutrinos are **unconstrained** (anarchical) or **inverted hierarchical**.
- If the present hint for normal hierarchy was confirmed, we would conclude that symmetry breaking plays a leading order role in constraining lepton flavour observables at low scale.
- In the **high-scale** case: the results do not change, except when the low- and high-scale analyses are **not equivalent**. The conditions for equivalence have been found.
- The complete set of additional predictions in the symmetric limit that can obtained at high-scale has been found. A **normal hierarchy** for the neutrinos can be obtained.
- If the present hint for normal hierarchy was confirmed, **a predictive symmetric limit could be close to what observed only because the low- and high-scale actions of the flavour symmetry are not equivalent**. Otherwise, symmetry breaking effects necessary play a leading order role in determining lepton flavour observables.