Extended Neutrino Sphere effects on Supernova ν Oscillations

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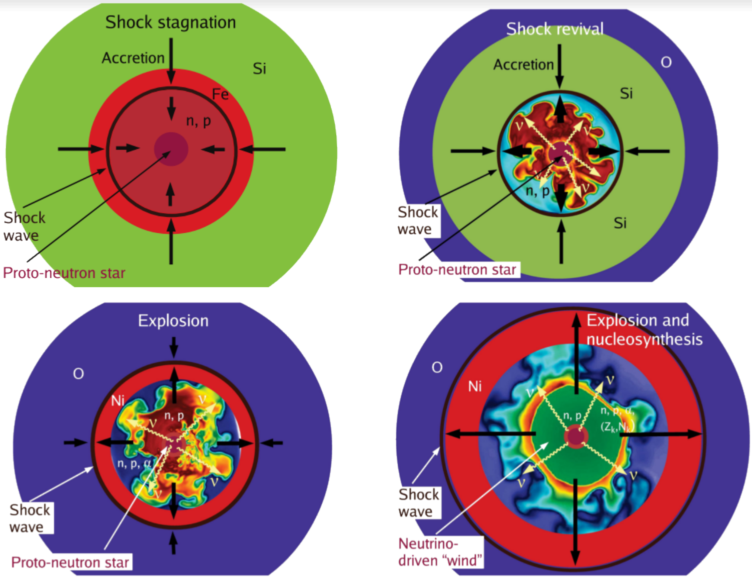
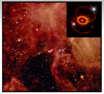
NOW 2018, Ostuni

September 11, 2018



Janka et al. 2012



Impact of neutrino oscillations

- Supernova explosion mechanism:

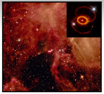
$$E(\nu_{\mu/\tau}) > E(\nu_e)$$

- Modify the neutrino signal:

$$\nu_e \rightarrow \nu_{\mu/\tau} \qquad \nu_{\mu/\tau} \rightarrow \nu_e$$

- Nucleosynthesis depends on the neutron fraction:

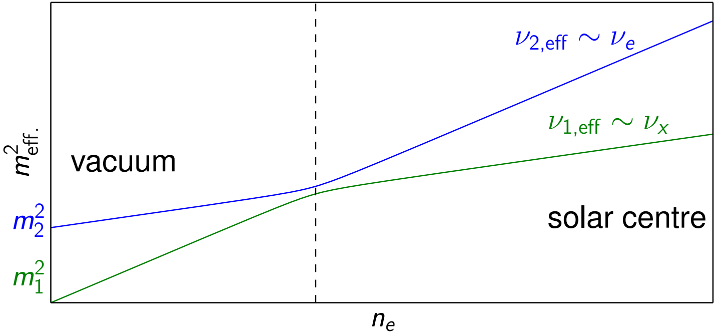
$$p + \bar{\nu}_e \rightarrow n + e^+ \qquad n + \nu_e \rightarrow p + e^-$$



Matter effect

Electron background shifts the energy eigenvalues.

$$H = \frac{\Delta m^2}{2E} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



Density matrix formalism

In the mean field approximation:

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix} = \frac{1}{2}(P_0 + \vec{P}\cdot\vec{\sigma}).$$

Similarly for the Hamiltonian:

$$H = \frac{1}{2}\begin{pmatrix} V_z & V_x - iV_y \\ V_x + iV_y & -V_z \end{pmatrix} = \frac{1}{2}\vec{V}\cdot\vec{\sigma}.$$

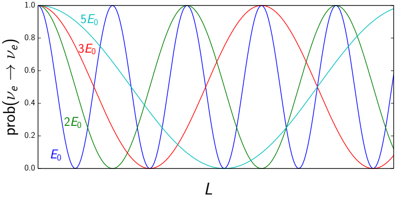
Equation of motion (in absence of collisions):

$$i\dot{\rho} = [H, \rho] \qquad \Leftrightarrow \qquad \dot{\vec{P}} = \vec{V} \times \vec{P}$$

Collective oscillations

Normal oscillations:

$$\mathrm{prob}(\nu_e \rightarrow \nu_e) \propto \cos^2(\Delta m^2 L/4E).$$

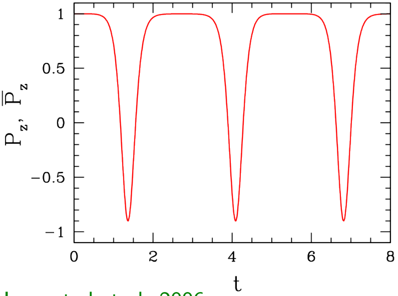


Neutrino background:

$$H_{\nu\nu} = \sqrt{2} G_F \int dp(\rho - \bar{\rho}).$$

Conversion independent of E.

Non-linear problem $\rightarrow$ hard to solve in a realistic setting.



Hannestad et al. 2006

Linear equations, RSLH and Smirnov, 2018

Observation

Considering the individual neutrino, its oscillations in a SN is a linear problem.

Idea

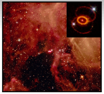
How much of the neutrino-neutrino refraction can we describe using linear equations.

Ultimate goal

General conclusions about the behaviour of neutrino oscillations in presence of neutrino-neutrino refraction.

Methods

- Solve equations from first principles, analytic and numeric.
- Describe complicated systems using effective potentials.



General equations

Probe neutrino in arbitrary neutrino and matter background.

$$H^{(p)} = \frac{1}{2}\begin{pmatrix} -c_{2\theta}\omega_p + V_e + V_\nu & s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B} \\ s_{2\theta}\omega_p + 2\bar{V}_\nu e^{-i\phi_B} & c_{2\theta}\omega_p - V_e - V_\nu \end{pmatrix}, \qquad (1)$$

$$V_\nu = \int d\mathbf{k}\, V_\nu^0(\mathbf{k})\left[\rho_{ee}(\mathbf{k}) - \rho_{\tau\tau}(\mathbf{k})\right],$$

$$\bar{V}_\nu e^{i\phi_B} = \int d\mathbf{k}\, V_\nu^0(\mathbf{k})\rho_{e\tau}(\mathbf{k})$$

$$V_\nu^0(\mathbf{k}) = \sqrt{2}G_F n(\mathbf{k})\left(1 - \mathbf{v}_{bg}\cdot\mathbf{v}_p\right)$$

General equations - rotated

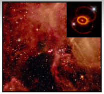
Rewrite the off-diagonal as $V' e^{i\phi'} = s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B}$. Apply the transformation $U = \mathrm{diag}\left(e^{i\phi'/2}, e^{-i\phi'/2}\right)$.

$$H^{(p)} = \frac{1}{2}\begin{pmatrix} V^r & V' \\ V' & -V^r \end{pmatrix},$$

where

$$V' = \sqrt{4\bar{V}_\nu^2 + 4 s_{2\theta}\omega_p \cos\phi_B \bar{V}_\nu + s_{2\theta}^2\omega_p^2},$$

$$V^r = V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p.$$



Conditions for a large conversion

Our Ansatz is that every case where a large conversion happens can be described in terms of at least one of these frameworks:

1. **Resonance**

   Vanishing diagonal:

   $$V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p = 0.$$

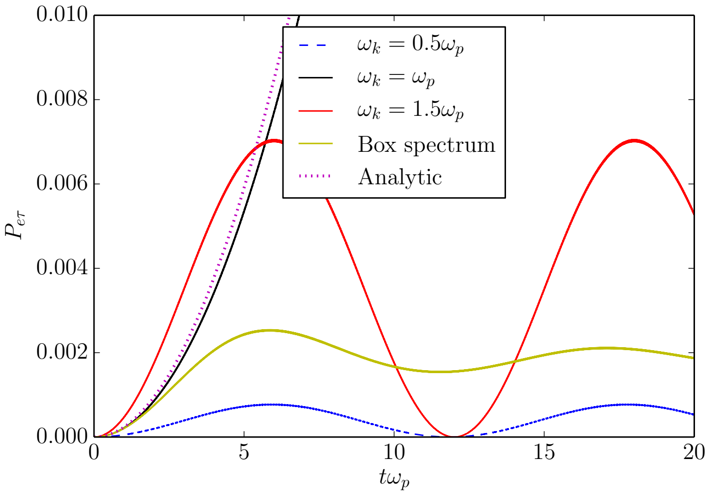
2. **Adiabatic conversion**

   Fast oscillations in V^r and V' can be removed by going to a rotating frame. This can result in a Hamiltonian describing adiabatic evolution.

3. **Parametric enhancement**

   Present if the period of oscillation equals the period of change of mixing angle.

Energy spectrum



RSLH and Smirnov, 2018

Neutrino emission

$\nu_{\mu/\tau}$ decouple first, then $\bar{\nu}_e$ and last ν_e.

$\nu_{\mu/\tau}$ have:
Number sphere:

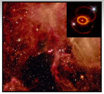
$$e^+e^- \to \nu_{\mu/\tau}\bar{\nu}_{\mu/\tau}$$

Energy sphere:

$$e\,\nu_{\mu/\tau} \to e\,\nu_{\mu/\tau}$$

Transport sphere:

$$N\,\nu_{\mu/\tau} \to N\,\nu_{\mu/\tau}$$



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$\bar{\nu}_e$ and ν_e are dominated by absorption and emission from nucleons. ($n_n > n_p$)
Mean free path:

$$\frac{1}{\lambda} \approx \frac{G_F^2}{\pi}(g_V^2 + 3g_A^2)E^2 n_N.$$

Emissivity:

$$j = \frac{1}{\lambda}\exp(-E/T)$$

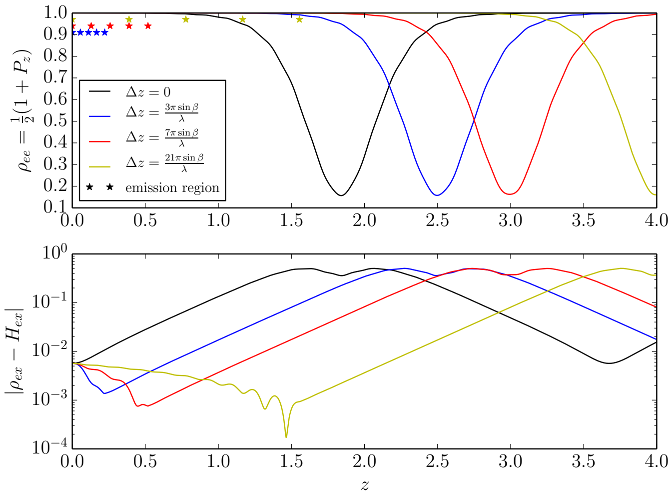
Extended source

Rough estimates:

- Neutrino sphere: ~ 10km.
- Width of neutrino sphere: ~ 1km.
- Oscillation length: $\sim \frac{1}{G_F n_e} \sim 10^{-8} - 10^{-7}$km.

Average over emission region suppresses oscillatory terms by $10^7 - 10^8$.
Parametric resonance is not removed as such.

Extended source - non-linear



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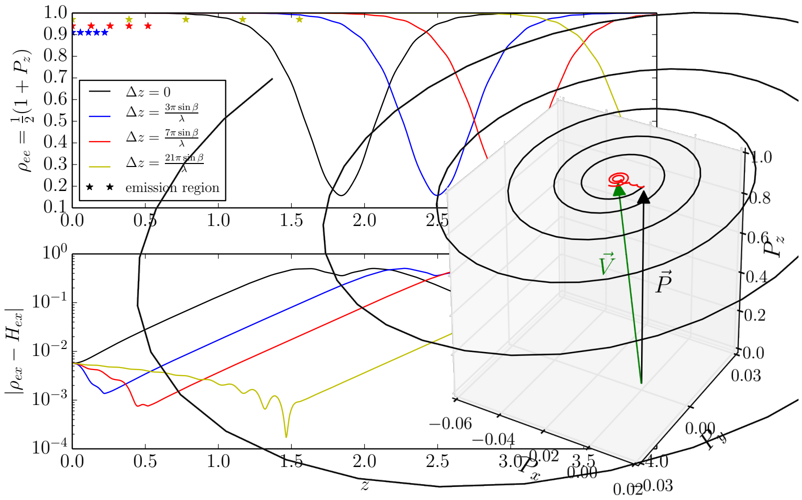
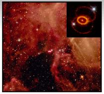




Changing background - only matter

Coordinate system with $\vec{V}$ along z-axis:

$$\rho_{12}(r) = \rho_{12,\mathrm{initial}} \exp\left(i \int_{r_e}^{r} \omega_m(r')dr' \right).$$

Average over emission point:

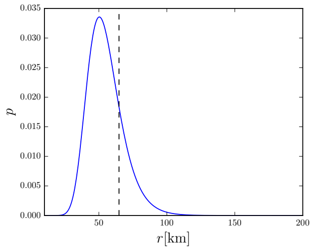
$$\langle \rho_{12}(r) \rangle = \int_0^r p(r_e) \frac{1}{2} \sin 2\theta_m(r_e) exp \left(i \int_{r_e}^{r} \omega_m(r')dr' \right) dr_e,$$

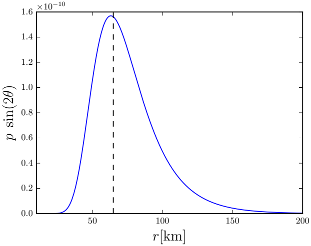
where

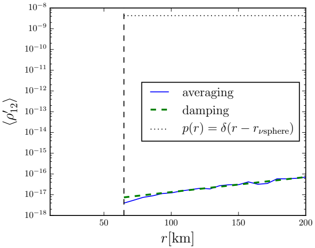
$$p(r_e) = \frac{N(r_e)}{N_0}$$

$$N(r_e) = \frac{1}{\lambda(r_e)} \exp(-E/T) \exp\left(- \int_{r_e}^{\infty} \frac{1}{\lambda(r')} dr' \right), \quad N_0 = \int dr_e N(r_e).$$

Changing background - only matter







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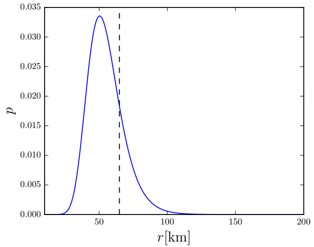
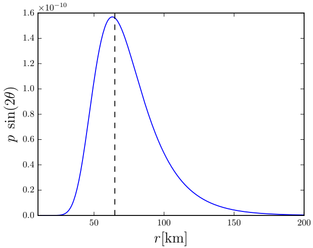
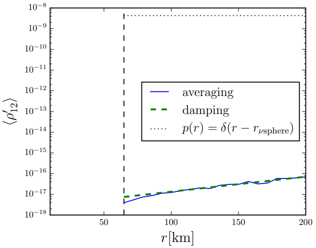






Include effect of damping, D:

$$\dot{\vec{P}} = \vec{V} \times \vec{P} - D\vec{P}_T.$$

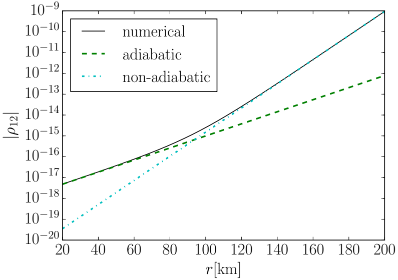
For V_z and D large and $P_z \approx 1$:

$$P_x \approx \frac{V_x V_z}{V_z^2 + D^2}, P_y \approx \frac{-V_x D}{V_z^2 + D^2}.$$

Bell et al. 1998, Hannestad et al. 2012

Effect of non-adiabaticity

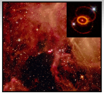
Solve $\dot{\vec{P}} = \vec{V} \times \vec{P} - D\vec{P}_T$ numerically:



Non-adiabatic effects for D=0:

$$\vec{P} = \begin{pmatrix} \frac{V_x}{V_z} + \frac{V_x \partial_r V_z}{V_z^3} \sin\left(\int V_z(r')dr'\right) \\ \frac{V_x \partial_r V_z}{V_z^3}\left(1 - \cos\left(\int V_z(r')dr'\right)\right) \\ 1 + \frac{V_x^2 \partial_r V_z}{V_z^4} \sin\left(\int V_z(r')dr'\right) \end{pmatrix}$$

$$|\rho_{12}| \approx \left|\frac{V_x \partial_r V_z}{2V_z^3}\right| \approx \left|\frac{V_x}{2V_z^2 r_0}\right|$$



Linear stability analysis Banerjee, Dighe, Raffelt 2011

Linear analysis demonstrate stability or instability.

$$\rho = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}$$

$$|S| = |P_x + iP_y| \ll 1, \quad s^2 + S^2 = 1 \Rightarrow s = P_z \approx 1.$$

Linearised equation:

$$i\dot{S} = (\omega + \lambda + \mu)S - \mu \int d\Gamma'(1 - v \cdot v')S',$$

Linear stability analysis

In Fourier space: $S = e^{-i\Omega t} Q$

$$\Omega Q = (\omega + \lambda + \mu) Q - \mu \int d\Gamma' (1 - v \cdot v') Q'$$

Unstable if $\mathrm{Im}(\Omega) \neq 0$. (See also Capozzi et al. 2017)
Discrete modes: solve matrix equation.
(Continious modes: Decompose in independent functions.)
Can also be formulated as a dipersion relation. (Izaguirre, Raffelt and Tamborra, 2016)

Simple model

Linearised equation: $(\omega = -1,\ \lambda = 30,\ \mu = 3)$

$$i\dot{\vec{S}} = \begin{pmatrix} -\omega + \lambda - \mu & \mu \\ -\mu & \omega + \lambda + \mu \end{pmatrix} \vec{S}$$

Eigenvalues:

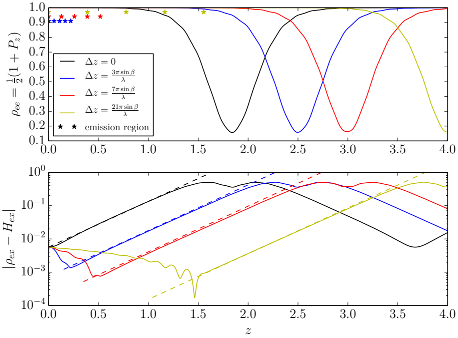
$$\Omega = \lambda \pm \sqrt{\omega(2\mu + \omega)}$$

Growth rate $= \mathrm{Im}(\Omega)$.

Emission point $= \frac{2}{3}\Delta z$.

Start value $= \frac{1}{\Delta z \lambda} \sin 2\theta_m$.

Does not work for large Δz.



Large Δz

- Take into account that μ changes inside the emission region.
- $\tilde{\mu}(z) = \mu \frac{z}{\Delta z}$ is so small for small z that no instability exists.

The fastest possible growth that one can expect is:

$$S = \frac{1}{2} \frac{\mu \sin 2\theta_m}{\Delta z V_e}$$

$$\exp\left(\frac{\Delta z}{3\mu} \left(2\mu \frac{z}{\Delta z} - 1 \right)^{3/2} \right).$$

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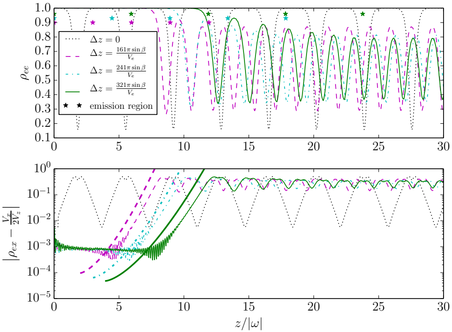
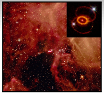
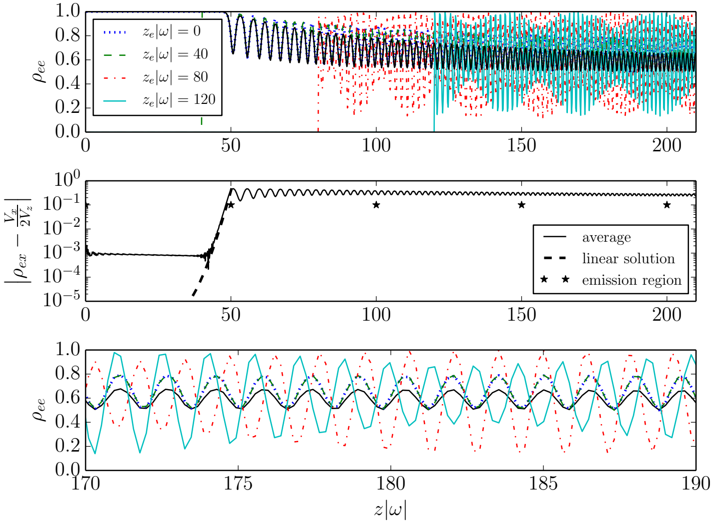




Very large Δz



Summary

- The non-negligible width of the neutrino sphere affects the neutrino state due to averaging over different emission points.
- The angle between the neutrino state and the Hamiltonian in polarization space is reduced by a factor 10^8 at the neutrino sphere by the averaging.
- A small adiabaticity violation increases the angle significantly as the neutrino propagates out through the supernova.
- The onset of neutrino conversion can be analysed using linear stability analysis, and for a given model, it can be calculated if conversion has the potentially to occur.

Summary

- The non-negligible width of the neutrino sphere affects the neutrino state due to averaging over different emission points.
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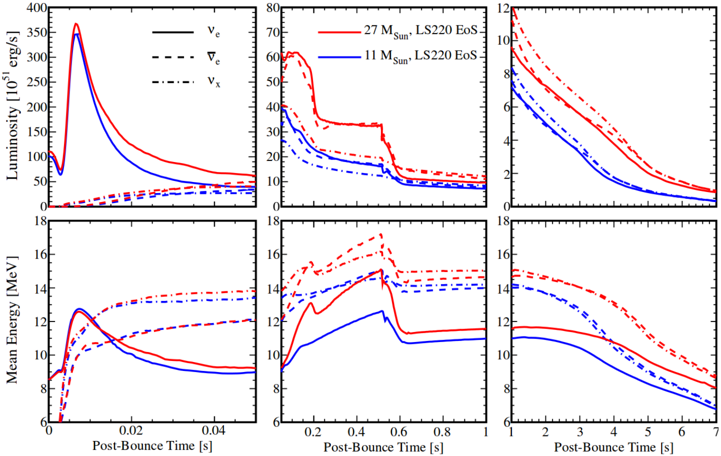
- The onset of neutrino conversion can be analysed using linear stability analysis, and for a given model, it can be calculated if conversion has the potentially to occur.

Thank you for your attention!

Neutrino emission

Deleptonisation Accretion Cooling



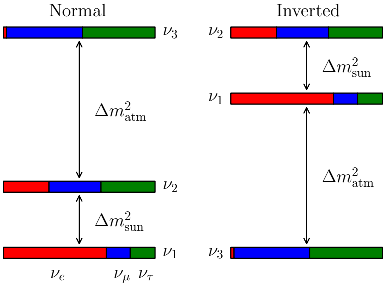
Lang et al. 2016

Neutrino mixing

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L^l \gamma^\mu l_L + h.c.$$

Interaction states and mass states are different:

$$\nu^l = U\nu^i.$$

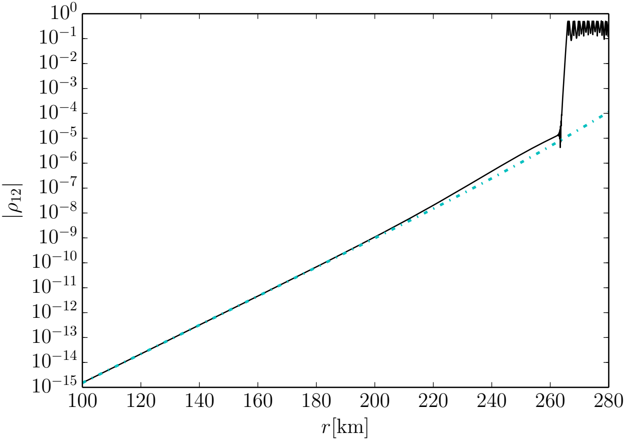


Mixing matrix:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Collective oscillations

Can collective oscillations still occur? YES!

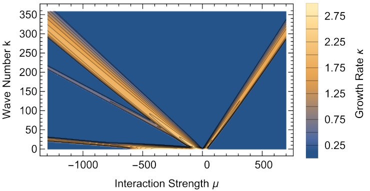


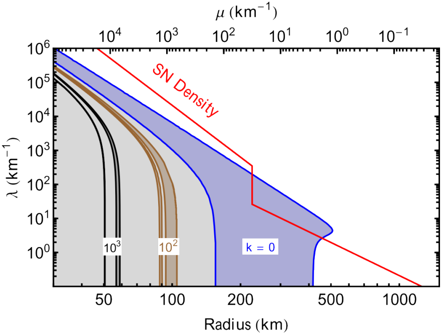
Multiple angles, in-homogeneous

Linear stability analysis of a more realistic model:

$$(\Omega + \mathbf{v}\cdot\mathbf{k})Q = (\omega + \lambda + \mu(\epsilon - \mathbf{v}\cdot\phi))Q - \mu\int d\Gamma'(1 - \mathbf{v}\cdot\mathbf{v}')g'Q'$$

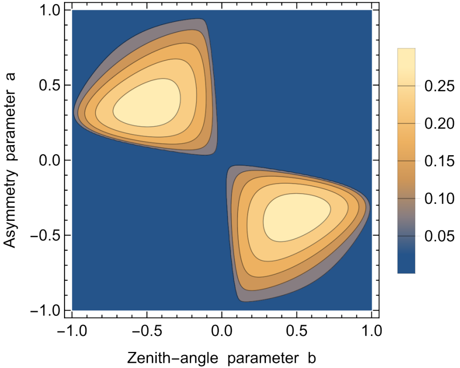
- μ and λ functions of r.
- Multi angle matter effect.
- Homogeneous mode: $k = 0$

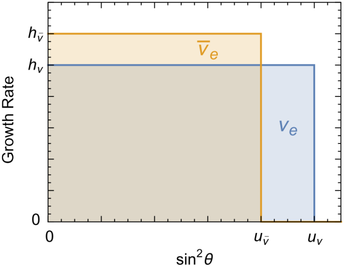




Chakraborty, RSLH, Izaguirre and Raffelt, 2015

Very fast flavour conversion R. F. Sawyer



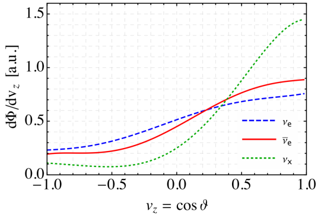


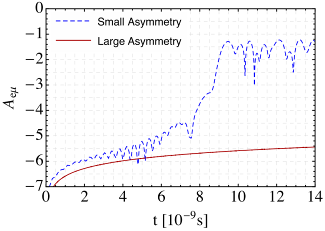
Chakraborty, RSLH, Izaguirre and Raffelt, 2016

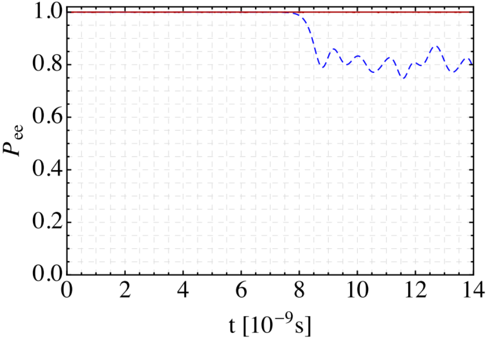
Conversion on meter-scale.

Can also occur in a supernova. Dasgupta, Mirizzi and Sen, 2016

Very fast flavour conversion







Dasgupta, Mirizzi and Sen, 2016