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Dissipative behavior of a quantum system interacting with a macroscopic medium

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Abstract

The modified Coleman–Hepp (AgBr) model describes the interaction between an ultrarelativistic quantum mechanical particle Q and an N -spin array D (a macroscopic medium in the $N \rightarrow \infty$ limit). We prove that the energy operator for D essentially behaves as a Wiener process in the weak-coupling, macroscopic limit, in a restricted state space. No assumptions are made on the spectrum of the Hamiltonian of the macroscopic system D . The mechanism of appearance of such a stochastic process and its relevance to issues like dissipation and irreversibility are briefly discussed.

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The derivation of a dissipative dynamics in quantum mechanics and quantum field theory is a long-standing problem and is, at present, the object of a widespread interest. An important boost on this subject was given in the fifties by van Hove [1], who clarified what are the main features of a quantum dissipative dynamics and derived a master equation via the famous “ $\lambda^2 T$ ” limit and the diagonal singularity.

Many other important contributions have been given to this issue, in the attempt to set up a general physical framework. There is a vast literature on this subject. In an interesting class of solvable models the environment is schematized with a collection of harmonic oscillators [2,3], and one can derive dissipative equations for the “object” particle. It is also true that many issues must still be clarified, both from the mathematical [4] and physical point of view. In particular, in a “quantum Langevin equation”, the random force is not Markovian [5] and gives rise to a colored noise [6,7].

On the other hand, it is commonly believed that a quantum mechanical measurement [8] occurs via a dephasing (decoherence) process [9–11]. From a technical viewpoint, “decoherence” means the elimination of the off-diagonal elements of the density matrix of the quantum system. Since a system described by a diagonal density matrix exhibits a purely stochastic behavior [12] one is led to expect a connection between dissipation, irreversibility and a quantum measurement process [13,14].

The analysis of solvable models has always proved to be very useful, in the present context [2,15]. The AgBr model [15,16] (see also other references in Ref. [17]), in particular, has played an important role in

the quantum measurement problem, because it is not trivial and yields very nice physical insight. In this Letter we shall base our discussion on this model and its modified version [17] that is able to take into account energy-exchange processes. We showed [14] that the model realizes van Hove's diagonal singularity [1] and displays the occurrence of an exponential regime at all times in the weak-coupling, macroscopic limit. In this way, a door is open to investigate the occurrence of a dissipative dynamics and its link with a quantum measurement process.

In this Letter, we shall concentrate on a particular aspect of the above-mentioned problems. Our main purpose is to derive a stochastic process from an underlying Hamiltonian dynamics and to clarify in which sense it is possible to identify a Wiener process for a dynamical variable of the microscopic system under investigation. As will be clear in the following treatment, we do neither require any conditions on the spectrum of D (that plays the role of environment system) nor perform any procedure like partial tracing in order to derive the stochastic process. To the authors' knowledge, it is the first time that the Heisenberg equations of motion (in operator form) provide us with a Wiener process in a restricted state space, in the weak-coupling, macroscopic limit, *without resorting to any assumptions* on the spectrum of the Hamiltonian. This is to be contrasted with other interesting attempts [18,2,3], where the environment system was endowed with a spectrum able to yield dissipation on the system under consideration.

The modified AgBr Hamiltonian [17] describes the interaction between an ultrarelativistic particle Q and a one-dimensional array (D -system), made up of N spin-1/2 objects. The array can be viewed as a caricature of a linear "photographic emulsion" of AgBr molecules, if one identifies the *down* state of the spin with the undivided molecule and the *up* state with the dissociated molecule (Ag and Br atoms). The particle and each molecule interact via a spin-flipping local potential. The total Hamiltonian for the $Q + D$ system reads

$$H = H_0 + H', \quad H_0 = H_Q + H_D, \tag{1}$$

where H_Q and H_D , the free Hamiltonians of the Q particle and of the "detector" D , respectively, and the interaction Hamiltonian H' are written as

$$H_Q = c\hat{p}, \quad H_D = \frac{1}{2}\hbar\omega \sum_{n=1}^N (1 + \sigma_3^{(n)}),$$

$$H' = \sum_{n=1}^N V(\hat{x} - x_n) \left[\sigma_+^{(n)} \exp\left(-i\frac{\omega}{c}\hat{x}\right) + \sigma_-^{(n)} \exp\left(+i\frac{\omega}{c}\hat{x}\right) \right]. \tag{2}$$

Here \hat{p} is the momentum of the Q particle, \hat{x} its position, V a real potential, x_n ($n = 1, \dots, N$) the positions of the scatterers in the array ($x_n > x_{n-1}$) and $\sigma_{i,\pm}^{(n)}$ the Pauli matrices acting on the n th site. The above Hamiltonian has attracted the attention of several researchers [19] due, in particular, to the presence of the free Hamiltonian of the array H_D , which makes the energy-exchange processes between Q and D physically meaningful. The original Hamiltonian [15] is reobtained in the $\omega = 0$ limit.

It is straightforward [20], if lengthy, to obtain the following (exact) expressions in the Heisenberg picture,

$$c\hat{p}(t) = c\hat{p} + \hbar\omega \sum_n \sin^2 \alpha_n(\hat{x}, t) \sigma_3^{(n)} + i\frac{\hbar\omega}{2} \sum_n \sin 2\alpha_n(\hat{x}, t) [\sigma_+^{(n)}(\hat{x}) - \text{H.c.}]$$

$$+ \sum_n [V(\hat{x} - x_n) - V(\hat{x} + ct - x_n)] [\sigma_+^{(n)}(\hat{x}) + \text{H.c.}], \tag{3}$$

$$\Delta H_D(t) \equiv H_D(t) - H_D = \sum_n \frac{\hbar\omega}{2} [\sigma_3^{(n)}(t) - \sigma_3^{(n)}]$$

$$= - \sum_n \hbar\omega \sin^2 \alpha_n(\hat{x}, t) \sigma_3^{(n)} - \sum_n i \frac{\hbar\omega}{2} \sin 2\alpha_n(\hat{x}, t) [\sigma_+^{(n)}(\hat{x}) - \text{H.c.}], \quad (4)$$

where

$$\alpha_n(\hat{x}, t) \equiv \int_0^t V(\hat{x} + ct' - x_n) dt' / \hbar, \quad \sigma_{\pm}^{(n)}(\hat{x}) \equiv \sigma_{\pm}^{(n)} \exp\left(\mp i \frac{\omega}{c} \hat{x}\right). \quad (5)$$

The operators (3) and (4) represent, respectively, the energy of the Q particle at time t and the energy stored in the array between time 0 and t .

We initially place the Q particle well outside D , moving towards D with speed c . The spin system is initially set in the ground state $|0\rangle_N$ of the free Hamiltonian H_D (all spins down). The initial state is written as

$$|\psi, 0\rangle_N \equiv |\psi\rangle \otimes |0\rangle_N = \int dx \psi(x) |x\rangle \otimes |0\rangle_N, \quad \int_{-\infty}^{+\infty} dx |\psi(x)|^2 = 1, \quad (6)$$

where, for simplicity, we choose ψ to be symmetrically distributed around the origin and with a compact support. A possible (and convenient) choice is $\psi(x) = (2a)^{-1/2} \theta(a - |x|) e^{ip_0 x / \hbar}$, and we require

$$a \ll x_1, \quad a \ll L \equiv x_N - x_1. \quad (7)$$

Notice that, due to the form of the free Hamiltonian of the particle H_Q , the wave packet does not disperse. The choice of the initial state $|\psi, 0\rangle_N$ is physically meaningful because the Q particle is initially outside D . We stress that it is possible to obtain exact expressions [17,14] for the propagator ${}_N\langle 0, \psi | \exp(-iHt/\hbar) | \psi, 0\rangle_N$ and the scattering matrix $S = \lim_{t \rightarrow \infty} \exp(iH_0 t / \hbar) \exp(-iHt/\hbar)$.

Define $q \equiv \sin^2(V_0\Omega/\hbar c)$ (the “spin-flip” probability [17], i.e. the probability of dissociating one AgBr molecule), and $V_0\Omega \equiv \int_{-\infty}^{\infty} V(x) dx$. In the following, we shall exclusively consider the weak-coupling, macroscopic limit,

$$N \rightarrow \infty, \quad q \simeq \left(\frac{V_0\Omega}{\hbar c}\right)^2 = O(N^{-1}), \quad (8)$$

which is equivalent to the requirement that the average number of excited molecules $\bar{n} = qN$ be finite. Observe that if we set

$$x_n = x_1 + (n-1)d, \quad L = x_N - x_1 = (N-1)d, \quad (9)$$

and let $d/L \rightarrow 0$ as $N \rightarrow \infty$, a summation over n can be replaced by a definite integration

$$q \sum_{n=1}^N f(x_n) \rightarrow \frac{qL}{d} \int_{x_1/L}^{x_N/L} f(z) dz \simeq \frac{\bar{n}}{L} \int_{x_1}^{x_N} f(y) dy. \quad (10)$$

(We are implicitly requiring that the function f be scale invariant [14].) Let us compute now

$$\begin{aligned} \langle c\hat{p}(t) \rangle &\equiv {}_N\langle 0, \psi | c\hat{p} | \psi, 0\rangle_N + \hbar\omega {}_N\langle 0, \psi | \sum_n \sin^2 \alpha_n(\hat{x}, t) \sigma_3^{(n)} | \psi, 0\rangle_N \\ &+ i \frac{\hbar\omega}{2} {}_N\langle 0, \psi | \sum_n \sin 2\alpha_n(\hat{x}, t) [\sigma_+^{(n)} \hat{x}] - \text{H.c.} | \psi, 0\rangle_N \end{aligned}$$

$$+{}_N\langle 0, \psi | \sum_n [V(\hat{x} - x_n) - V(\hat{x} + ct - x_n)] [\sigma_+^{(n)}(\hat{x}) + \text{H.c.}] | \psi, 0 \rangle_N. \tag{11}$$

The first term is nothing but cp_0 , the initial energy of the Q particle, while the last two terms vanish due to the action of $\sigma_{\pm}^{(n)}$. It is instructive to work out explicitly the second term. To this end, it is convenient to work in the Fermi–Yang approximation $V(y) = V_0\Omega\delta(y)$. (One can make the less restrictive hypothesis that V has a compact support, but the final formulas are more involved [20]). We shall henceforth restrict our attention to the situation in which Q is inside D . One obtains

$$\begin{aligned} \hbar\omega {}_N\langle 0, \psi | \sum_n \sin^2 \alpha_n(\hat{x}, t) \sigma_3^{(n)} | \psi, 0 \rangle_N &= -\hbar\omega \langle \psi | \sum_n \sin^2 \alpha_n(\hat{x}, t) | \psi \rangle \\ &= -\hbar\omega \int dx |\psi(x)|^2 \sum_n \sin^2 \left(\frac{V_0\Omega}{c\hbar} \int_{x-x_n}^{x+ct-x_n} dy \delta(y) \right) \\ &= -\hbar\omega \int dx |\psi(x)|^2 \sum_n \sin^2 \left(\frac{V_0\Omega}{c\hbar} \theta(x+ct-x_n)\theta(x_n-x) \right) \\ &\rightarrow -\hbar\omega \frac{\bar{n}}{L} \int dx |\psi(x)|^2 \int_{x+ct-x_N}^{x+ct-x_1} dz \theta(z)\theta(ct-z) \\ &= -\hbar\omega \frac{\bar{n}}{L} (ct-x_1)\theta(a+ct-x_1)\theta(x_N-ct-a) + \text{b.e.}, \end{aligned} \tag{12}$$

where the arrow denotes the weak-coupling, macroscopic limit (8)–(10), the expression in the far RHS holds only when Q is inside D (notice that $|x| \leq a \ll x_1 \leq x_n, \forall n$, so that $\theta(ct-z) = 1$), and b.e. is a shorthand notation for “border effects”, namely terms appearing for $|ct-x_1|, |ct-x_N| \leq a$: These terms can be computed exactly [20], but we will not consider them in this Letter. In conclusion

$$\langle c\hat{p}(t) \rangle = cp_0 - \hbar\omega \frac{\bar{n}}{L} (ct-x_1) + \text{b.e.} \tag{13}$$

The energy of the Q particle decreases linearly with respect to t inside D , which reminds us of the exponential behavior of the propagator ${}_N\langle 0, \psi | \exp(-iHt/\hbar) | \psi, 0 \rangle_N$ [14]. Notice, however, the presence of ω in this expression: What is seen here is an energy-dissipative process.

In order to clarify the stochastic nature of the system, let us now look at the expectation value of the operator (4) in the state (6). By applying a technique identical to the one used in (12) one easily obtains

$$\langle \Delta H_D(t) \rangle \equiv {}_N\langle 0, \psi | \Delta H_D(t) | \psi, 0 \rangle_N = \hbar\omega \frac{\bar{n}}{L} (ct-x_1) + \text{b.e.}, \tag{14}$$

in agreement with (13) (the energy lost by Q must be stored in D). Consider now

$$\begin{aligned} &\langle \Delta H_D(t_1) \Delta H_D(t_2) \rangle \\ &= (\hbar\omega)^2 {}_N\langle 0, \psi | \left(\sum_n \sin^2 \alpha_n(\hat{x}, t_1) \sigma_3^{(n)} \right) \left(\sum_n \sin^2 \alpha_n(\hat{x}, t_2) \sigma_3^{(n)} \right) | \psi, 0 \rangle_N \\ &+ \left(i \frac{\hbar\omega}{2} \right)^2 {}_N\langle 0, \psi | \left(\sum_n \sin 2\alpha_n(\hat{x}, t_1) [\sigma_+^{(n)}(\hat{x}) - \text{H.c.}] \right) \\ &\times \left(\sum_n \sin 2\alpha_n(\hat{x}, t_2) [\sigma_+^{(n)}(\hat{x}) - \text{H.c.}] \right) | \psi, 0 \rangle_N + \text{vanishing terms.} \end{aligned} \tag{15}$$

A calculation similar to the previous one yields, in the weak-coupling, macroscopic limit (8)–(10),

$$\langle \Delta H_D(t_1) \Delta H_D(t_2) \rangle \longrightarrow \left(\hbar \omega \frac{\bar{n}}{L} \right)^2 (ct_1 - x_1)(ct_2 - x_1) + (\hbar \omega)^2 \frac{\bar{n}}{L} [c \min(t_1, t_2) - x_1] + \text{b.e.}, \quad (16)$$

where we have neglected the size of the wave packet as compared to the size of D (see (7)). Let us consider now the operator

$$\hat{\Sigma}(t) = \Delta H_D(t) - \langle \Delta H_D(t) \rangle. \quad (17)$$

From Eqs. (14) and (16) one obtains

$$\langle \hat{\Sigma}(t) \rangle = 0, \quad (18)$$

$$\langle \hat{\Sigma}(t_1) \hat{\Sigma}(t_2) \rangle = (\hbar \omega)^2 \frac{\bar{n}}{L} [c \min(t_1, t_2) - x_1] + \text{b.e.} = (\hbar \omega)^2 \frac{\bar{n}}{L} c \min(\tau_1, \tau_2) + \text{b.e.}, \quad (19)$$

where $\tau_\ell = t_\ell - x_1/c$ ($\ell = 1, 2$) is the time that (the center of) the wave packet of Q spends in D .

This property is reminiscent of a Wiener process [12]. However, in order to shed light on the presence and the nature of the stochastic process, one must compute the multi-time correlation functions. The calculation is rather involved and will be presented in full generality elsewhere [20]. Let us sketch the most salient steps. Define the characteristic functional

$$\begin{aligned} \phi[\beta] &\equiv \left\langle \exp \left(\int dt \beta(t) \hat{\Sigma}(t) \right) \right\rangle \equiv_N \langle 0, \psi | \exp \left(\int dt \beta(t) \hat{\Sigma}(t) \right) | \psi, 0 \rangle_N \\ &= \int dx |\psi(x)|^2_N \langle 0 | \prod_n \exp \left(-\hbar \omega \int dt \beta(t) \{ \sin^2 \alpha_n(x, t) \sigma_3^{(n)} \right. \\ &\quad \left. + (i/2) \sin 2\alpha_n(x, t) [\sigma_+^{(n)}(x) - \sigma_-^{(n)}(x)] \right) | 0 \rangle_N \exp \left(- \int dt \beta(t) \hbar \omega \frac{\bar{n}}{L} (ct - x_1) \right), \end{aligned} \quad (20)$$

where we are only interested in the case in which the wave packet of the Q particle is completely inside D , so that the border effects can be neglected. The disentanglement of the exponential is straightforward but somewhat involved, and requires some care. The final result [20] is

$$\phi[\beta] = \int dx |\psi(x)|^2 \exp \left(- \sum_n Y(a_n, b_n) \right) \exp \left(- \int dt \beta(t) \hbar \omega (\bar{n}/L) (ct - x_1) \right), \quad (21)$$

$$Y(a, b) = a\sqrt{b^2 + 1} + \ln \left(\frac{2\sqrt{b^2 + 1}}{(\sqrt{b^2 + 1} + b) + (\sqrt{b^2 + 1} - b)e^{2a\sqrt{b^2 + 1}}} \right),$$

$$a_n \equiv -\frac{\hbar \omega}{2} \int dt \beta(t) \sin 2\alpha_n(x, t), \quad a_n b_n \equiv -\hbar \omega \int dt \beta(t) \sin^2 \alpha_n(x, t).$$

Since, in the weak-coupling, macroscopic limit (8)–(10), both a_n and b_n are of order $1/\sqrt{N}$, one gets

$$\begin{aligned} \phi[\beta] &\longrightarrow \exp \left(+\frac{1}{2} (\hbar \omega)^2 \frac{\bar{n}}{L} \int dt dt' \beta(t) [c \min(t, t') - x_1] \beta(t') \right) \\ &= \exp \left(+\frac{1}{2} (\hbar \omega)^2 \frac{\bar{n}}{L} \int dt dt' \beta(t) c \min(\tau, \tau') \beta(t') \right) \quad (\tau \equiv t - x_1/c). \end{aligned} \quad (22)$$

The characteristic functional turns out to be Gaussian, which proves that $\hat{\Sigma}(t)$ behaves as a Gaussian stochastic process, and that, due to (19), it is nothing but a Wiener process within the restricted state space spanned by $|\psi, 0\rangle_N$. All the above results are exact.

We have seen that the operator $\hat{\Sigma}(t)$, which is essentially the energy operator $H_D(t)$ for the N -spin array D , serves as a “noise operator” in the $N \rightarrow \infty$ limit (8), in the sense of Eqs. (18) and (19). Furthermore, we have explicitly shown that there appears a Wiener stochastic process, which is characterized by the Gaussian white noise properties [12] or by the above characteristic functional (22), in this model. Although the appearance of a stochastic process of some sort could probably be expected on the basis of the stochastic behavior of the propagator ${}_N\langle 0, \psi | \exp(-iHt/\hbar) | \psi, 0 \rangle_N$ [14], the emergence of the Gaussian white noise is remarkable, for such a nontrivial Hamiltonian like (2). It is worth stressing again that the stochastic behavior, i.e. the exponential decay form of the propagator shown in [14], is independent of ω and depends only on the probability of spin-flipping. On the contrary, the presence of ω and therefore the presence of an energy-exchange process is essential for the derivation of the Wiener process, through which the energy of the system is dissipated. We understand that the weak-coupling, macroscopic limit (8)–(10), that is closely related to van Hove’s limit, plays a crucial role in this respect: It corresponds to a kind of coarse graining and scale-change procedures. Another important fact, not to be dismissed, is that we have exclusively considered the dynamics within a restricted state space spanned by $|\psi, 0\rangle_N$. We emphasize, once again, that the stochastic process is derived *without* any hypothesis on the spectrum of H and *without* tracing over the states of the macroscopic system D .

There are several interesting open problems that deserve further investigation. For instance, it is well known that the reduced dynamics of a (sub)system in interaction with a larger system (playing the role of reservoir) is well described in terms of quantum dynamical semigroups [21]. Moreover, one should be able to derive a master (or Langevin) equation for some dynamical variables of the subsystem. A promising candidate for such a dynamical variable, in the present model, is obviously the energy–momentum of the Q particle. (Notice that, in the ultrarelativistic ansatz of the AgBr model, the momentum of Q is nothing but its energy.) There are also some details that make us think that the Schrödinger equation *itself*, under suitable conditions, like weak-coupling and macroscopicity, could play the role of a Langevin-type equation. These problems are now under investigation.

References

- [1] L. van Hove, *Physica* 21 (1955) 517;
E.B. Davies, *Commun. Math. Phys.* 39 (1974) 91;
G.L. Sewell, *Quantum theory of collective phenomena* (Clarendon Press, Oxford, 1986).
- [2] V.B. Magalinskij, *Zh. Eksp. Teor. Fiz.* 36 (1959) 1942 [*Sov. Phys. JEPT* 9 (1959) 1381];
G.W. Ford, M. Kac and P. Mazur, *J. Math. Phys.* 9 (1965) 504;
P. Ullersma, *Physica* 32 (1966) 27;
R. Zwanzig, *J. Stat. Phys.* 9 (1973) 215.
- [3] A.O. Caldeira and A.J. Leggett, *Phys. Rev. Lett.* 46 (1981) 211; *Physica A* 121 (1983) 587; *Phys. Rev. A* 31 (1985) 1059;
R.P. Feynman and F.L. Vernon, *Ann. Phys. (N.Y.)* 24 (1963) 118.
- [4] L. Accardi, A. Frigerio and Y.G. Lu, *Comm. Math. Phys.* 131 (1990) 537;
L. Accardi, Y.G. Lu and I. Volovich, *Quantum Probability and Related Topics* 9 (1994) 15.
- [5] G.W. Ford, J.T. Lewis and R.F.O’Connell, *Phys. Rev. A* 37 (1988) 4419.
- [6] M. Mizutani, S. Muroya and M. Namiki, *Phys. Rev. D* 37 (1988) 3033.
- [7] C.W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991);
R. Blasi and S. Pascazio, *Phys. Rev.* 53A (1996) 2033.
- [8] J. von Neumann, *Die mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).
- [9] J.A. Wheeler and W.H. Zurek, eds., *Quantum theory and measurement* (Princeton Univ. Press, Princeton, 1983);
P. Busch, P.J. Lahti and P. Mittelstaedt, *The quantum theory of measurement* (Springer, Berlin, 1991);
M. Namiki and S. Pascazio, *Phys. Rep.* 232 (1993) 301.
- [10] S. Machida and M. Namiki, *Prog. Theor. Phys.* 63 (1980) 1457; 1833;
M. Namiki, *Found. Phys.* 18 (1988) 29;
M. Namiki and S. Pascazio, *Phys. Rev. A* 44 (1991) 39.
- [11] W.H. Zurek, *Phys. Rev. D* 24 (1981) 1516; *D* 26 (1982) 1862;
J.P. Paz, S. Habib and W.H. Zurek, *Phys. Rev. D* 47 (1993) 488.

- [12] Selected papers on noise and stochastic processes, ed. N. Wax (Dover, New York, 1954);
C.W. Gardiner, *Handbook of stochastic methods* (Springer, Berlin, 1990).
- [13] A.J. Leggett, Proc. 4th Int. Symp. on Foundations of quantum mechanics, eds. M. Tsukada et al. (Phys. Soc. Japan, Tokyo, 1993) p. 10.
- [14] H. Nakazato, M. Namiki and S. Pascazio, Phys. Rev. Lett. 73 (1994) 1063; Int. J. Mod. Phys. B 10 (1996) 247.
- [15] K. Hepp, Helv. Phys. Acta 45 (1972) 237.
- [16] J.S. Bell, Helv. Phys. Acta 48 (1975) 93;
S. Machida and M. Namiki, Proc. Int. Symp. on Foundations of quantum mechanics, eds. S. Kamefuchi et al. (Phys. Soc. Japan, Tokyo, 1984) p. 136;
S. Kudaka, S. Matsumoto and K. Kakazu, Prog. Theor. Phys. 82 (1989) 665.
- [17] H. Nakazato and S. Pascazio, Phys. Rev. Lett. 70 (1993) 1; Phys. Rev. A 48 (1993) 1066.
- [18] A.O. Caldeira, H.A. Cerdeira and R. Ramaswamy, Phys. Rev. A 40 (1989) 3438.
- [19] C.P. Sun, Phys. Rev. A 48 (1993) 898;
K. Hiyama and S. Takagi, Phys. Rev. A 48 (1993) 2568;
X.J. Liu and C.P. Sun, Phys. Lett. A 198 (1995) 371;
Y.Q. Li and Y.X. Chen, Phys. Lett. A 202 (1995) 325.
- [20] R. Blasi, H. Nakazato, M. Namiki and S. Pascazio, in preparation.
- [21] G. Lindblad, Comm. Math. Phys. 48 (1976) 119;
V. Gorini, A. Kossakowski and E.C.G. Sudarshan, J. Math. Phys. 17 (1976) 821;
R. Alicki and K. Lendi, Lecture notes in physics, Vol. 286. Quantum dynamical semigroups and applications (Springer, Berlin, 1987);
A.S. Holevo, Proc. Steklov Inst. Math. 191 (1989) 131.