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Understanding Physical Knowledge
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CONTENTS

R. Lupacchini and V. Fano, *Understanding physical knowledge*..... 1

PART I – MATHEMATICAL MACHINERY

G. Fano, *An elementary version of the mathematics of quantum mechanics*.....17

G. Boniolo and F. de Felice, *The formal structure of the relativistic theories*..... 65

PART II – CONCEPTUAL ANALYSIS

P. Facchi and S. Pascazio, *Quantum measurements, decoherence and irreversibility*...101

G. Tarozzi, *Perspectives on the EPR paradox and the measurement problem*.....121

C. Timpson and H. Brown, *Entanglement and relativity*.....147

S. Bergia and F. Toscano, *Cosmology as a scientific discipline*.....167

V. Fano, *Realism vs conventionalism in the philosophy of space-time physics*.....189

M. Ghins, *Realism in common sense and in theoretical physics*.....209

M. L. Dalla Chiara and R. Giuntini, *The logic of quantum mechanics*.....231

A. Ekert and R. Lupacchini, *Physical knowledge and mathematical understanding*.....243

Quantum measurements, decoherence and irreversibility

ABSTRACT. We outline and discuss the basic ideas and approaches to the quantum measurement problem. The issue of decoherence and the emergence of classical properties are analyzed, focusing in particular on the long-standing problem of irreversibility. We also give a few examples of mesoscopic systems and discuss their conceptual relevance.

1. *Introduction*

Fundamental equations in physics are time-reversal invariant. Quantum mechanics is no exception: the Schrodinger equation and the whole structure of the theory are time reversible. There is a noteworthy exception, though: the act of measurement, for which one postulates that the wave function collapses into a statistical mixture of states described by a suitable density matrix. This process is non-unitary and irreversible.

A quantum measurement is performed by a macroscopic apparatus, to which the collapse is ultimately ascribed. One should notice, however, that there is no clear-cut difference between ‘microscopic’ and ‘macroscopic’ systems: the distinction between these two classes is often roughly made on the basis of the system ‘size’ and number of elementary constituents. However, the size of a system is certainly not the only criterion to distinguish between these two cases. For example, the difference between the microscopic and the macroscopic world becomes subtler (and mind boggling) if one considers phenomena of persistence of the phase correlations of the wave function that describes the whole system or its subsystems (or even their elementary constituents). This topic has recently become very fashionable and goes under the general name of ‘decoherence:’ in a few words, decoherence is the study of the loss of phase correlations due to the interactions undergone by the system. It goes without saying that such a phenomenon becomes very interesting in the context of systems that are neither microscopic nor macroscopic. ‘Mesoscopic’ systems are at the borderline between these two situations: one expects then the presence of phase-correlation phenomena as well as semiclassical properties. Thanks to recent technological development, it is possible nowadays to investigate many new situations that lie at the borderline between the microscopic and the macroscopic domains.

In this article, rather than concentrating our attention on general features and definitions, we will give a brief (and personal) survey of the quantum measurement problem, focusing on decoherence and irreversibility, leading to the emergence of classical properties.

2. Preliminaries: the ‘collapse’ of the wave function

We first introduce notation and some preliminary notions. Consider a quantum system Q , whose states ψ are unit vectors belonging to the Hilbert space \mathcal{H} . The evolution of ψ is governed by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi, \quad (1)$$

where H , the Hamiltonian, is a self-adjoint operator: $H = H^\dagger$. The formal solution of the above equation is (in Dirac’s notation)

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle \equiv U(t)|\psi(0)\rangle \quad (2)$$

where the evolution operator U , due to the self-adjointness of H , is unitary

$$U(t)U^\dagger(t) = U^\dagger(t)U(t) = 1, \quad \forall t. \quad (3)$$

The above property of the evolution operator encloses the time symmetric features of the theory.

Let F be an observable, represented by a linear self-adjoint operator \hat{F} acting on \mathcal{H} . We will henceforth identify the observable and the corresponding operator and drop the hat on the latter. We can always decompose the state ψ into the eigenfunctions of F and write

$$|\psi\rangle = \sum_i c_i |\chi_i\rangle, \quad c_i = \langle \chi_i | \psi \rangle, \quad (4)$$

where

$$F|\chi_i\rangle = \lambda_i |\chi_i\rangle, \quad (5)$$

λ_i being the (real) eigenvalues of F and $|\chi_i\rangle$ the corresponding eigenfunctions. For simplicity, we assume that the eigenvalues are not degenerate, that is $\lambda_i \neq \lambda_j$ for $i \neq j$ (maximal observable).

One of the fundamental postulates of quantum mechanics states that if one performs a measurement of the observable F on the system in state ψ one finds any one of the states $|\chi_i\rangle$ with probability $|c_i|^2$. We shall call the process

$$|\psi\rangle \longrightarrow |\chi_k\rangle \quad (\text{with probability } |c_k|^2) \quad (6)$$

‘projection rule’ or ‘postulate,’ yielding a ‘naive’ wave function collapse.

Von Neumann [Von Neumann 1932] improved the above description of the collapse, making it self-contained, by introducing the density matrix ρ :

$$\rho = |\psi\rangle\langle\psi| = \sum_i \sum_j c_i c_j^* |\chi_i\rangle\langle\chi_j| \longrightarrow \rho_F = \sum_k |c_k|^2 \pi_k, \quad (7)$$

where $\pi_k \equiv |\chi_k\rangle\langle\chi_k|$ is the projection operator onto the k th eigenstate of F . This expression describes well a process in which all the phase correlations among different eigenstates are erased, so that we obtain a sum of exclusive probabilities of finding each eigenstate. We emphasize that *no* unitary evolution law (2) can yield the process (6) or (7).

However, (7) is still not satisfactory, because in some cases this description has mind-boggling features. Consider, for example, the case $c_1 = c_2 = 1/\sqrt{2}$, corresponding to a dichotomic observable F . In this case, (7) becomes

$$\rho \longrightarrow \rho_F = \frac{1}{2}(\pi_1 + \pi_2). \quad (8)$$

On the other hand, by defining $\chi_{\pm} = (\chi_1 \pm \chi_2)/\sqrt{2}$, we easily see that (8) turns into

$$\rho \longrightarrow \rho_G = \frac{1}{2}(\pi_+ + \pi_-) = \rho_F, \quad (9)$$

where $\pi_{\pm} \equiv |\chi_{\pm}\rangle\langle\chi_{\pm}|$. Note that $\rho_F = \rho_G$ is an identity, and observe that (8) describes a measurement of the observable F , while (9) describes a measurement of another observable G with eigenstates χ_{\pm} . In general, $[F, G] \neq 0$, so that (7) describes the incompatible measurements of two noncommuting observables at the same time: two distinct macroscopic setups (or preparation procedures) yield the same density matrix.

This puzzling result was first pointed out, to our knowledge, by Watanabe [Watanabe], and is deeply rooted into the well-known phenomenon of the nonunique decomposability of mixed states [d'Espagnat 1971, Busch, Lahti and Mittelstaedt 1991], which can also be used to point out that some (statistical) interpretations concerning the behavior of EPR-correlated particles [Einstein, Podolsky and Rosen 1935] lead to inconsistencies. We will come back to this point later and show that Watanabe's objection can be circumvented.

3. Von Neumann and Wigner

The unitarity of evolutions and the superposition principle (which are two milestones in quantum mechanics) have other interesting and far-reaching consequences in this context. Since a quantum measurement is performed by another system D (detector), endowed with macroscopic features and interacting with the object system Q, one can explore the consequences of the introduction of the detector in the description of the measurement process.

Assume that the detector can be described by a wave function Φ (... a rather delicate assumption!). Since the detector is designed so as to yield the result of the measurement after the interaction with the quantum system Q, the evolution is

$$|\chi_i\rangle \otimes |\Phi_0\rangle \longrightarrow |\chi_i\rangle \otimes |\Phi_i\rangle, \quad (10)$$

where $|\Phi_0\rangle$ and $|\Phi_i\rangle$ are the initial and final detector states, the latter registering the measurement outcome i of the measurement. In other words, the above equation represents an instrument that changes its state from $|\Phi_0\rangle$ to $|\Phi_i\rangle$, according to the object eigenstate $|\chi_i\rangle$. This is one of the simplest way to model a detector.

Von Neumann and Wigner strongly asserted that quantum mechanics, in particular the superposition principle, should be strictly applied to the measurement process. If the object system is in state (4) the evolution of the total system is

$$|\Psi\rangle \equiv |\psi\rangle \otimes |\Phi_0\rangle = \sum_i c_i |\chi_i\rangle \otimes |\Phi_0\rangle \longrightarrow \sum_i c_i |\chi_i\rangle \otimes |\Phi_i\rangle \equiv |\Psi_{\text{fin}}\rangle \quad (11)$$

for the whole measurement process. This is often called the Von Neumann measurement process.

However, the phase correlation among different Q eigenstates is still present in $|\Psi_{\text{fin}}\rangle$, as we easily understand by writing the corresponding density matrix

$$\rho_{\text{tot}} = |\Psi_{\text{fin}}\rangle\langle\Psi_{\text{fin}}| = \sum_k |c_k|^2 \pi_k \otimes |\Phi_k\rangle\langle\Phi_k| + \sum_i \sum_{j \neq i} c_i c_j^* |\chi_i\rangle\langle\chi_j| \otimes |\Phi_i\rangle\langle\Phi_j|. \quad (12)$$

Obviously, the above is still a projection operator, representing a pure state with non-vanishing off-diagonal components (the last term in the right hand side). On the other hand, the expectation value of *any* observable which refers to the system alone, $A_S \otimes \mathbf{1}_D$, reads

$$\langle A_S \rangle = \text{Tr}(\rho_{\text{tot}} A_S \otimes \mathbf{1}_D) = \text{Tr}_S(A_S \text{Tr}_D \rho_{\text{tot}}) = \text{Tr}_S(A_S \rho), \quad (13)$$

where $\rho = \text{Tr}_D \rho_{\text{tot}}$. Thus, after the measurement has taken place, the state of the system *alone* is completely described by the reduced density matrix

$$\rho = \text{Tr}_D \rho_{\text{tot}} = \sum_k |c_k|^2 \pi_k = \rho_F \quad (14)$$

defined in (7). All phase correlations become unobservable if one looks only at the system, because they are encoded in the entanglement. The same happens for the detector.

As is well known, Von Neumann and Wigner (more or less explicitly) maintained that the measuring process is not completed by the interaction of Q with D, which is instead followed by the so-called Von Neumann's chain of measurements, connecting Q to the observer via many steps. Eventually the theory brings in an 'Abstraktes Ich' or 'consciousness' at the end of the chain, which should be responsible for the wave function collapse by measurement. On the basis of these arguments, Wigner claimed that quantum mechanics is incomplete. This theory provoked the famous paradoxes of Schrödinger's cat and Wigner's friend, on which we shall not elaborate because Stephen Hawking might reach for his gun.

We should also remark that one implicitly requires that the detector states $|\Phi_i\rangle$ in (11) are subject to the orthogonality condition

$$\langle \Phi_i | \Phi_j \rangle = \delta_{ij}. \quad (15)$$

This derives from the idea that $\{|\Phi_i\rangle\}$ is a set of eigenstates of a macroscopic observable of D, like a pointer position. However, Wigner, Araki and Yanase [Wigner 1952, Araki and Yanase 1960, Yanase 1961] showed that a measurement process of an observable which does not commute with a conserved (additive) quantity cannot be described by (11) with the orthogonality condition (15). In spite of this, they also proved that an approximate measurement of such an observable is possible in the original sense of a Von Neumann process, up to any desired accuracy, if we use an adequate measuring apparatus endowed with a huge number of degrees of freedom. From this point of view, it is possible to find a measuring apparatus that yields a measurement process of the Von Neumann type, for which the following asymptotic orthogonality holds:

$$\langle \Phi_i | \Phi_j \rangle = \delta_{ij} + (1 - \delta_{ij}) O(\epsilon), \quad \epsilon \xrightarrow{N \rightarrow \infty} 0, \quad (16)$$

where N is a parameter corresponding to the number of degrees of freedom of the detector.

Although the above condition is rather natural for a *bona fide* detector (which is designed so as to *measure* an observable, yielding an outcome that can be discriminated from the other possible outcomes), it appears much less natural when applied to a generic environment. We will come back to this subtle point later (Sec. 7).

Finally, it is worth noting that, strictly speaking, Eq. (11) is only a ‘spectral decomposition,’ yielding no dephasing and no collapse of the wave function.¹ It is then clear that the underlying dynamics is unitary and entails no loss of phase coherence (decoherence) whatsoever. In this sense, it cannot be regarded as a satisfactory solution of the quantum measurement problem.

We started this section by assuming that the detector can be described by a wave function Φ . The conclusions obtained here are still valid if one relaxes this (strong) hypothesis and assumes that the detector is initially described by a density matrix (mixed state).

4. Partial tracing and irreversibility

The technique of partial tracing, applied in the previous section to the Von Neumann process (11), yields a completely reduced density matrix for the system, provided that the orthogonality condition (15) or (16) holds for the detector states. However, it is somewhat questionable whether this can be considered as a *bona fide* collapse of the wave function. Indeed, from a genuine physical viewpoint, no irreversibility is implied: partial tracing (as well as Von Neumann’s projection rule) is very useful from a computational point of view, but does not actually correspond to a physical operation. It is more to be viewed as the computation of an average over the detector states (namely, the unobserved degrees of freedom).

The evolution (11)-(12) of the total system (Q+D) is always unitary, and therefore reversible, for each elementary process. Irreversibility appears only as a consequence of an additional mathematical manipulation on the total density matrix. Whether such a solution is viewed as a satisfactory one is just, after all, a matter of attitude toward physics and physical theories. We feel that the difference between these two possible attitudes, as well as the philosophy underlining the two approaches, is profound.

We will not further elaborate on these issue, discussed in [Namiki, Pascazio and Nakazato 1997]. We will show, on the other hand, that the off-diagonal part of the total density matrix does not vanish in a Von Neumann process, even when the orthogonality of the D states (16) is valid. To this end, first decompose the final state density matrix ρ_{tot} into the sum of its diagonal and off-diagonal parts

$$\begin{aligned}\rho_{\text{tot}} &= \rho_{\text{diag}} + \rho_{\text{off}}, \\ \rho_{\text{diag}} &= \sum_k |c_k|^2 \pi_k \otimes |\Phi_k\rangle\langle\Phi_k|, \quad \rho_{\text{off}} = \sum_i \sum_{j \neq i} c_i c_j^* |\chi_i\rangle\langle\chi_j| \otimes |\Phi_i\rangle\langle\Phi_j|.\end{aligned}\quad (17)$$

By making use of idempotency $\rho_{\text{tot}}^2 = \rho_{\text{tot}}$, we easily compute

$$\rho_{\text{diag}}^2 - \rho_{\text{diag}} = \rho_{\text{off}} - \rho_{\text{off}}^2 - \rho_{\text{diag}}\rho_{\text{off}} - \rho_{\text{off}}\rho_{\text{diag}}.\quad (18)$$

¹The notion of spectral decomposition was introduced by Wigner [Wigner 63], who considered the Stern-Gerlach decomposition of an initial spin state, where each component of the spin becomes associated with a different wave packet. This notion can be generalized and given a full Hamiltonian description [Pascazio and Namiki 1994].

By calculating the partial trace with respect to the D states before taking the limit for $N \rightarrow \infty$, we obtain

$$\mathrm{Tr}_D \rho_{\mathrm{off}} = O(\epsilon), \quad (19)$$

$$\begin{aligned} \mathrm{Tr}_D \rho_{\mathrm{off}}^2 &= -\mathrm{Tr}_D (\rho_{\mathrm{diag}}^2 - \rho_{\mathrm{diag}}) + O(\epsilon) \\ &= \sum_k |c_k|^2 (1 - |c_k|^2) \pi_k + O(\epsilon). \end{aligned} \quad (20)$$

This means that even though its trace vanishes, ρ_{off} itself does not vanish, even in the infinite N limit, because $\mathrm{Tr}_D \rho_{\mathrm{off}}^2 \neq 0$. This argument, not well known to everybody, is due to Furry [Furry 1966, Namiki, Pascazio and Nakazato 1997].

It is clear that if one agrees with the idea that ‘decoherence’ or ‘collapse’ are *bona fide* irreversible processes, then the Von Neumann-Wigner approach does not provide the solution to the problem. This is physically obvious: in a process of the type (11) or (12) coherence is always present and can be brought back to light by a good experimental physicist, by means of suitable disentanglement techniques (and accurate control of unwanted environmental decohering effects). One must mention, in this context, the beautiful neutron interference experiments performed by Rauch’s and Werner’s groups [Rauch and Werner 2000] evidencing that the apparent disappearance of interference does not always mean loss of coherence: the supports of two wave packets of a single neutron may not overlap (so that the corresponding quantum states are orthogonal), but a beautiful interference pattern can be brought to light by making use of a sophisticated momentum post-selection technique. A lack of coherence implies the absence of interference, but the *vice versa* is not true: the absence of interference does not necessarily signify a lack of coherence. Similar phenomena are known in classical optics [Born and Wolf 1999] (although, admittedly, the quantum flavor is different, for experiments are performed on single quantum systems).

5. Environment: general ideas

We now briefly outline the main features of the environment approach to decoherence and the quantum measurement problem. For a review, see [Giulini *et al* 1996, Zurek 1991].

Let the initial state of a quantum system be given again by (4) and let $|E_0\rangle$ be the initial state of the ‘environment’ E, namely of the other quantum systems that are located close to the system investigated and that interact with it. The evolution yields

$$|\chi_i\rangle \otimes |E_0\rangle \longrightarrow |\chi_i\rangle \otimes |E_i\rangle, \quad (21)$$

where $|E_i\rangle$ is the final state of the environment. This process is analogous to (10): in a few words, the environment works as an *imperfect* detector, by changing its state from $|E_0\rangle$ to $|E_i\rangle$, according to the system’s eigenstate $|\chi_i\rangle$. [We will understand later in which sense E acts as an ‘imperfect’ detector: look at the comments that follow Eq. (24).]

By applying the superposition principle to (21) we obtain

$$|\Psi\rangle \equiv |\psi\rangle \otimes |E_0\rangle = \sum_i c_i |\chi_i\rangle \otimes |E_0\rangle \longrightarrow \sum_i c_i |\chi_i\rangle \otimes |E_i\rangle \equiv |\Psi_{\mathrm{fin}}\rangle, \quad (22)$$

which corresponds to (11). (We use the same notation as in Sec. 3 since no confusion arises.)

The phase correlation among the different eigenstates of the system is still present, as one sees by writing the density matrix

$$\rho_{\text{fin}} = |\Psi_{\text{fin}}\rangle\langle\Psi_{\text{fin}}| = \sum_k |c_k|^2 \pi_k \otimes |E_k\rangle\langle E_k| + \sum_i \sum_{j \neq i} c_i c_j^* |\chi_i\rangle\langle\chi_j| \otimes |E_i\rangle\langle E_j|. \quad (23)$$

Compare with (12).

Usually, at this stage, one computes the partial trace with respect to the environment states and obtains the density matrix (14) for the system alone. However, one should notice that we are not explicitly requiring the orthogonality condition for the environment states $|E_i\rangle$ in (21)-(23)

$$\langle E_i | E_j \rangle = \delta_{ij} + (1 - \delta_{ij}) O(\epsilon), \quad \epsilon \xrightarrow{N \rightarrow \infty} 0. \quad (24)$$

Equation (24) often becomes valid in physically relevant situations, essentially when the environment states orthogonalize as a consequence of the continuous interaction with the system. However, this orthogonalization takes place during a certain elapse of time and a thorough analysis should yield the correct couplings and timescales. For smaller couplings and/or shorter timescales, it is sensible to assume, in general, that the process (24) has not occurred yet, and the ϵ -deviations in (24) are important and ought to be taken into account. In this sense, the process described above cannot be considered as a ‘measurement’ in the Von Neumann-Wigner sense. We notice, incidentally, that the condition (24) is often required in many ‘environment’ approaches to quantum measurements, often somewhat lightheartedly: at most, one can say that the environment acts as an ‘imperfect’ detector, for which the above orthogonality condition does not hold or holds only rather approximately. In general terms, the environment can be viewed as a measuring apparatus without an explicit ‘meter.’ We will come back to the subtleties involved in the requirements (15), (16) and (24) in Section 7.

6. *Decoherence via dephasing*

The description of the previous section can be improved by noticing that an experiment involves the accumulation of many individual events, pertaining to the same macroscopic conditions (‘preparation’). Different Q systems, endowed with the same wave function, interact with different local subsystems of the environment, which are in general different from each other and can be labelled by an index ℓ , accounting for internal motion and other rapidly changing factors. We therefore write

$$|\chi_i\rangle \otimes |E_0^{(\ell)}\rangle \longrightarrow S_i^{(\ell)} |\chi_i\rangle \otimes |E_i\rangle, \quad (25)$$

where we explicitly wrote the numerical factor $S_i^{(\ell)}$ due to the interaction between Q and E. Such a term can be included in the definition of $|E_i\rangle$, but due to its primary role in our discussion, it is convenient to factor it out. We assumed for simplicity that $S_i^{(\ell)}$ is a c-number (or, in other words, that the state of Q is not changed by the interaction and the final state of E depends on ℓ only through its phase) and focus on its phase. These simplifying hypotheses make the analysis simpler and enable one to focus on the main

ideas (the assumption on the E-state can be easily relaxed; the assumption on the Q-state would lead us to discuss the distinction between ‘destructive’ and ‘nondestructive’ measurements and is outside the scope of this review). Our discussion is clearly inspired by scattering theory, according to the philosophy of the many Hilbert space approach to quantum measurements [Machida and Namiki 1981, Namiki, Pascazio and Nakazato 1997], although the present analysis deals in general with dephasing processes, that do not necessarily entail a ‘measurement,’ namely a recording process by the environment: this is indeed the reason why we need not require the orthogonality condition (24).

By introducing the statistical average over ℓ

$$\overline{(\dots)} \equiv \frac{1}{N_p} \sum_{\ell=1}^{N_p} (\dots)^{(\ell)}, \quad (26)$$

where N_p is the total number of particles detected in one experimental run, we get

$$\bar{\rho}_{\text{tot}} = \sum_{i,j} c_i c_j^* \Delta_{ij} |\chi_i\rangle \langle \chi_j| \otimes |E_i\rangle \langle E_j|. \quad (27)$$

In words, we are considering the *accumulated* distribution over many experimental results performed under the same *macroscopic* conditions, although the *microscopic* details of the environment can be different from case to case. This is obviously in line with the fundamental principles of quantum mechanics. We have defined

$$\Delta_{ij} \equiv \overline{S_i S_j^*}, \quad (28)$$

so that all the effects of the environment are simply contained in Δ_{ij} . If we write

$$\Delta_{ij} = d_i \delta_{ij} + \eta_{ij} (1 - \delta_{ij}), \quad (29)$$

then a nonvanishing value of the second (‘off-diagonal’) addendum represents the degree of remaining coherence. A complete dephasing is characterized by

$$d_i = 1, \quad \eta_{ij} = 0, \quad (30)$$

so that

$$\bar{\rho}_{\text{tot}} = \sum_i |c_i|^2 \pi_i \otimes |E_i\rangle \langle E_i|. \quad (31)$$

In this case the factor $S_i^{(\ell)}$ yields a random sequence with respect to ℓ . This yields complete decoherence (namely a complete loss of phase coherence) and can be viewed as a ‘collapse’ of the wave function.

Let us now endeavor to understand when and in which sense the environment acts as a measuring device. A general quantal description of the process (21) is given by

$$|\chi_i\rangle \otimes |E_0\rangle \longrightarrow \sum_{\ell} b_i^{(\ell)} |\chi_i\rangle \otimes |E_i^{(\ell)}\rangle, \quad (32)$$

where $\sum_{\ell} |b_i^{(\ell)}|^2 = 1$ and ℓ refers to the additional microscopic degrees of freedom, which are unobservable. Notice that, like in (25), the microscopic details of E are explicitly

taken into account and observe that the index i takes a given value in (32). (There is a subtle difference between (25) and (32): the former hinges on the philosophy of classical statistical mechanics, the latter on a more quantal description, but we shall not elaborate on this here.) In terms of the density matrix,

$$\rho_{\text{tot}} = \sum_{i,j} c_i c_j^* |\chi_i\rangle \langle \chi_j| \otimes \sum_{\ell,m} b_i^{(\ell)} b_j^{(m)*} |E_i^{(\ell)}\rangle \langle E_j^{(m)}|. \quad (33)$$

Let us write

$$|E_i^{(\ell)}\rangle = |E_i\rangle \otimes |\epsilon_\ell\rangle, \quad (34)$$

where $|E_i\rangle$ refers to some macroscopic variable (that characterizes the macroscopic state of the environment and takes identical values for different ℓ s), while $|\epsilon_\ell\rangle$ (accounting for the microscopic degrees of freedom) changes for different ‘events’ (namely, repetitions of the experiment under the same macroscopic conditions). By inserting (34) into (33) one gets

$$\rho_{\text{tot}} = \sum_{i,j} c_i c_j^* |\chi_i\rangle \langle \chi_j| \otimes |E_i\rangle \langle E_j| \otimes |\varphi_i\rangle \langle \varphi_j|, \quad (35)$$

with

$$|\varphi_i\rangle \equiv \sum_{\ell} b_i^{(\ell)} |\epsilon_\ell\rangle. \quad (36)$$

By tracing away the unobservable microscopic degrees of freedom $|\epsilon_\ell\rangle$, we get (27), with

$$\Delta_{ij} = \langle \varphi_j | \varphi_i \rangle = \sum_{\ell} b_i^{(\ell)} b_j^{(\ell)*}, \quad (37)$$

which is to be compared with (28). In the last equality in (37) we choose, without loss of generality, orthogonal $|\epsilon_\ell\rangle$ s, $\langle \epsilon_\ell | \epsilon_m \rangle = \delta_{\ell m}$, whence $\Delta_{ii} = \sum_{\ell} |b_i^{(\ell)}|^2 = 1$ and $\delta_i = 1$ in (29).

Notice also that, whenever $\langle \varphi_j | \varphi_i \rangle = \delta_{ij}$, then $\eta_{ij} = 0$. In this case one can say that the environment is endowed with a ‘pointer,’ $|\varphi_i\rangle$, that ‘displays’ different results for different values of i . In the latter case, the environment plays the role of a *bona fide* detector, endowed with macroscopic features and with a pointer yielding the results of the measurement.

Irreversibility appears as a consequence of the average over different microscopic configurations, introduced in (26) and (37). It is an irreversibility of the same nature as that encountered in classical statistical mechanics [Tolman 1980, Huang 1987]. It is epistemic (and not ontologic) and consists in the idea that it is extremely improbable that coherence can be recovered, due to the huge number of additional degrees of freedom involved. Decoherence appears therefore as a consequence of the macroscopicity of the environment (or measuring apparatus, if the environment is endowed with a pointer). The emergence of ‘classical’ properties is therefore ascribed, *à la* Boltzmann, to the very large number of freedoms characterizing the macroscopic system interacting with the microscopic one [Peres 1980, Berry 2001]. No *bona fide* loss of coherence can be expected when only a small number (say 2 or 3) of quantum systems interact, without any other system appearing in the picture, for phase correlations can always be brought to light, in principle, by a good experimental physicist.

7. Critique of the orthogonality conditions (15), (16) and (24)

Let us comment on the meaning of conditions (15), (16) and (24). Roughly speaking, (15) stems from the idea that it is possible to bring to completion a spectral decomposition (in Wigner's sense [Wigner 1963]). This is clearly an assumption: even in the simple case of a Stern-Gerlach apparatus, the spin decomposition is not free from leakage effects, so that the wave packets associated with different spin states are never completely orthogonal (as one can easily check by looking at any spin decomposition experiments, even the most recent ones).

The approximate orthogonality condition (16) is more realistic than (15), but should nevertheless be considered with care. Not only it is model dependent, but requires that a parameter denoting a macroscopic feature of the apparatus tends to infinity.

The requirement (24) is conceptually different: unlike a *bona fide* detector, a generic environment is not designed so as to carry out the measurement of a quantum observable. In this sense, the property (24) seems to require a sort of 'astuteness' of the environment, almost as if it 'knew' in advance that it is supposed to yield a quantum measurement. Different environmental states will tend to decohere and become orthogonal, but the timescales over which such a process occurs depends on many factors and should be analyzed in great detail. Such effects are model dependent and are not considered here, not even in the analysis (32)-(37).

All the above-mentioned assumptions are delicate and must be analyzed case by case. By contrast, the orthogonality condition is taken for granted in other approaches to quantum measurement and decoherence, such as the many-world interpretation [Everett III 1957, De Witt and Graham 1973]. This is probably to be regarded as an intrinsic weak point of this approach. One should also stress, however, that in many practical cases the quantitative differences due to the ϵ corrections are negligible.

We conclude by noticing that if E states are *not* orthogonal, Eq. (23) [as well as (27) and (31)] must be handled with care, for they 'conceal' a certain degree of additional (residual) coherence.

8. Resolution of Watanabe's problem

Let us reconsider Watanabe's problem [Watanabe], introduced at the end of Sec. 2. However, we now take the states of the detector (or of the environment) into account. We consider again, for simplicity, the dichotomic case [$c_1 = c_2 = 1/\sqrt{2}$ in (12)]. If a dephasing process has taken place, one obtains (in the notation of Sec. 3)

$$\bar{\rho}_{\text{tot}} = \sum_i |c_i|^2 \pi_i \otimes |\Phi_i\rangle\langle\Phi_i| \quad (38)$$

and in the dichotomic case

$$\rho_{\text{tot}} = \frac{1}{2} (\pi_1 \otimes |\Phi_1\rangle\langle\Phi_1| + \pi_2 \otimes |\Phi_2\rangle\langle\Phi_2|). \quad (39)$$

[Obviously, if one traces over the detector states, one reobtains (8).] The inconsistency pointed out by Watanabe does not occur anymore, owing to the presence of the D states.

It is impossible (try!) to reexpress the above density matrix in a form that involves the rotated states (9) and that is still diagonal.

The physical meaning of this resolution of Watanabe's problem is obvious. Formulae (8) and (9) describe an 'unpolarized beam' of identical particles. This is identical (in the sense that it has identical quantum mechanical properties and yields identical results) from whichever direction one looks at it. On the other hand, although (39) still describes an unpolarized beam, it associates (namely, statistically 'correlates') each state of Q with a corresponding state of D. In other words, there is a signature of the measurement (or preparation, if D is regarded as an 'emitter') procedure of the unpolarized beam. This cannot be erased by any mathematical mean.

Incidentally, since the states of D *must* be explicitly included into the description of quantum measurements in order to avoid Watanabe's objection, one may say that quantum mechanics, unlike classical physics, is not consistent with *naïve realism*. Indeed, we need not explicitly introduce any detector variables or states in order to discuss the measurement problem in classical physics.

9. Other approaches

Many different solutions have been proposed in order to solve the problem of quantum measurements. No solution (not even those sketched above) is, in our opinion, completely satisfactory from a philosophical point of view. We shall not try to give an exhaustive list here. Besides the environment theories mentioned earlier, there are other noteworthy approaches [Jauch 1964, Emch 1972, Yaffe 1982, Kakazu and Matsumoto 1990] that investigate the existence and features of 'classical' properties for some macroscopic apparatus: the possible observables of a macrosystem are restricted to a commutable set, so that for such a 'classical' system the physical states fall into equivalence classes which cannot be distinguished by any observation performed on the system. In particular, pure and mixed states turn out, under appropriate conditions, to belong to the same equivalence class, and are therefore 'equivalent' from a physical point of view.

This obviously 'solves' the measurement problem, because the states (11)-(12) and (38) belong to the same equivalence class, and are therefore physically indistinguishable. It must be noted, however, that the conceptual difficulties inherent to quantum measurements are not entirely resolved. The measurement problem is simply recast in a different form, being shifted to a different level of description. Nevertheless, the formal approach is very elegant and shows that it is possible to define observables for macroscopic systems that approximate a classical behaviour in some limit.

Diósi [Diósi 1988], Belavkin [Belavkin 1989] and many other authors [Gisin and Percival 1992, 1993, Carmichael 1999] have focussed their attention on the derivation of a master equation [Gorini, Kossakowski and Sudarshan 1976, Lindblad 1976, Alicki and Lendi 1987]. These studies are interesting and noteworthy. In this case, the system investigated is shown to undergo a stochastic process (due to the interaction with its environment) that makes its dynamics nonunitary. A good account on the emergence of classical features in this context can be found in [Sewell 2002].

The many-worlds interpretation [Everett III 1957, DeWitt and Graham 1973] outlines a theory of measurement within the framework of (11), by making use of the orthogonal

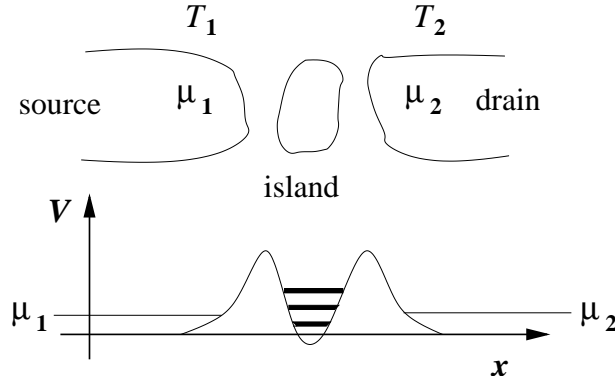


Figure 1: A quantum dot. $T_{1,2}$ are temperatures and $\mu_{1,2}$ chemical potentials. Below we showed the potential as a function of a longitudinal coordinate x parallel to the current through the dot. We also drew the metastable discrete levels of the island.

decomposition (15). In this interpretation each branch state, specified by a different measurement outcome, together with the measuring apparatus, branches into its own world. We will neither comment on this idea, nor on other approaches that propose a modification of the dynamical laws of quantum mechanics.

10. Mesoscopic systems: quantum dots and fullerene molecules

Issues like quantum measurement and (to some extent) irreversibility were considered rather academic until a few decades ago. This prejudice has disappeared during the last few years, mainly because of the advent of new technologies and the possibility of performing experiments that were unrealizable until a few years ago. It is impossible to give an exhaustive list of the most recent experimental results in this area, so we will just give two examples, both in the mesoscopic (namely neither macroscopic nor microscopic) domain.

One of the most interesting and intriguing systems displaying a quantal behavior in spite of its (almost) macroscopic size is the quantum dot [Landauer 1957, 1981, Buttiker *et al* 1985]. A quantum dot is a small electron droplet (to be called ‘island’) isolated from its two leads (which we will call ‘source’ and ‘drain’) by tunnelling barriers. Island, source and drain are all electrodes, made of metallic material (which, due to the low temperature necessary to successfully operate the device, is often in a superconducting state). See Figure 1. Source and drain are characterized by a temperature T and a chemical potential μ . These are typical macroscopic (thermodynamical) quantities. On the other hand, due to electron confinement in the (small) isolated region, the quantum dot has discrete metastable energy levels, that are able to influence the behavior and properties of the wave functions of single electrons in the island. The interest of the quantum dots lies in the unique features of the electric current flowing from the source to the drain through the island. Due to the quantum features of the dot, such a current depends critically on the metastable level structure of the island, as well as on genuine macroscopic parameters like μ and T . The device can be operated in different conditions (ballistic regime, Coulomb blockade) and is endowed with very interesting mesoscopic properties. Decoherence sets

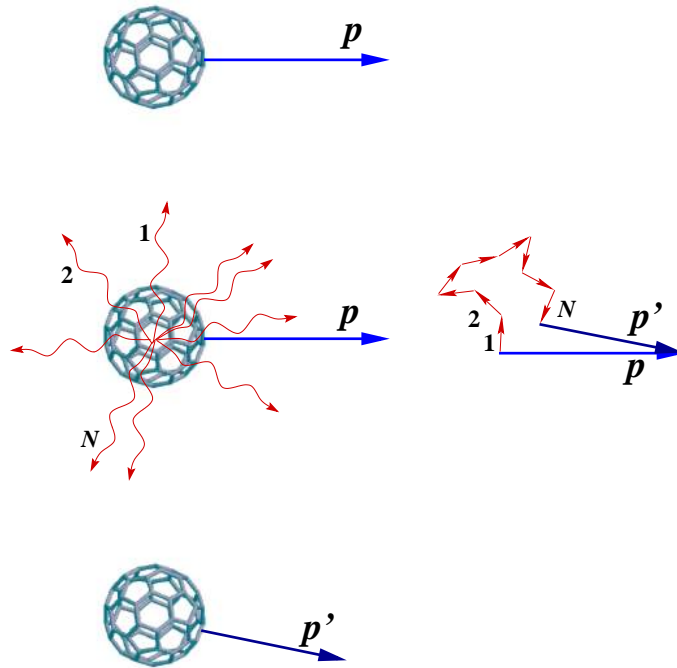


Figure 2: A molecule of fullerene emitting thermal blackbody-like radiation during its travel toward a screen in a double-slit experiment. N (thermal) photons are emitted and the momentum of the molecule changes accordingly. In some cases, one can reasonably assume that the molecule makes a random walk in momentum space. (From [Facchi, Mariano and Pascazio 2002].)

physical bounds on its operation and is, in this context, a hot topic of investigation. The present experimental applications of quantum dots are many and diverse.

Other systems of great interest are the fullerenes. These are Carbon molecular structures whose spatial shape is roughly spherical. The quantum (double-slit) interference of the fullerene molecules C_{60} and C_{70} has been very recently observed in a series of pioneering experiments performed in Vienna [Arndt *et al* 1999]. The double-slit experiment is probably the simplest, most familiar and most fundamental quantum mechanical example: it has been carried out with many elementary particles, such as photons [Mandel and Wolf 1995], neutrons [Rauch and Werner 2000], electrons [Tonomura 1994], atoms [Carnal and Mlynek 1991] and small molecules [Schöllkopf and Toennies 1994, Chapman *et al* 1995]. All these systems are ‘elementary,’ in the sense that they are made up of one or at most a small number of sub-constituents and *can be described by a wave function*. Also, their internal structure is (supposed to be) elementary (although it is already rather complicated for small molecules).

By contrast with the above mentioned particles, a molecule of fullerene such as C_{60} is made up of 60 nuclei of Carbon and 360 electrons, for a total of about 10^3 constituents. Although fullerenes are fully quantum mechanical systems, they also have macroscopic-like features and emit thermal (blackbody) radiation. See Figure 2.

A summary of the features of fullerenes and the physical phenomena that characterize

them is given in Table 1. We have divided them into micro, meso and macroscopic properties. However, it is necessary to stress that the boundary between these regimes is fuzzy. While everybody (we expect) would agree on the micro and macroscopic list, the ‘mesoscopic’ properties are subtler and their classification somewhat dependent on individual taste.

Among the microscopic features we notice the existence of a wave function (for the motion of the center of mass), a microscopic size, a de Broglie wavelength and the occurrence of very elementary processes. Among the mesoscopic ones there are the number of elementary constituents, high symmetry and structural stability (leading to decoupling from the environment and making possible double- and multiple-slit interference experiments), distribution of roto-vibrational modes, polarizability and van der Waals interactions with the slits of the diffraction grating and important infrared (IR) fluorescence spectra. Finally, among the macroscopic features we emphasize multiphoton absorption processes, black-body emission spectra, surface curvature effects, a quick and remarkable mixing of states, an internal temperature (!) and bulk properties. *All* these features have been tested and brought to light in different physical experiments.

The analysis of the interference experiments for large molecules brings to light new and unexpected phenomena [Grisenti, Schöllkopf, Toennies, Hegerfeldt and Köhler 1999, Facchi, Mariano and Pascazio 2002] in which the *quantitative* estimates of decoherence effects play a primary role.

Table 1: Properties of fullerene molecules.

microscopic	wave function of the center of mass size (radius) $r_0 = 3.5\text{\AA}$ de Broglie wavelength $\lambda_{DB} = 10^{-12}\text{m}$ fragmentation $\text{C}_{60} \rightarrow \text{C}_{58} + \text{C}_2$ ionization $\text{C}_{60} \rightarrow \text{C}_{60}^+ + e^-$
mesoscopic	10^3 constituents high symmetry and structural stability leading to decoupling from environment distribution of rotational and vibrational modes polarizability van der Waals interactions with slits of grating in a diffraction experiment IR fluorescence
macroscopic	multiphoton absorption blackbody-like thermal emission Mie’s theory (curvature effects in photon emission) emissivity α statistical mixing of states/modes temperature bulk properties: optical bands, excitons, plasmons

11. *Measurements, decoherence and irreversibility*

Although it is easy to define the quantum mechanical *coherence*, which is intuitively related to the possibility of creating a superposition in a Hilbert space, it is not obvious what the *lack* of quantum coherence is. Most physicists would say that such a lack of coherence can be given a meaning only in a statistical sense (but there are noteworthy historical exceptions: for a survey, see [Namiki, Pascazio and Nakazato 1997]).

As we have seen, the notion of decoherence is intimately related to those of measurement and irreversibility. One can speak of ‘measurement’ in quantum mechanics when the off-diagonal terms of the density matrix disappear. This is an irreversible process, in a statistical sense, and can be obtained either by tracing away the states of the environment/detector or by computing an average of some sort on the microscopic unobserved freedoms of the environment/detector. We have looked in detail at both procedures, emphasizing their differences and conceptual limits. Does this solve the quantum measurement problem? Does it provide any clues into the long-standing issue of irreversibility? Not really. We believe that Von Neumann and Wigner (as well as other founder fathers of quantum theory), when writing Eq. (6), were referring to the occurrence and/or outcomes of *single* quantum mechanical events. This problem has no solution in the present framework. Nobody can predict in which state an individual quantum system will collapse. Many people may claim to have a consistent interpretation for single quantum events (or the accumulation thereof), but interpretations depend on personal taste, *predictions* do not. One can even advocate that interpretations are unnecessary in quantum mechanics [Fuchs and Peres 2000]. For this reason, we feel that it is much more sensible (and humble) to regard the present status of the research on the topics discussed in this article on the same footing as phenomenology and engineering. Both phenomenology and engineering are noble sciences, based on experiments, data interpretation, predictions and humbleness, and always hinge upon simple and neat principles.

For an elementary quantum system it is not too difficult to set bounds on coherence. However, if the picture becomes more complicated and the (microscopic or mesoscopic) system interacts with its environment, ‘exploring’ it (e.g. by emitting photons in the course of its evolution), its branch waves will gradually ‘decohere,’ hindering *de facto* the possibility of conceiving a coherent superposition in the Hilbert space. In this sense, coherence (viewed as ability to interfere) simply means isolation from the environment—a lesson that experimental physicists know much better (and since much longer) than their theoretical colleagues, who do not (and never had to) care about ‘isolating’ a beam in order to make it interfere.

We are left with an interesting philosophical question. Once we have more or less understood what decoherence is and how it is related to irreversibility, we may go back to the starting point (Sec. 2) and ask: when is a quantum state ‘pure’? When can a system be described by a wave function that obeys the Schrödinger equation? For example, one wonders whether it is entirely obvious that a mesoscopic system such as a quantum dot or a molecule of fullerene can be described by a single wave function that obeys a Schrödinger equation. The answer to these questions must be, in our opinion, a *quantitative* one, in order to be considered satisfactory by phenomenologists and engineers.

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