Proc. Waseda Int. Sympo. on Fundamental Physics —New Perspectives in Quantum Physics—
J. Phys. Soc. Jpn. Vol. 72 (2003) Suppl. C pp. 30–33
©2003 The Physical Society of Japan

Quantum Zeno Subspaces and Decoherence

Paolo Facchi and Saverio Pascazio

Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, I-70126 Bari, Italy

(Received February 3, 2003)

A profound consequence of the quantum Zeno effect is the partitioning of the total Hilbert space into quantum Zeno subspaces, among which any transition is hindered. Such a phenomenon can be due to frequent nonselective measurements, frequent unitary kicks or strong continuous coupling. Its practical relevance for halting decoherence is discussed and the problem of timescales is considered.

KEYWORDS: quantum Zeno subspaces, decoherence, inverse quantum Zeno effect

1. Introduction

The quantum Zeno effect[1,2] is usually ascribed to repeated projections[3] made on a quantum system. However, it is a more general phenomenon, that can be readily understood in terms of the dynamical time evolution of quantum systems.[4] For instance, a spontaneous emission process can often be viewed as a very effective measurement, for it is irreversible and leads to an entanglement of the state of the system (the emitting atom or molecule) with the state of the apparatus (the electromagnetic field); at the same time, it is a *bona fide* dynamical process, that can be described by the Schrödinger equation. For these reasons, the main physical features of the Zeno effect would still be apparent if one would formulate the measurement process in more realistic terms, introducing a physical apparatus, a Hamiltonian and a suitable interaction with the measured system, with no explicit use of projection operators.[5,6]

More to this, once one has realized that the quantum Zeno effect (QZE) is a mere consequence of the dynamics, it turns out that neither a *bona fide* detection scheme, nor irreversibility are necessary requisites: in fact, *any* interactions—and not only those that can be considered as measurement processes of some sort—that considerably affect the system (in a sense to be made more precise in the following) provoke QZE. Therefore, the QZE appears in a much broader context than its original formulation, whenever a strong disturbance dominates the time evolution of the quantum system.[4]

The aim of this article is to discuss a profound consequence of the QZE, of great practical relevance, as it may "protect" a system from decoherence. We show that, whenever a disturbance dominates the time evolution, the system is forced to evolve in a set of orthogonal ("Zeno") subspaces of the total Hilbert space. This phenomenon will be discussed in Sec. 3 for the case of nonselective (projective) measurements, then extended in Sec. 4 to the case of unitary kicks[7] and finally in Sec. 5 to (unitary) continuous interactions.[8] The quantum Zeno subspaces are completely determined by the disturbance: they are nothing but its invariant subspaces; in the latter two cases (unitary kicks and continuous coupling) they are the eigenspaces of the interaction. An example is considered in Sec. 6, in order to elucidate the key issue of the relevant timescales.

2. The Quantum Zeno Effect

Let us outline Misra and Sudarshan's formulation[1] of the QZE. Let Q be a quantum system, whose states belong to the Hilbert space $\mathcal{H}$ and whose evolution is described by the superoperator

$$\hat{U}_t\rho = U(t)\rho U^\dagger(t), \qquad U(t) = \exp(-iHt), \tag{1}$$

where ρ is the density matrix of the system and H a time-independent self-adjoint Hamiltonian. Let P be a projection operator onto a proper subspace $\mathcal{H}_P = P\mathcal{H}$ of $\mathcal{H}$. We prepare Q in the initial state $\rho_0 = P\rho_0 P$ at time 0 and perform a series of P-observations at time intervals $\tau = t/N$, in order to ascertain whether the system is still in the subspace $\mathcal{H}_P$. If Q is found in $\mathcal{H}_P$ at every step, its state at time t reads

$$\rho^{(N)}(t) = \frac{V_N(t)\rho_0 V_N^\dagger(t)}{p^{(N)}(t)}, \quad V_N(t) = \left[PU\left(\frac{t}{N}\right)\right]^N, \tag{2}$$

where

$$p^{(N)}(t) = \mathrm{Tr}\left[V_N(t)\rho_0 V_N^\dagger(t)\right] \tag{3}$$

is the survival probability in $\mathcal{H}_P$. Equations (2)–(3) are the formal statement of the QZE, according to which very frequent observations modify the dynamics of the quantum system: under general conditions, if N is sufficiently large, all transitions outside $\mathcal{H}_P$ are inhibited. [Notice that the dynamics (2)–(3) is *not reversible*.] Consider, indeed, the $N \to \infty$ limit ("continuous observation") and assume, for simplicity, that the Hamiltonian is bounded, i.e. $\|H\| < \infty$. Then, it is easy to evaluate the limit

$$\mathcal{V}(t) \equiv \lim_{N\to\infty} V_N(t) = P\exp(-iPHPt)\,. \tag{4}$$

Equation (4) is also valid for an unbounded Hamiltonian, if $\mathcal{H}_P \subset D(H)$. Then the resulting Hamiltonian PHP is self-adjoint and $\mathcal{V}(t)$ is unitary in $\mathcal{H}_P$. When the above condition does not hold, one can always formally write the limiting evolution for a lower bounded Hamiltonian in the form (4), but has to define the meaning of PHP and study its self-adjointness.[9,10]

In conclusion, the final state of Q is

$$\rho(t) = \lim_{N\to\infty} \rho^{(N)}(t) = \mathcal{V}(t)\rho_0\mathcal{V}^\dagger(t) \tag{5}$$

and the probability to find the system in $\mathcal{H}_P$ is

$$p(t) = \lim_{N\to\infty} p^{(N)}(t) = \mathrm{Tr}\left[\mathcal{V}(t)\rho_0\mathcal{V}^\dagger(t)\right] = \mathrm{Tr}\left[\rho_0 P\right] = 1, \tag{6}$$

where we made use of the property $\mathcal{V}^\dagger(t)\mathcal{V}(t) = P$. If the particle is "continuously" observed, in order to check whether it has survived inside $\mathcal{H}_P$, it will never make a transition to $\mathcal{H}_P^\perp$ (QZE).

Two important remarks are now in order: first, notice that the system *does not* remain in its initial state, after the series of very frequent measurements, but evolves in the subspace $\mathcal{H}_P$. Second, the limiting *Zeno dynamics* within $\mathcal{H}_P$ is given by the unitary group (4) and is engendered by the new Hamiltonian PHP, that replaces the original one H. Therefore, starting from the dynamics (2), which is irreversible and probability-nonconserving, one ends up with a fully unitary evolution. Reversibility is recovered in the limit.

3. The Quantum Zeno Subspaces

The extension[8)] of Misra and Sudarshan's theorem to nonselective measurements is straightforward and leads to the appearance of the quantum Zeno subspaces. The measurements are "nonselective" when the measuring apparatus does not select the different outcomes, but only destroys the phase correlations between some states, provoking the transition from a pure state to a mixture.[11)] Let

$$\{P_n\}_n, \qquad P_nP_m = \delta_{mn}P_n, \qquad \sum_n P_n = \mathbb{1} \tag{7}$$

be an orthogonal resolution of the identity and $\mathcal{H}_n = P_n\mathcal{H}$ the relative subspaces, so that $\mathcal{H} = \bigoplus_n \mathcal{H}_n$. The associated nonselective measurement is described by the superoperator

$$\hat{P}\rho = \sum_n P_n\rho P_n \tag{8}$$

and the evolution after N measurements in a time t is governed by the superoperator

$$\hat{V}_t^{(N)} = \left(\hat{P}\hat{U}_{t/N}\right)^N. \tag{9}$$

We perform a first, preparatory measurement, so that the initial state is

$$\hat{P}\rho_0 = \sum_n P_n\rho_0 P_n. \tag{10}$$

By assuming again that $\|H\| < \infty$ and by noting that for $n_1 \neq n_k$

$$\lim_{N\to\infty} P_{n_N}U\left(\frac{t}{N}\right)\cdots P_{n_k}\cdots U\left(\frac{t}{N}\right)P_{n_1}U\left(\frac{t}{N}\right) = 0, \tag{11}$$

on can show that the limiting superoperator

$$\hat{\mathcal{V}}_t \equiv \lim_{N\to\infty} \hat{V}_t^{(N)}, \tag{12}$$

reads

$$\rho(t) = \hat{\mathcal{V}}_t\rho_0 = \sum_n \mathcal{V}_n(t)\rho_0\mathcal{V}_n^\dagger(t), \tag{13}$$

where

$$\mathcal{V}_n(t) = \lim_{N\to\infty}\left[P_nU(t/N)P_n\right]^N = P_n\exp(-iP_nHP_nt) \tag{14}$$

are unitary within the subspaces $\mathcal{H}_n$.

The components $\mathcal{V}_n(t)\rho_0\mathcal{V}_n^\dagger(t)$ make up a block diagonal matrix: the initial density matrix is reduced to a mixture and any interference between different subspaces $\mathcal{H}_n$ is destroyed. Moreover,

$$p_n(t) = \mathrm{Tr}\left[\rho(t)P_n\right] = \mathrm{Tr}\left[\rho_0P_n\right] = p_n(0), \quad \forall n, \tag{15}$$

where we made use of

$$\sum_n \mathcal{V}_n^\dagger(t)\mathcal{V}_n(t) = \sum_n P_n = \mathbb{1}. \tag{16}$$

In words, probability is conserved in each subspace and no probability "leakage" between any two subspaces is possible: the total Hilbert space splits into invariant *quantum Zeno subspaces* $\mathcal{H}_n$ and the different components of the density matrix evolve independently within each sector. Misra and Sudarshan's seminal result (6) is reobtained when $p_n(0) = 1$ for some n, in (15): the initial state is then in one of the invariant subspaces and the survival probability in that subspace remains unity.

For a generic unbounded Hamiltonian, with the necessary precautions on the meaning of operators and boundary conditions, the Zeno evolution (14) can be written

$$\mathcal{V}_n(t) = P_n\exp(-iH_Zt), \tag{17}$$

where

$$H_Z = \hat{P}H = \sum_n P_nHP_n \tag{18}$$

is the *Zeno Hamiltonian.*

4. Unitary Kicks

The formulation of the preceding section hinges upon projections *à la* von Neumann. Projections are instantaneous processes, yielding the collapse of the wave function (an ultimately nonunitary process). However, one can obtain the QZE without making use of nonunitary evolutions,[12)] by exploiting Wigner's idea of spectral decomposition.[13)] In this section we further elaborate on this issue, obtaining the QZE by means of a *generic* sequence of frequent *instantaneous unitary* processes, that need not be spectral decompositions. We will only give the main results, as additional details and a complete proof, which is related to von Neumann's ergodic theorem,[14)] can be found elsewhere.[7)]

Consider the dynamics of a quantum system Q undergoing N "kicks" U_{kick} (instantaneous unitary transformations) in a time interval t

$$U_N(t) = \left[U_{\mathrm{kick}}U\left(\frac{t}{N}\right)\right]^N. \tag{19}$$

In the large N limit, since the dominant contribution is $U_{\rm kick}^N$, one considers the sequence of unitary operators

$$V_N(t) = U_{\rm kick}^{\dagger N} U_N(t) = U_{\rm kick}^{\dagger N}\left[U_{\rm kick} U\left(\frac{t}{N}\right)\right]^N \tag{20}$$

and its limit

$$\mathcal{U}(t) \equiv \lim_{N\to\infty} V_N(t). \tag{21}$$

One can show that

$$\mathcal{U}(t) = \exp(-iH_{\rm Z}t), \tag{22}$$

where the Zeno Hamiltonian $H_{\rm Z}$ is formally given by (18), P_n being now the spectral projections of $U_{\rm kick}$

$$U_{\rm kick} = \sum_n e^{-i\lambda_n} P_n. \quad (e^{-i\lambda_n} \neq e^{-i\lambda_l}, \text{ for } n \neq l). \tag{23}$$

In conclusion

$$\begin{aligned} U_N(t) &\sim U_{\rm kick}^N \mathcal{U}(t) = U_{\rm kick}^N \exp(-iH_{\rm Z}t) \\ &= \exp\left(-i\sum_n N\lambda_n P_n + P_n H P_n t\right). \end{aligned} \tag{24}$$

The unitary evolution (19) yields therefore a Zeno effect and a partition of the Hilbert space into Zeno subspaces, like in the case of repeated projective measurements discussed in Sec. 3. The appearance of the Zeno subspaces is a direct consequence of the wildly oscillating phases between different eigenspaces of the kick.

It is superfluous to stress the analogy of the approach outlined in this section with the seminal papers on quantum maps and quantum chaos.[15]

5. Continuous Coupling

The formulation of the preceding sections hinges upon instantaneous processes, that can be unitary or nonunitary. However, as explained in the Introduction, the basic features of the QZE can be obtained by making use of a continuous coupling, when the external system takes a sort of steady "gaze" at the system of interest. The mathematical formulation of this idea is contained in a theorem[8,16] on the (large-K) dynamical evolution governed by a *generic* Hamiltonian of the type

$$H_K = H + KH_{\rm c}, \tag{25}$$

which again need not describe a *bona fide* measurement process. H is the Hamiltonian of the quantum system investigated and $H_{\rm c}$ can be viewed as an "additional" interaction Hamiltonian performing the "measurement." K is a coupling constant.

Consider the time evolution operator

$$U_K(t) = \exp(-iH_K t). \tag{26}$$

In the $K \to \infty$ limit, the dominant contribution is $\exp(-iKH_{\rm c}t)$. One therefore considers the limiting evolution operator

$$\mathcal{U}(t) = \lim_{K\to\infty} \exp(iKH_{\rm c}t)\, U_K(t), \tag{27}$$

that can be shown to have the form

$$\mathcal{U}(t) = \exp(-iH_{\rm Z}t), \tag{28}$$

where the Zeno Hamiltonian $H_{\rm Z}$ is again formally given by (18), P_n being now the eigenprojection of $H_{\rm c}$ belonging to the eigenvalue η_n

$$H_{\rm c} = \sum_n \eta_n P_n, \qquad (\eta_n \neq \eta_m, \quad \text{for } n \neq m)\ . \tag{29}$$

In conclusion, the limiting evolution operator is

$$\begin{aligned} U_K(t) &\sim \exp(-iKH_{\rm c}t)\,\mathcal{U}(t) \\ &= \exp\left(-i\sum_n Kt\eta_n P_n + P_n H P_n t\right), \end{aligned} \tag{30}$$

whose block-diagonal structure is explicit. Compare with (24). The above statements can be proved by making use of the adiabatic theorem.[17] It is also worth emphasizing interesting links with the quantum evolution in the strong coupling limit.[18]

The idea of formulating the Zeno effect in terms of a "continuous coupling" to an external apparatus has often appeared in the literature of the last two decades.[5,19,20] However, the first quantitative estimate of the link with the formulation in terms of projective measurements is rather recent.[4,6]

6. Decoherence and Timescales

One of the main potential applications of the quantum Zeno subspaces concerns the possibility of "freezing" decoherence and probability leakage due to the interaction of the system of interest (e.g., a qubit in quantum computation) with its environment. Let us look at an elementary example. Consider a qubit

$$\mathcal{H}_0 = \{|a\rangle, |b\rangle\} \tag{31}$$

and let one of its states, say $|b\rangle$, interact with a reservoir, yielding decoherence. The effective Hamiltonian[16] ($\Lambda > 0$)

$$H_{\rm dec} = \lambda\left(|b\rangle\langle c| + |c\rangle\langle b|\right) + (\omega_c - i\Lambda)|c\rangle\langle c| \tag{32}$$

describes the spontaneous emission of state $|b\rangle$ into a structured (i.e., non-flat) continuum (represented by state $|c\rangle$) with spectral density[4]

$$\kappa(\omega) = \frac{\lambda^2}{\pi}\frac{\Lambda}{(\omega-\omega_c)^2+\Lambda^2}. \tag{33}$$

We have chosen this Hamiltonian because it is rich enough to yield information about both the Zeno and inverse Zeno regimes. We will only consider the small coupling situation $\lambda \ll \omega_c, \Lambda$. The decay rate of state $|b\rangle$ reads

$$\gamma = 2\pi\kappa(0) = 2\lambda^2\frac{\Lambda}{\omega_c^2+\Lambda^2}, \tag{34}$$

while $\tau_{\rm Z} = \left[\int d\omega\, \kappa(\omega)\right]^{-\frac{1}{2}} = \lambda^{-1}$ is the Zeno time,[21] that is the convexity of the initial quadratic region.

This example is relevant for quantum computation, if one aims at protecting a given subspace ($\mathcal{H}_0$) from decoherence, by inhibiting spontaneous emission. We consider only the scheme analyzed in Sec. 5 and look at the effects of a suitable continuous coupling H_c one eigenspace of which is $\mathcal{H}_0$ (Zeno subspace). This can be achieved by resonantly coupling state $|c\rangle$ to a fourth level $|M\rangle$ with the effective Hamiltonian[4)]

$$\begin{aligned} H_K &= H_{\rm dec} + H_M + K H_{\rm c}, \\ H_M &= (\omega_c - i\Lambda)|M\rangle\langle M|, \quad H_{\rm c} = |c\rangle\langle M| + |M\rangle\langle c|. \end{aligned} \tag{35}$$

(A somewhat related example, in terms of pulsed kicks, has been recently considered.[22)]) It turns out that $\mathcal{H}_0 = P_0\mathcal{H}$ is the eigenspace of H_c belonging to the eigenvalue $\eta = 0$. Moreover, $P_0 H_{\rm dec} P_0 = 0$, hence $\mathcal{H}_0$ is protected from decoherence when $K \to \infty$. In fact, the modified decay rate of $|b\rangle$ can be shown to be[4, 20)]

$$\gamma_{\rm eff}(K) = \pi\,[\kappa(K) + \kappa(-K)] = \gamma \frac{\kappa(K) + \kappa(-K)}{2\kappa(0)} \tag{36}$$

and when the Rabi frequency K is sufficiently large, the spontaneous emission from level $|b\rangle$ is hindered like

$$\gamma_{\rm eff}(K) \sim \lambda^2 \frac{\Lambda}{K^2}, \quad K \to \infty. \tag{37}$$

However, these conclusions are valid for *very large* couplings: in order to get an effective protection of level $|b\rangle$, one needs

$$K \gtrsim \omega_c, \Lambda, \tag{38}$$

that is a coupling of the order of the bandwidth. More to this, it is easy to see that, if $|\omega_c| > \Lambda/\sqrt{3}$, an inverse quantum Zeno effect[23, 24)] takes place and the requirement for obtaining the QZE becomes even more stringent.[24)] Indeed, under the above condition, $\kappa(\omega)$ is a convex function for $\omega \simeq 0$, that is $\kappa(K)+\kappa(-K) \geq 2\kappa(0)$, hence $\gamma_{\rm eff}(K) \geq \gamma$. Therefore, by increasing K one does not obtain the desired effect, but its opposite: decoherence is *enhanced* rather than hindered. In such a case, in order to get a QZE, a coupling larger than the bandwidth is not sufficient: one must require also

$$K > \sqrt{3\omega_c^2 - \Lambda^2}. \tag{39}$$

Couplings are inversely proportional to timescales. For this reason, the real problem is that of the relevant timescales yielding QZE (rather than inverse QZE). Conditions (38)–(39) can be very demanding for a real system subject to dissipation[21, 24)] and the model (32) yields then nice insight into some recent examples.[25)] We will elaborate on this point in a future paper, in view of possible applications.

1) B. Misra and E. C. G. Sudarshan: J. Math. Phys. **18** (1977) 756.
2) A. Beskow and J. Nilsson: Arkiv für Fysik **34** (1967) 561; L. A. Khalfin: JETP Letters **8** (1968) 65.
3) J. von Neumann: *Mathematical Foundation of Quantum Mechanics* (Princeton University Press, Princeton, 1955); G. Lüders: Ann. Phys. (Leipzig) **8** (1951) 322.
4) P. Facchi and S. Pascazio: Progress in Optics, ed. E. Wolf (Elsevier, Amsterdam, 2001) Vol. **42**, Ch. 3, p. 147.
5) A. Peres: Am. J. Phys. **48** (1980) 931; K. Kraus: Found. Phys. **11** (1981) 547; A. Sudbery: Ann. Phys. **157** (1984) 512.
6) L. S. Schulman: Phys. Rev. A **57** (1998) 1509.
7) P. Facchi, D. A. Lidar and S. Pascazio: quant-ph/0303132 (2003).
8) P. Facchi and S. Pascazio: Phys. Rev. Lett. **89** (2002) 080401.
9) C. N. Friedman: Indiana Univ. Math. J. **21** (1972) 1001; K. Gustafson: quant-ph/0203032 (2002).
10) P. Facchi, V. Gorini, G. Marmo, S. Pascazio and E. C. G. Sudarshan: Phys. Lett. A **275** (2000) 12; P. Facchi, S. Pascazio, A. Scardicchio and L. S. Schulman: Phys. Rev. A **65** (2002) 012108.
11) J. Schwinger: Proc. Nat. Acad. Sc. **45** (1959) 1552.
12) S. Pascazio and M. Namiki: Phys. Rev. A **50** (1994) 4582; T. Petrosky, S. Tasaki and I. Prigogine: Phys. Lett. A **151** (1990) 109; Physica A **170** (1991) 306.
13) E. P. Wigner: Am. J. Phys. **31** (1963) 6.
14) M. Reed and B. Simon: *Functional Analysis I* (Academic Press, San Diego, 1980) p. 57.
15) G. Casati, B. V. Chirikov, J. Ford and F. M. Izrailev: in *Stochastic Behaviour in Classical and Quantum Hamiltonian Systems*, ed. G. Casati and J. Ford, Lecture Notes in Physics (Springer-Verlag, Berlin) **93** (1979) 334; M. V. Berry, N. L. Balazs, M. Tabor and A. Voros: Ann. Phys. **122** (1979) 26. In this context, see M. V. Berry: Ann. N. Y. Acad. Sci. **755** (1995) 303; B. Kaulakys and V. Gontis: Phys. Rev. A **56** (1997) 1131; P. Facchi, S. Pascazio and A. Scardicchio: Phys. Rev. Lett. **83** (1999) 61; J. C. Flores: Phys. Rev. B **60** (1999) 30; **62** (2000) 16291(R); S. A. Gurvitz: Phys. Rev. Lett. **85** (2000) 812; J. Gong and P. Brumer: Phys. Rev. Lett. **86** (2001) 1741; A. Luis: J. Opt. B **3** (2001) 238.
16) P. Facchi: quant-ph/0202174 (2002); P. Facchi and S. Pascazio: quant-ph/0202127 (2002).
17) J. E. Avron and A. Elgart: Commun. Math. Phys. **203** (1999) 445, and references therein.
18) M. Frasca: Phys. Rev. A **58** (1998) 3439.
19) R. A. Harris and L. Stodolsky: Phys. Lett. B **116** (1982) 464; A. Venugopalan and R. Ghosh: Phys. Lett. A **204** (1995) 11; M. P. Plenio, P. L. Knight and R. C. Thompson: Opt. Comm. **123** (1996) 278; M. V. Berry and S. Klein: J. Mod. Opt. **43** (1996) 165; A. Luis and L. L. Sánchez-Soto: Phys. Rev. A **57** (1998) 781; K. Thun and J. Peřina: Phys. Lett. A **249** (1998) 363; A. D. Panov: Phys. Lett. A **260** (1999) 441; J. Řeháček, J. Peřina, P. Facchi, S. Pascazio and L. Mišta: Phys. Rev. A **62** (2000) 013804; P. Facchi and S. Pascazio: Phys. Rev. A **62** (2000) 023804; B. Militello, A. Messina and A. Napoli: Phys. Lett. A **286** (2001) 369; A. Luis: Phys. Rev. A **64** (2001) 032104.
20) E. Mihokova, S. Pascazio and L. S. Schulman: Phys. Rev. A **56** (1997) 25.
21) P. Facchi and S. Pascazio: Phys. Lett. A **241** (1998) 139.
22) G. S. Agarwal, M. O. Scully and H. Walther: Phys. Rev. Lett. **86** (2001) 4271.
23) A. M. Lane: Phys. Lett. A **99** (1983) 359; W. C. Schieve, L. P. Horwitz and J. Levitan: Phys. Lett. A **136** (1989) 264; B. Elattari and S. A. Gurvitz: Phys. Rev. A **62** (2000) 032102; A. G. Kofman and G. Kurizki: Nature **405** (2000) 546.
24) P. Facchi, H. Nakazato and S. Pascazio: Phys. Rev. Lett. **86** (2001) 2699.
25) G. M. Palma, K. A. Suominen and A. K. Ekert: Proc. Roy. Soc. Lond. A **452** (1996) 567; L. M. Duan and G. C. Guo: Phys. Rev. Lett. **79** (1997) 1953; P. Zanardi and M. Rasetti: Phys. Rev. Lett. **79** (1997) 3306; Mod. Phys. Lett. B **11** (1997) 1085; D. A. Lidar, I. L. Chuang and K. B. Whaley: Phys. Rev. Lett. **81** (1998) 2594; L. Viola, E. Knill and S. Lloyd: Phys. Rev. Lett. **82** (1999) 2417; A. Beige, D. Braun, B. Tregenna and P. L. Knight: Phys. Rev. Lett. **85** (2000) 1762.