Effective string action from Lorentz invariance of confining gauge theories

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The effective string picture of the confining flux tube

- Long color flux tubes joining quark sources in the rough phase of any confining gauge theory behave as string-like objects

  Lüscher Symazik & Weisz 1980

- They are described by the transverse coordinates $X^i(\xi_0, \xi_1)$ $(i = 1, D - 2;\ \ 0 \le \xi_1 \le R\ ;\ 0 \le \xi_0 \le L)$ which are the collective modes describing their position
- the X^i's can be seen as the Nambu-Goldstone modes of the spontaneously broken translation invariance in the transverse directions
- in a general confining vacuum we do not expect other massless fields

- The confining string representation of the Polyakov loop correlation function is given by the functional integral

$$\langle P(0)\, P^{\star}(R)\rangle_{T=1/L} = \int \prod_{i=1}^{D-2} \mathcal{D}X^i \, e^{-S[X^i]}$$

- This is only expected to be valid to any finite order of the perturbation expansion in the parameter $1/(\sigma RL)$
- Decays of highly excited states through glueball radiation are not included in this description
- The Polyakov loop correlator and the corresponding string partition function differ by non-perturbative corrections of the order e^{-mL} (m= mass of the lightest glueball)

What do we know about the effective string action?

- Free string limit:
  $S[X^i] = \sigma\, RL + \sigma \int d^2\xi \, \frac{1}{2}(\partial_\alpha X \cdot \partial^\alpha X) + \dots$
- Two main consequences
  1. *Lüscher term,* in the confining, static interquark potential

$$V(r) = \sigma\, r + \mu - \frac{\pi}{24}\frac{D-2}{r}$$

  2. *Quantum broadening of the flux tube:* the mean area w^2 of its cross-section grows logarithmically with the interquark distance R

$$w^2 = \frac{1}{2\pi\sigma}\log(R\,\Lambda)$$

- The Lüscher term is simply the Casimir, or zero point energy E_o of a string of length r with fixed ends:
- ➩ normal modes: $\frac{\pi\, n}{r}, \quad n = 1, 2, \dots$
- ➩ $E_o = (D-2) \sum_n \frac{\pi\, n}{2r} = (D-2)\, \frac{\pi}{2}\, \zeta(-1) = -\frac{\pi}{24} \frac{D-2}{r}$
- ❋ First uncontroversial observations in the 90's in 3D $\mathbb{Z}_2$ gauge theory
- ❋ Very challenging in non-Abelian gauge theories. Reliable results using the exponential variance reduction algorithm Lüscher and Weisz 2001
- ❋ Quantum broadening of the flux tube in SU(2) 3D gauge theory observed only recently FG, M.Pepe and .U-J Wiese 2010 :

What else do we know about the effective action?

- assuming translation invariance $X^i \to X^i + a^i$, the most general expression of the effective string action has the following expansion in derivatives of X^i, up to field redefinitions,

$$S[X^i] = \sigma\, RL + \sigma \int d^2\xi \Big[\tfrac{1}{2}(\partial_\alpha X \cdot \partial^\alpha X) \quad \text{free string limit}$$
$$+ c_2(\partial_\alpha X \cdot \partial^\alpha X)^2 + c_3(\partial_\alpha X \cdot \partial_\beta X)(\partial^\alpha X \cdot \partial^\beta X) \quad \text{first non-Gaussian correction}$$
$$+ c_4(\partial_\alpha X \cdot \partial^\alpha X)^3 + c_5(\partial_\alpha X \cdot \partial_\beta X)^2(\partial_\gamma X \cdot \partial^\gamma X) \quad \text{second non-Gaussian corr.}$$
$$+ c_6(\partial_\alpha \partial_\beta X \cdot \partial^\alpha \partial^\beta X)(\partial_\gamma X \cdot \partial^\gamma X) \quad \text{first term different from the Nambu-Goto string expansion}$$
$$+ O(\partial^8 X^4) \Big] + \text{boundary terms}$$

- It defines a string partition function as a perturbative expansion in the parameter $1/\sqrt{\sigma}R$ which has presumably a finite radius of convergence $\sqrt{\sigma}r \geq \sqrt{\sigma}/T_c$

- In 2004 Lüscher and Weisz noted that comparison of the string partition function on a cylinder (Polyakov correlator) with the sum over closed string states in a Lorentz invariant theory yields strong constraints (open-closed string duality):

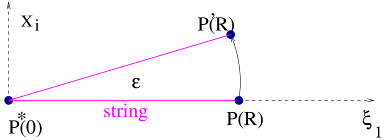
$$(D-2)c_2 + c_3 = \frac{D-4}{8}$$ Lüscher & Weisz, 2004

$$c_2 + c_3 = -\frac{1}{8}$$ Aharony & Karzbrun, 2009

- ⇨ $c_2 = \frac{1}{8} \quad c_3 = -\frac{1}{4}$
- ⇨ The $1/\sigma RL$ expansion of the Nambu Goto action
  $S_{NG} = \sigma \int d^2\xi \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha X \cdot \partial_\beta X)}$
  $(= \sigma \int d^2\xi \sqrt{1 + \partial_\alpha X \cdot \partial_\alpha X} \text{ in } D = 3)$
  satisfies these two constraints

Spontaneous breakdown of Lorentz invariance

- The formation of a confining flux tube spontaneously breaks the transverse translational as well as the Lorentz (or rotational) invariance of the bulk space-time
- The confining string action can be thought as the effective low energy action built integrating over all the massive degrees of freedom
- Even if the complete $SO(1, D-1)$ invariance is broken by the classical configuration around which one expands, the effective action should still respect this symmetry through a non-linear realisation



Aharony & Field, 2010 : non-linear realization of Lorentz symmetry

$\delta_\epsilon^{i,\alpha} X^j = -\epsilon\,\delta^{ij}\xi_\alpha - \epsilon\, X^i \partial_\alpha X^j$ ➪ $[\delta_\epsilon^{i\,\alpha}, \delta_\eta^{j\,\alpha}] X^k = \epsilon\eta(\delta_{jk} X^i - \delta_{ik} X^j)$

➪ $\delta S[X^i] = -\epsilon\sigma \int d^2\xi$
$\left[(1+4c_3)\partial_\beta X^i(\partial_\alpha X\cdot\partial^\beta X) + (1-8c_2)X^i(\partial_\alpha\partial_\beta X\cdot\partial^\beta X) + \dots\right]$

- One may implement systematically this recipe: FG 2011
- in D=3 the most general terms with only first derivatives are $\sum_{k=0}^{\infty} c_k(\partial_\gamma X\cdot\partial^\gamma X)^k$

➪ Lorentz (or rotational) invariance requires ; $k\,c_k + (k-\frac{3}{2})\,c_{k-1} = 0$

➪ The solution with initial condition $c_1 = \frac{1}{2}$ is the binomial $c_k = \binom{\frac{1}{2}}{k}$

➪ $S[\partial X] = \sigma\int d^2\xi\sqrt{1+(\partial_\gamma X\cdot\partial^\gamma X)} = S_{NG}$

Boundary terms (in open strings)

- Boundary conditions: $X^i(\xi_0, 0) = 0$
- The first non-trivial boundary term $S_1 = c\sqrt{\sigma}\int d\xi_0\,\partial_1 X\cdot\partial_1 X$ is incompatible with Lorentz invariance because
  $\delta_\epsilon^{i\,1} S_1 = -c\sqrt{\sigma}\epsilon\int d\xi_0\,\partial_1 X_i + \text{higer order terms} \neq 0$
- The first boundary term compatible with Lorentz invariance is
  $S_{boundary} = \frac{b}{\sqrt{\sigma}}\int d\xi_0\,\partial_1\partial_0 X\cdot\partial_1\partial_0 X + \dots$
- Lorentz invariance generates an infinite sequence of terms
  $S_{boundary} = \frac{1}{\sqrt{\sigma}}\int d\xi_0 \sum_{k=0}\Big[b_k\partial_1\partial_0 X\cdot\partial_1\partial_0 X(\partial_1 X\cdot\partial_1 X)^k + c_k(\partial_1\partial_0 X\cdot\partial_1 X)^2(\partial_1 X\cdot\partial_1 X)^k\Big]$

➩ $b_n + b_{n+1} = 0 \;,\; (n+1)\,c_n + n\,c_{n+1} = 0 \;,\; b_n + c_n + c_{n+1} = 0$

➩ $S_{boundary} = \frac{b}{\sqrt{\sigma}}\int d\xi_0\left[\frac{\partial_1\partial_0 X\cdot\partial_1\partial_0 X}{1+\partial_1 X\cdot\partial^1 X} + \frac{(\partial_1\partial_0 X\cdot\partial_1 X)^2}{(1+\partial_1 X\cdot\partial^1 X)^2}\right]$

➩ contribution to a rectangular Wilson loop $W(R, L)$
$\langle S_{boundary}\rangle_W = -\frac{\pi^3 b}{60\sigma}\left(\frac{L}{R} + \frac{R}{L}\right)\frac{L}{R^3} E_4\left(i\frac{L}{R}\right) + \dots$

- $S[X^i] = S_{NG} + S_{boundary} +$
  $c_6 \int d^2\xi (\partial_\alpha \partial_\beta X \cdot \partial^\alpha \partial^\beta X)(\partial_\gamma X \cdot \partial^\gamma X) + O(\partial^8 X^6)$
- in $D = 3$ dimensions the c_6 term is a total derivative
- there are reasons to believe that in $D > 3$ the Lorentz symmetry is anomalous unless $c_6 = \frac{26-D}{48\pi\sigma}$ (Polchinski & Strominger, 1991)

$S[X^i] = \sigma RL + \sigma \int d^2\xi \Big[\frac{1}{2}(\partial_\alpha X \cdot \partial^\alpha X)$ free string limit

$-\frac{1}{4}(\partial_\alpha X \cdot \partial^\alpha X)^2 + \frac{1}{8}(\partial_\alpha X \cdot \partial_\beta X)(\partial^\alpha X \cdot \partial^\beta X)$ first non-Gaussian correction

$-\frac{1}{16}(\partial_\alpha X \cdot \partial^\alpha X)^3 + \frac{1}{8}(\partial_\alpha X \cdot \partial_\beta X)^2(\partial_\gamma X \cdot \partial^\gamma X)$ second non-Gaussian corr.

$\frac{26-D}{48\pi\sigma}(\partial_\alpha \partial_\beta X \cdot \partial^\alpha \partial^\beta X)(\partial_\gamma X \cdot \partial^\gamma X)$ first bulk term different from the Nambu-Goto string expansion

$+O(\partial^8 X^4) \Big] + \frac{b}{\sqrt{\sigma}} \int d\xi_0 \, \partial_1 \partial_0 X \cdot \partial_1 \partial_0 X + \dots$

- in the above expansion b is the first free parameter of the effective action. It is the first term where one could see a dependence on the gauge group.

Conclusions

1. There are universal effects in Wilson loops and Polyakov correlators that are well understood and accurately explained in terms of an underlying confining string
2. Lorentz invariance puts strong constraints on the effective action of the confining string
3. The free-string limit as well as the first non-Gaussian correction of the confining string are universal and agree with numerical data