Chiral Properties of Strong Interactions in a Magnetic Background

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21 September 2011

INFN

SM&FT 2011 - Bari

In collaboration with: Massimo D'Elia

Based on: Phys. Rev. D 83, 114028 (2011)

Outline

- 1) L-QCD in external (background) magnetic field.
- 2) Magnetic catalysis and chiral symmetry breaking.
- 3) Numerical results and comparison with other models.
- 4) Conclusions.

1) L-QCD in external (background) magnetic field.

Why studying QCD in a magnetic background?

Theoretical point of view: B is a further direction in parameter space to probe QCD phase diagram and vacuum properties.

Phenomenological point of view: heavy ion collision experiments, CME, cosmological phase transitions, magnetars.

Our aim:

Study how vacuum (*i.e.* zero temperature) chiral properties, such as chiral condensates, depend on the external magnetic field. Focus on the particular case of a constant and uniform z-oriented field.

1) L-QCD in external (background) magnetic field.

Existing lattice studies.

Chromo-Magnetic field:
[P. Cea and L. Cosmai, JHEP 0508, 079 (2005)],
[P. Cea, L. Cosmai and M. D'Elia, JHEP 0712, 097 (2007)]

Chiral properties and CME (Chiral Magnetic Effect):
[P. V. Buividovich et al., Phys. Rev. D 80, 054503 (2009)],
[M. Abramczyk et al., arXiv:hep-lat/0911.1348], [Buividovich et al., arXiv:hep-lat/1003.2180]

Finite temperature and phase diagram:
[D'Elia, Sanfilippo and Mukherjee; arXiv:hep-lat/1005.5365v2] ,
[G. Endrodi; talk at Lattice2011]

1) L-QCD in external (background) magnetic field.

Discretization details and numerical setup.

QCD with $N_f = 2$ mass-degenerate flavours: u and d quark.

$$q_u = +(2/3)|e| \qquad q_d = -(1/3)|e|$$

Partition function for rooted staggered fermions:

$$Z = \int \mathcal{D}U \mathcal{P}[m, U, B, Q] = \int \mathcal{D}U \, e^{-S_G} \det M^{1/4}[m, B, q_u] \det M^{1/4}[m, B, q_d]$$

the staggered fermion matrix M in presence of the background is:

$$M_{i,j}[B, q] = \hat{m}\delta_{i,j} + \frac{1}{2}\sum_{\mu=1}^{4} \eta_{i,\mu}(U'_{i,\mu}(B, q)\delta_{i,j-\hat{\mu}} - U'^{\dagger}_{i-\hat{\mu},\mu}(B, q)\delta_{i,j+\hat{\mu}})$$

where

$$U'_{i,\mu}(B, q) = u_{i,\mu}(B, q) \; U_{i,\mu}$$

To have $\mathbf{B} = B\hat{z}$ we choose $A^{\mathrm{EM}}_{\mu}(\mathrm{x}) = (0, xB, 0, 0)$ so that:

$$u_{i,\mu}(B, q) = 1 \text{ unless } \mu = y \rightarrow u_{i,y}(B, q) = e^{iea^2 i_x B}$$

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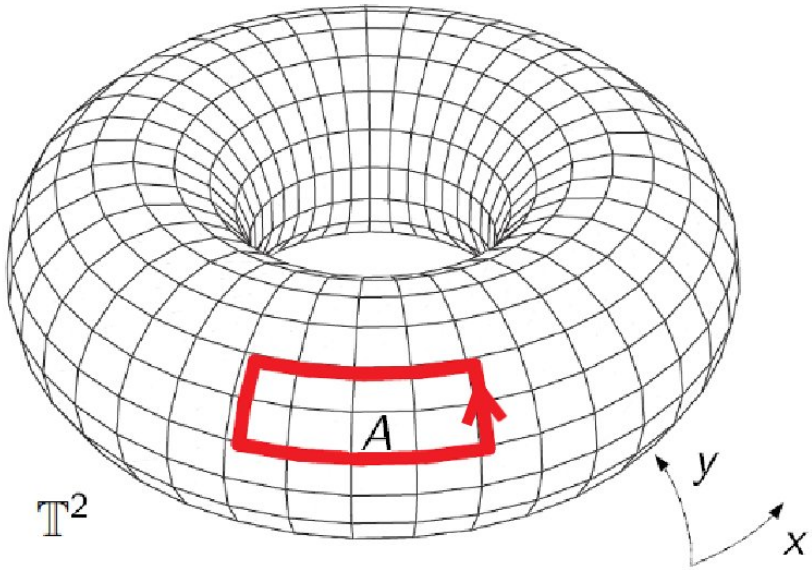

Effect: quantization of (|q|B)

$$q\Phi_A(B) = q\Phi_{\mathbb{T}^2 - A}(B) + 2\pi b$$

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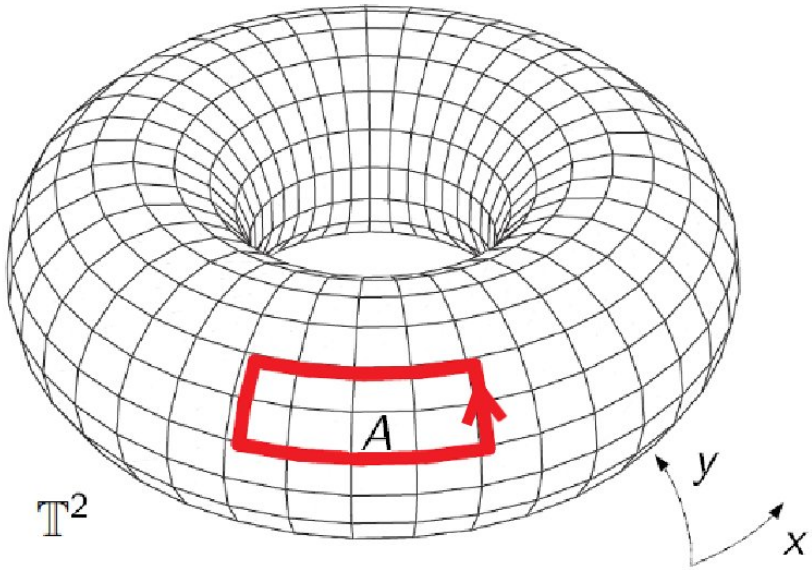

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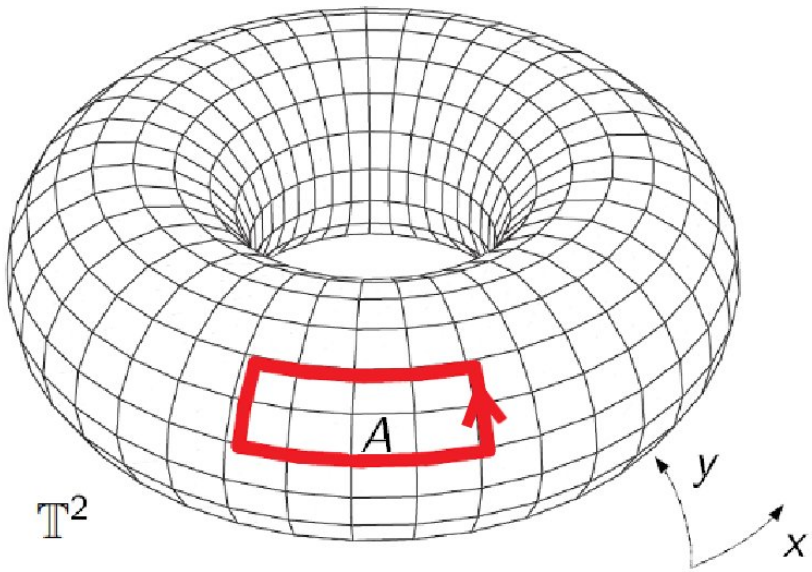

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which implies

$$(qB) = \frac{2\pi b}{l_x l_y} = \frac{2\pi b}{a^2 L_x L_y}$$

So that

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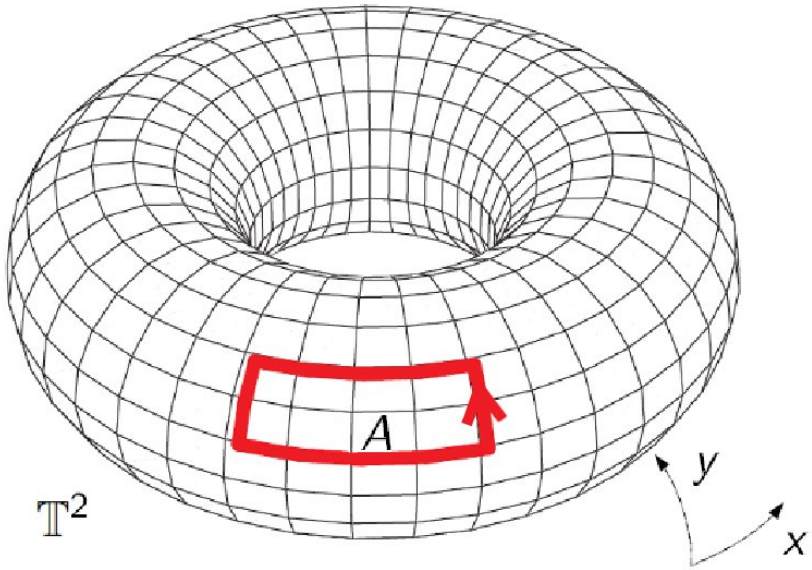

More details in [Al-Hashimi and Wiese; arXiv:0807.0630]

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Algorithm: Rational Hybrid Monte Carlo (RHMC)

Adimensional parameters:

- lattice size $L_x = L_y = L_z = L_t = 16$
- beta gauge $\beta = 5.30$
- bare mass $\hat{m} = 0.01335$
- field quanta $b \in [0, L_x L_y] \equiv [0, 256]$

Dimensional parameters:

- lattice spacing $a \simeq 0.3\,\mathrm{fm} \rightarrow (1/a \simeq 660 MeV)$
- temperature $T \simeq 40\mathrm{MeV} \rightarrow$ below χ and deconf. transitions.
- pion mass $m_\pi \simeq 200\mathrm{MeV}$
- field range $(|e|B) \in (180 - 700\ MeV)^2 \sim (10^{14} - 10^{15})\,T$

Warning!

Because of 2π periodicity of the magnetic-phase and B inversion symmetry we naively expect:

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2) Magnetic catalysis and chiral symmetry breaking.

Magnetic catalysis $\equiv$ increase of χ-SB due to the magnetic field.

How does $\Sigma(B) = <\overline{\psi}\psi>$, the quark condensate, depends on B?
We have $N_f = 2$ flavours so

$$\Sigma(B) = <\bar{u}u> + <\overline{d}d> = \Sigma_u(B) + \Sigma_d(B).$$

Let's define their's relative increments: $r_{u/d}(B) = \frac{\Sigma_{u/d}(B) - \Sigma(0)}{\Sigma(0)}$.

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Residual renormalization effects on r:

- multiplicative due to additive mass dependent $\rightarrow$ non-negligible (order of 10%)
- residual B dependent $\rightarrow$ negligible if $|e|B$ away from UV cutoff

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Since $q_u \neq q_d$ we expect $r_u \neq r_d$!
Many models predicts particular functional forms for $r(B)$.

What about full (L)QCD?

2) Magnetic catalysis and chiral symmetry breaking.

Contributions to magnetic catalysis.

In general one can try to compute:

$$\Sigma_j(m, m', B, B') = \int \mathcal{D}U \, \mathcal{P}[m', U, B'] \, \mathrm{Tr}(M^{-1}[m, B, q_j])$$

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Contributions to magnetic catalysis.

- Valence contribution (or pseudo-quenched):

$$\Sigma^{\text{val}}_{u/d}(B) = \int \mathcal{D}U\, \mathcal{P}[m, U, 0]\, \text{Tr}(M^{-1}[m, B, q_j])$$

- Dynamical contribution:

$$\Sigma^{\text{dyn}}_{u/d}(B) = \int \mathcal{D}U\, \mathcal{P}[m, U, B]\, \text{Tr}(M^{-1}[m, 0, q_j])$$

- Total contribution:

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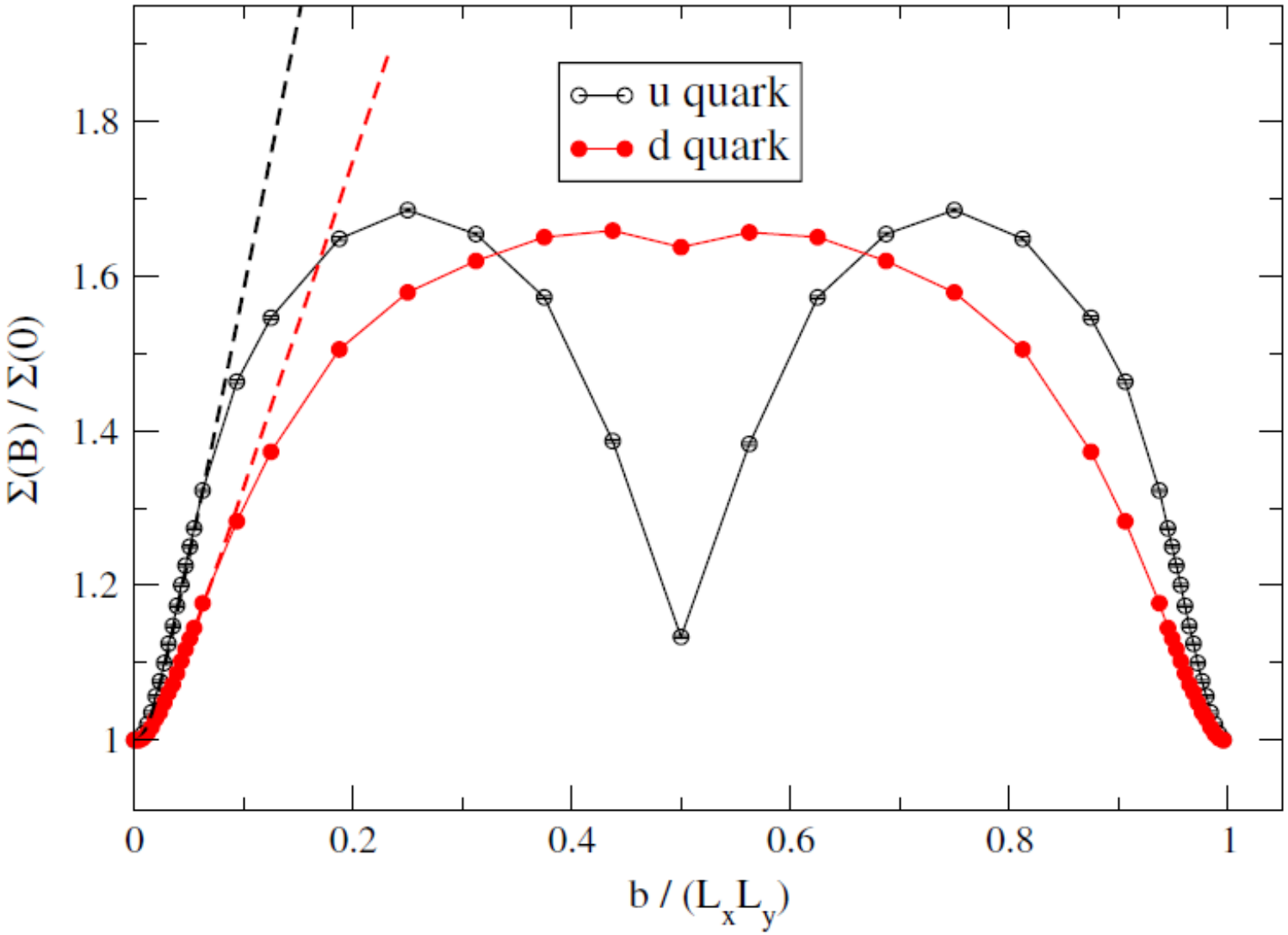
Assuming that the effects of B on $\mathcal{P}$ and on the observable are small perturbations:

$$r_{u/d}(B) = r^{\text{val}}_{u/d}(B) + r^{\text{dyn}}_{u/d}(B) + \mathcal{O}(B^4)$$

3) Numerical results and other model comparisons.

Periodicity and saturation effects. [Phys.Rev. D 83, 114028 (2011)]



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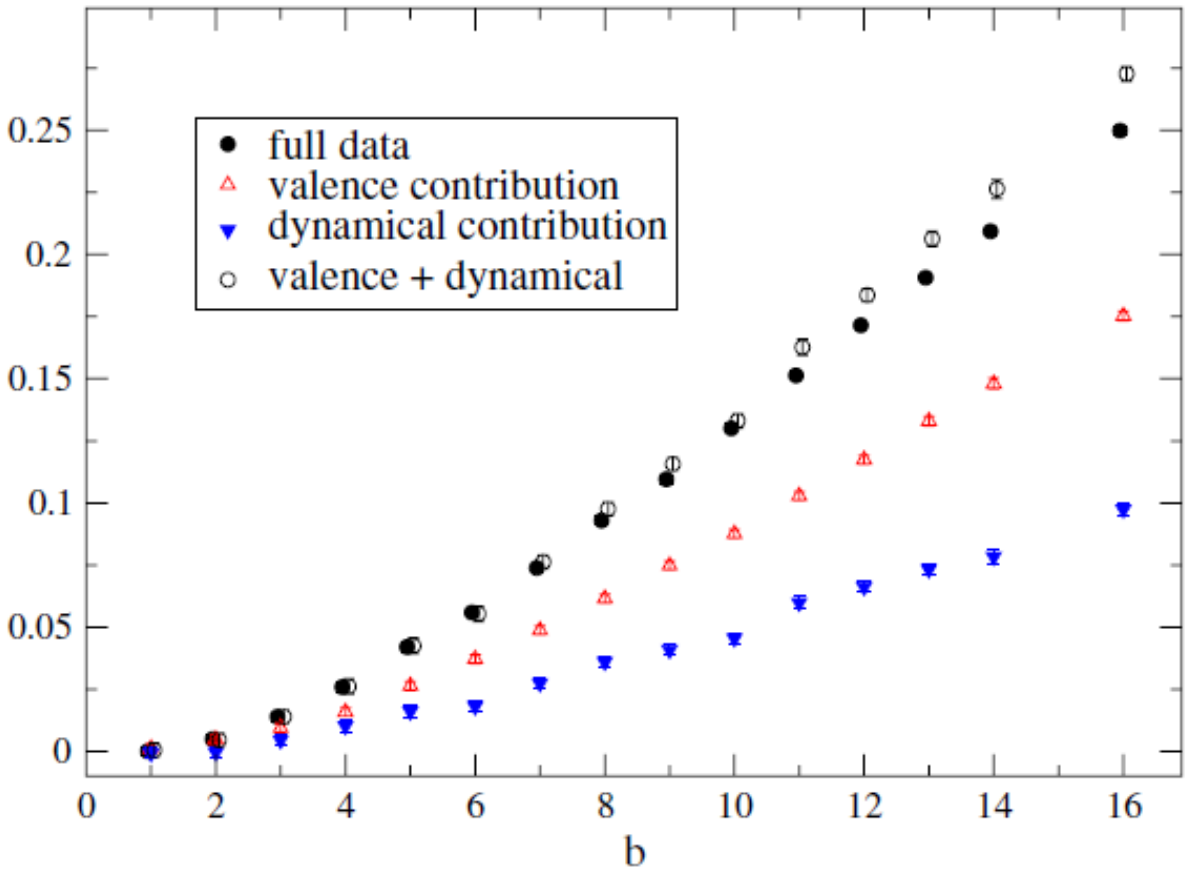
Valence and Dynamical contributions to catalysis.
[Phys.Rev. D 83, 114028 (2011)]



- $r(B) \simeq r^{\mathrm{dyn}}(B) + r^{\mathrm{val}}(B)$
- $r^{\mathrm{dyn}}(B) \simeq 0.4 \cdot r(B)$

3) Numerical results and other model comparisons.

Some remarks about *val* and *dyn* contribution.

- Additivity of *val* and *dyn* to produce the total signal:

  signature that the magnetic field B acts as a small perturbation.

- We are starting to see deviation when saturation effects

  are significant and when $\sqrt{|e|B}$ approaces to the UV cutoff.

- Unquenching increases the signal ($\sim 40\%$ of the total).

  $r^{\text{val}}(B) \simeq 0.6\, r(B)$ and $r^{\text{dyn}}(B) \simeq 0.4\, r(B)$

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Review of some model predictions.

- NJL model (et al.); [Klevansky and Lemmer; Phys.Rev. D 39, 3478 (1989)]
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  $r(B) \propto (|e|B) \cdot \mathcal{I}(\frac{m_\pi^2}{(|e|B)}) \quad \rightarrow \quad$ OK up to b=13 $\quad (|e|B) \sim (650 MeV)^2$
- quenched SU(2); [Buividovich et al.; Phys.Lett. B 682,484 (2010)]
  $r(B) \propto (|e|B) \quad \rightarrow \quad$ Never fits
- quenched SU(3); [Braguta et al.; arXiv:hep-lat/1011.3795]
  $r(B) \propto (|e|B)^\nu \ \nu \sim 1.6 \rightarrow$ OK up to b=8 and $\nu \sim 2$ $(|e|B) \sim (510 MeV)^2$

3) Numerical results and other model comparisons.

Parameter Fit with 1-loop χPT.

$$r(B) = \frac{\log(2)(|e|B)}{16\pi^2 {F_\pi}^2}\mathcal{I}(\frac{{m_\pi}^2}{(|e|B)})$$

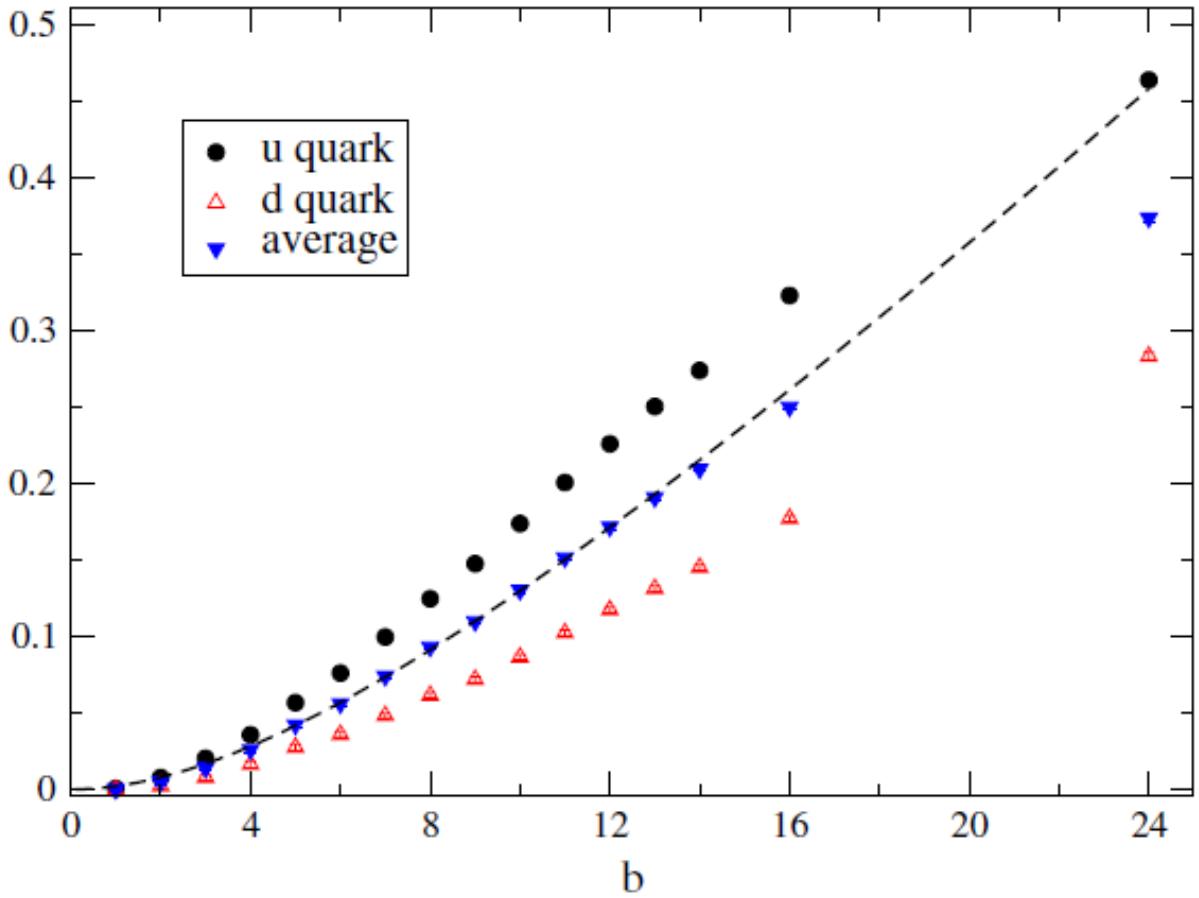
where

$$\mathcal{I}(y) = \frac{1}{\log(2)}\left(\log(2\pi) + y\log(\frac{y}{2}) - y - 2\log\left(\Gamma(\frac{1+y}{2})\right)\right)$$

$b_{\min}$	$b_{\max}$	χ^2/ndf	m_π (MeV)	F_π (MeV)
1	8	0.63	457(59)	54.7(5.6)
1	9	0.93	392(39)	61.7(4.2)
1	10	0.89	374(27)	63.8(2.9)
1	11	0.80	369(20)	64.3(2.1)
1	12	0.79	359(15)	65.6(1.6)
1	13	0.97	344(14)	67.2(1.4)
1	14	1.40	328(14)	69.1(1.4)
1	16	1.92	310(13)	71.1(1.3)

3) Numerical results and other model comparisons.

Parameter Fit with 1-loop χPT.



4) Conclusions.

Studying $N_f = 2$ (L)QCD with a magnetic background:

- we studied magnetic catalysis, i.e. the increase of chiral symmetry breaking induced by the magnetic field.
- we have explicitly shown periodicity and saturation effects.
- dynamical fermions provide about 40% of the total contribution to magnetic catalysis.
- the best fitting functional form: 1-loop χPT with $m_\pi \neq 0$.