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Stable states of a relativistic bilocal stochastic oscillator: a new quark-lepton model

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Abstract. We analyse a stochastic linearised extension of the Yukawa–Takabayasi–Feynman bilocal oscillator model and show that: (a) the external Poincaré group P commutes with an internal extension of the Lorentz group, i.e. $U(1) \otimes SO(6,2)$; (b) the corresponding internal fundamental spinor representation of the associated D_4 algebra yields eight quarks and eight leptons, which correspond to heuristic proposals of Nambu and Salam.

All recent attempts to classify elementary quarks and leptons start from the heuristic introduction of assumed gauge groups and Yang–Mills interactions, completed with *ad hoc* Higgs multiplets. These theories, illustrated by Salam (1968) and Weinberg (1967), justifiably claim important success, but leave open the question of the origin of the new internal 'charges', as well as the cause of relations of the type $Q = T + Y/2$ which connect quantum numbers associated with couplings of a different physical nature.

The aim of the present paper is to revive and develop an alternative line of research started by Yukawa (1950a, b, 1953, 1956). In this type of model, particles are extended time-like hypertubes in space–time which can be represented, in the first approximation, by bilocal structures that yield internal quantised states corresponding to quarks and leptons. This model has been studied in quantum form by Feynman *et al* (1971) and essentially developed by Takabayasi (1965a, b, 1968, 1979).

Until now, various attempts along this line have failed to produce satisfactory results (Katayama 1963, de Broglie *et al* 1963). The new step taken here is to introduce (in the trail of recent developments in the stochastic interpretation of quantum mechanics (Vigier 1979, Cufaro Petroni and Vigier 1979)) the following idea: to add to the space–time coordinates x_1 and x_2 of the two points, bound together by a relativistic harmonic oscillator potential, supplementary stochastic motions δx_1 and δx_2, which reflect the action of an isotropic constant thermostat. They, as well as their corresponding momenta δp_1 and δp_2, satisfy the relations $\langle \delta x_{1,2} \rangle = \langle \delta p_{1,2} \rangle = 0$, where $\langle \ldots \rangle$ represents averages taken on four-dimensional volume elements in configuration space.

Before we do this, let us briefly revisit Feynman's presentation of Yukawa's model, in order to discuss its linearisation and the connection between external Poincaré and internal homogeneous Lorentz groups of motion. Feynman's equation for the two-body bound state is (Feynman *et al* 1971)

$$[2(\Box_{x_1}+\Box_{x_2})-(\omega^2/16)(x_1-x_2)^2+m_0^2]\phi(x_1,x_2)=0. \tag{1}$$

By introducing the centre-of-mass and relative variables (Kim *et al* 1978) $Q=\frac{1}{2}(x_1+x_2)$ and $q=(\frac{1}{2}\sqrt{2})(x_1-x_2)$, it can be written in the form (for $\omega=1$)

$$[\Box_Q+m_0^2+\tfrac{1}{2}(\Box_q-q^2)]\phi(Q,q)=0 \tag{2}$$

separable in the Q and q variables; writing $\phi(Q,q)=\psi_{\rm E}(Q)\psi_{\rm I}(q)$ one obtains the equations

$$(\Box_Q+m_0^2+\epsilon)\psi_{\rm E}(Q)=0 \tag{3}$$

and

$$\tfrac{1}{2}(\Box_q-q^2)\psi_{\rm I}(q)=\epsilon\psi_{\rm I}(q). \tag{4}$$

Equation (3) describes the external motion of the bilocal system: it is a Klein–Gordon equation, and its solution of the form $\psi_{\rm E}(Q)=\exp(-{\rm i}PQ)$ (with $P^2=M^2=m_0^2+\epsilon$) corresponds to the free motion of the system. The external wavefunction $\psi_{\rm E}(Q)$ transforms under the usual external Poincaré group $\mathrm{P}=\mathrm{T}\otimes\mathrm{SO}(3,1)$, since under a translation $x_{1,2}\to x_{1,2}+a$ we have $Q\to Q+a$. The case of the internal wavefunction $\psi_{\rm I}(q)$ is different, as the relative coordinate q remains invariant under translation. Accordingly, it transforms under homogeneous Lorentz transformation. For the scalar case, considered in Feynman *et al* (1971) there is no problem. The problem arises when we linearise equations (3) and (4) into

$$(\gamma\hat{P}-M)\psi_{{\rm E},\alpha}(Q)=0 \tag{3a}$$

and

$$(\gamma\hat{p}+\gamma q+{\rm i}\sqrt{2}\epsilon)\psi_{{\rm I},\alpha}(q)=0 \tag{4a}$$

to obtain external and internal spinors, $\psi_{{\rm E},\alpha}(Q)$ and $\psi_{{\rm I},\alpha}(q)$. Here $\hat{P}$ and $\hat{p}$ correspond to the operators ${\rm i}\partial_Q$ and ${\rm i}\partial_q$, respectively. As is known, (Takabayasi 1979) it is not possible to linearise (4), unless we impose the supplementary condition $q\hat{p}\psi_{{\rm I},\alpha}(q)=0$. If this is done, then we can introduce commuting external and internal SO(3,1) transformations by utilising Chevalley's (Chevalley 1946, Halbwachs and Souriau 1964) left and right translations $\mathrm{SO}(3,1)_{\rm L,R}$, of the homogeneous Lorentz group SO(3,1). This bilateral group BilSO (3,1) is equal to $\mathrm{SO}(3,1)_{\rm L}\otimes\mathrm{SO}(3,1)_{\rm R}=(\mathrm{SO}(3,1)\otimes\mathrm{SO}(3,1))/C$, where C is the centre of the whole group. Such commuting left and right Lorentz translations (having the same Casimir operators) have already been used by de Broglie *et al* (1963) in their rotator particle model. Now, $\mathrm{SO}(3,1)_{\rm L}$ and $\mathrm{SO}(3,1)_{\rm R}$ (which can be represented by three-dimensional complex rotations) correspond to external and internal transformations respectively, i.e. to angular momentum projections on Einstein tetrads associated with the observer and the particle. Moreover, one sees that for $\Omega=0$ the model reduces to the rotator model of de Broglie *et al* (1963).

Now we introduce our stochastic motion, as was done by Guéret *et al* (1979) (denoted I hereafter). The variable Q becomes $Q+\delta Q$, but nothing is changed in the average motion of Q, since the internal temperature does not modify the centre-of-mass motion. The relative variable q goes into $q+\delta q$, but then equations (4) and (4*a*) transform according to the (6+2)-dimensional group of motion, introduced in I to describe the motion of a relativistic oscillator embedded in a random stochastic thermostat.

To include the stochastic behaviour in the motion described by equation (4), which evidently stems from the Hamiltonian

$$H_{\mathrm{I}}=(1/2m)(p^2+\omega^2q^2), \tag{5a}$$

we note that the stochastic contributions δq and δp behave like new independent variables, so that we can consider the total set of variables as describing two independent points (i.e. q and δq) in a (6+2)-dimensional configuration space. Of course, if one assumes an isotropic constant thermostat, we have $\langle q\rangle=q$ and $\langle p\rangle=p$ along with $\langle\delta q\rangle=\langle\delta p\rangle=0$, where the $\langle\ \rangle$ represents averages taken on four-dimensional volume elements in configuration space. As is known, any motion in our new 16-dimensional phase space implies an assumption on the connection of the sets (q, p) and $(\delta q, \delta p)$. If we limit ourselves to the descriptions of small (regular plus random) motions at the bottom of an arbitrary potential well, (i.e. Γ_0 harmonic oscillations) we can generalise H_{I} (in equation (5a)) into

$$H'_{\mathrm{I}}=(1/2m)[(p+\delta p)(p+\delta p)+\omega^2(q+\delta q)(q+\delta q)] \tag{5b}$$

which yields the corresponding Liouville equation. This yields an average motion described by the average Hamiltonian $\langle H'_{\mathrm{I}}\rangle$ which can be calculated. Indeed since we have $\langle AB\rangle=\langle A\rangle\langle B\rangle$ for any pair of independent variables A, B (with $A\neq B$) in phase space we obtain (with $\langle q\rangle=q$ and $\langle p\rangle=p$)

$$H_{\mathrm{I}}=\langle H'_{\mathrm{I}}\rangle=(1/2m)[p^2+\delta p^2+\omega^2(q^2+\delta q^2)] \tag{6}$$

which describes motions in our 2(6+2)-dimensional phase space and has been analysed from Cartan's point of view by Guéret *et al* (1973), denoted II hereafter.

The Hamiltonian (6) and its linearised form given in II are invariant under the symplectic group Sp(12, 4) and admit as the general symmetry group $\mathrm{U}(6,2)\supset \mathrm{SU}(1,1)\otimes\mathrm{SO}(6,2)$, where SO(6, 2) now contains $\mathrm{SO}(3,1)_{\mathrm{R}}$, as its subgroup which acts on the two distinct spinor representations, 8 and $\bar{8}$ (along with a third, vector representation 8′), interchangable under the discrete automorphisms of the corresponding D_4 Lie algebra analysed by Cartan (1938).

We note here that the introduction of internal stochastic motions (represented by δq) is mathematically equivalent to doubling the number of space–time coordinates, i.e. to move into an extended configuration space. In particular, the new internal time coordinate represents the relative time projection of the random part of the motion on the particle's rest mass frame. This solves the age-old problem of interpreting the new internal times which necessarily appear in bilocal or extended particle models.

If one then further transforms $\mathrm{H_I}$ to a Feynmann–Gell'Mann type of equation, i.e. to $H'_{\mathrm{I}}=(1/2m)P^2$, which can be linearised into $H''_{\mathrm{I}}=(1/2m)(\Gamma p)$, with the help of the 16-dimensional matrices Γ calculated in I, we can assume that the wavefunctions ψ_{E} and ψ_{I} are simultaneously spinors, or vectors, and classify all the various fermionic particles into the two spinor families $\psi'_{\mathrm{I}}(8)$ and $\psi''_{\mathrm{I}}(\bar{8})$, the corresponding antiparticles belonging to opposite values of U(1) in $\mathrm{G}=\mathrm{U}(1)\otimes\mathrm{SO}(6,2)$, which leaves invariant H'_{I} and H''_{I}. The Weyl–Cartan algebra of SO(6, 2) explicitly calculated by Guéret *et al* in II and Vigier (1976) (denoted III) yields (along with the Casimir invariant operators) four diagonal commuting operators H_i $(i=1,2,3,4)$ and 24 'raising' and 'lowering' operators E_α and $E_{-\alpha}$ with $[E_\alpha, E_{-\alpha}]=\frac{1}{8}\alpha_iH_i$, where α_i denotes a root and $H_1=M_{12}$,

$H_2 = M_{34}$, $H_3 = M_{56}$ and $H_4 = M_{78}$. Moreover, $x_1^2 + \ldots + x_6^2 - x_7^2 - x_8^2 = \text{constant}$ in $E_{6,2}$. We thus obtain for $J = 1/2^+$ (external spinors) eight states of the same 'colour' denoted yellow (i.e. y), corresponding to the finite non-unitary representation (8) (see table 1) with four quarks and four leptons belonging respectively to the representations (4) and ($\bar{4}$) of the subgroup SO(6) = SU(4) with opposite H_4 values, each state is characterised by the eigenvalue H_0 of U(1) and one spinor component of the ket $E_\alpha = |H_1, H_2, H_3, H_4)$ determined by Cartan (1938). These H-values are given in the first five columns of table 1; the last columns give, as in III, the usual quantum numbers: $T_3 = (H_1 - H_2)/2$, $Y = (\frac{1}{3})(H_1 + H_2 - 2H_3)$, $Z = -(\frac{1}{2})(H_1 + H_2 + H_3)$, $Q = T_3 + (Y/2) - (2Z/3) - (H_4/2) = H_1 + H_4$, along with $S = H_1 + H_2$ and C(charm) $= (3H_4/2) - Z$ which we have determined (following Yukawa's (1956) and Okubo's (1978) suggestions, that corresponds to Yang–Mills gauge fields) from a choice of compact subgroups of G_I preserved in typical superpositions (combinations) of our basic oscillating states.

Table 1.

	H_0	H_1	H_2	H_3	H_4	T_3	Y	Z	S_w	C	Q	Particle	SU(3)	SU(2)
ξ_{123}	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+\frac{3}{4}$	−1	0	0	c^y	singlet	doublet $H_3 = -\frac{1}{2}$
$-\xi_3$	−1	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{2}{3}$	$-\frac{1}{4}$	+1	+1	+1	s^y		
$-\xi_1$	−1	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	+1	+1	u^y	triplet	doublet $H_3 = +\frac{1}{2}$
$-\xi_2$	−1	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	+1	0	d^y		
ξ_4	−1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{3}{4}$	+1	0	0	ν_τ	singlet	doublet $H_3 = +\frac{1}{2}$
ξ_{124}	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{2}{3}$	$+\frac{1}{4}$	−1	−1	−1	τ^-		
ξ_{234}	−1	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	−1	−1	e^-	triplet	doublet $H_3 = -\frac{1}{2}$
ξ_{314}	−1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	−1	0	ν_e		

This choice of internal H_i combinations to label the quantum numbers results in a unique way from our dynamical model for two reasons. The first is that the corresponding Yang–Mills fields must be associated with particular compact subgroups embedded in our general non-compact dynamical group. The second is that these subgroups must yield the SU(2) CSU(3) CSU(4) embedding which gives the correct generalisation of the Gell'Mann–Okubo formulae. Moreover, following Cartan, the choice of the H_i to label the spinor components cannot be avoided. Indeed the SO(6, 2) 28 generators contains 16 simultaneously diagonalisable commuting operators, out of which 12 are Casimir operators whose eigenvalues label the representations and four (i.e. the H_i's) differentiate the spinor (vector) components.

Of course, the corresponding antiparticles will have $J = 1/2^-$ and the opposite H_i-values, since they correspond to internal mirror motions in our scheme (Flato *et al* 1965). Table 1 yields a unique combination of Salam's (1974, 1976) F_e-type fermions with four yellow quarks q^y and four leptons l^y. Curiously, these quarks are just the 'Yukawon' first proposed by Yukawa and discussed in the literature by de Broglie *et al*

(1963). They can be mapped on Salam's (1974) and Pati and Salam's (1973, 1974a, b, 1975) integer quark classification.

Table 2 with $j = 1/2^+$ yields the (blue) $\bar{8}$ octet of particles (the antiparticles being obtained as for 8) which corresponds to F_μ-type fermions. The corresponding SU(3) quark triplet corresponds to Sakata's well known model.

The last fundamental octet of (coloured) gluon vector particles (table 3) splits into a SU(4) sextet (i.e. two SU(3) triplets) and two SU(4) singlets which ensure (II) quark–lepton transition from 8 to $\bar{8}$ and vice versa. It has $J = 1^+$. This also maps on Salam's (1974) Pati and Salam's (1975, 1976) and Pati *et al*'s (1976) proposals.

We now mention some consequences of our model.

A. The model evidently contains the essential part of the strong-interaction results predicted by Nambu (Han and Nambu 1965), Pati and Salam in their integer-charged

Table 2.

	H_0	H_1	H_2	H_3	H_4	T_3	Y	Z	S_w	C	Q	Particle	SU(3)	SU(2)
ξ_0	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	$-\frac{3}{4}$	$+1$	$+\frac{3}{2}$	$+1$	c^b	singlet	doublet $H_3 = +\frac{1}{2}$
ξ_{12}	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{2}{3}$	$+\frac{1}{4}$	-1	$+\frac{1}{2}$	0	s^b		
ξ_{23}	-1	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	$+\frac{1}{2}$	0	d^b	triplet	doublet $H_3 = -\frac{1}{2}$
ξ_{34}	-1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	$+\frac{1}{2}$	$+1$	u^b		
$-\xi_{1234}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+\frac{3}{4}$	-1	$-\frac{3}{2}$	-1	M^-	singlet	doublet $H_3 = -\frac{1}{2}$
ξ_{34}	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$+\frac{2}{3}$	$-\frac{1}{4}$	$+1$	$-\frac{1}{2}$	0	M^e		
ξ_{14}	-1	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	ν_μ	triplet	doublet $H_3 = +\frac{1}{2}$
ξ_{24}	-1	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	-1	μ^-		

Table 3.

	H_0	H_1	H_2	H_3	H_4	T_3	Y	Z	S_w	C	Q	SU(3)	SU(4)
x^4	0	0	0	0	$+1$	0	0	0	0	$+\frac{3}{2}$	$+1$	singlet	singlet
x^1	0	$+1$	0	0	0	$+\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$	$+\frac{1}{2}$	$+1$		
x^2	0	0	$+1$	0	0	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$	$+\frac{1}{2}$	0	triplet	
x^3	0	0	0	$+1$	0	0	$-\frac{2}{3}$	$-\frac{1}{2}$	0	$+\frac{1}{2}$	0		sextet
$x^{3'}$	0	0	0	-1	0	0	$+\frac{2}{3}$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0		
$x^{2'}$	0	0	-1	0	0	$+\frac{1}{2}$	$-\frac{1}{3}$	$+\frac{1}{2}$	-1	$-\frac{1}{2}$	0	triplet	
$x^{1'}$	0	-1	0	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$+\frac{1}{2}$	-1	$-\frac{1}{2}$	-1		
$x^{4'}$	0	0	0	0	-1	0	0	0	0	$-\frac{3}{2}$	-1	singlet	singlet

quark model. Indeed, if we assume that strong interactions preserve C (i.e. SU(3)) and H_0 (fermion) numbers, we see that boson multiplets are built with coloured $q\bar{q}$ combinations of the SU(3) triplets contained in $\bar{8}$ and 8: the corresponding F_e and F_μ hadronic multiplets resulting from their multiplication by SU(3) singlets, i.e. $F_e(F_\mu) = q(q\bar{q})$. The SU(3) gauge group contains the usual $J=1^-$ SU(3) $q\bar{q}$ multiplets of uncoloured gluons belonging to $\bar{8}\otimes\bar{8}$ and $8\otimes 8$, plus Pati's and Salam's triplets of X_e and X_μ $\bar{q}l$ particles. Like Salam's 'prodigal' model, the model predicts (III) strong qq and (strongly reduced) ll interactions, including $q \to \bar{l}+l+\bar{l}$ strong decays of free quarks. To these strong interactions, one must add, as a consequence of Cartan's triality principle (i.e. $8\otimes\bar{8}=8'$, $\bar{8}\otimes 8'=8$ and $8\otimes 8'=\bar{8}$), strong interactions resulting from the two SU(3) triplets and singlets contained in 8′. In principle, evidently they strongly mix F_e-type and F_μ-type fermions, but one sees that, if the corresponding masses are high enough, there is (Pati and Salam 1974a, b) no observable mixing of the F_e and F_μ worlds, except through weak and electromagnetic interactions. One thus guarantees that normal hadrons (including K^0 and $\bar{K}^0$) may be considered predominantly as made up of e-quark type only and forbid transitions of the type $K^0 \to e^- + \mu^+, e^+ + e^-, \mu^+ + \mu^-$. Finally, since all particles have non-zero bare mass, strong Lagrangians are invariant under U(1), which ensures parity conservation, i.e. G_I (strong) = U(1) ⊗ SU(3). As is well known, this implies the existence of a very light pseudoscalar boson, i.e. Weinberg's (1978) and Peccei and Quinn's (1977a, b) 'axion', which might have already been observed in anomalous redshifts recently discussed in the literature (Arp 1971, 1973, Pecker 1976). Moreover, the introduction of a random part in all gauge groups G_I implies (Vigier 1962) that the corresponding Yang–Mills fields must have an effective non-zero mass, so that the model (as will be later discussed) implies the existence of the corresponding Higgs multiplets.

B. The model contains as a frame for weak interactions the maximal compact subgroup $U(1)\otimes U(1)\otimes SU_L(4)$, of which we identify the subgroup $U(1)\otimes SU_L(2)$ which preserves the 'weak' leptonic charge H_3 with the weak-interaction gauge group of Weinberg and Salam, which appears, in terms of our internal motions, as the only and most probable candidate for a correct unification of weak and electromagnetic interactions. Indeed, with this assumption, one sees immediately that tables 1 and 2 now apparently agree with known facts of all types of strong–weak–electromagnetic interactions. Indeed, starting with $(\bar{8})_L$, we can assume that it connects the usual SU(4) basic 4-representation of quarks u^y, d^y, s^y, c^y with the conjugate representation 4 of e-leptons in order to cancel the triangle anomalies. Since, independently of the $Q=f(H_i)$ definition, we must have a lepton EM charge sequence of the type (−1, 0, 0, −1), the associated quark quartet *must* have the charges (0, 1, 1, 0), so that the model indeed implies the Nambu (1965) and Salam (1974, 1976) integer-charge assumption. Moreover, since isobasic spin and strangeness are now defined in the same way (III) for quarks and leptons, μ^- and ν_μ cannot be introduced in the same SU(4) quartet since they would generate strangeness, changing neutral currents and lepton number non-conservation. As a consequence, since experiment shows that τ can only be a sequential lepton (i.e. excited electron), we associate e-leptons with yellow quarks and μ-leptons with blue quarks. The electric charge now exactly corresponds to $Q=H_1+H_4=(\frac{1}{2})(\lambda_3+\sqrt{\frac{1}{3}}\lambda_8-\sqrt{\frac{2}{3}}\lambda_{15})+H_4$, where λ_i denote the usual (Amati *et al* 1964) SU(4) generating matrices. Moreover, since the H_3-conserving Weinberg–Salam (WS) group is exactly embedded in $SU_L(4)$, as assumed recently by Yang (1977, 1978), we have only L-multiplets and quarks and leptons must have the same $(1-i\gamma_5)/2$ current parity. Weak CP-violation thus follows in this scheme from Okubo's suggestion (1968a, b) that weak Yang–Mills fields imply maximum parity violation, i.e. that we have $H_w = ig(j_w + 1_w)W_\mu + \text{HC}$, with CP-parity -1, the W_μ having strong interactions among themselves. It is also interesting to note that, in our model, any breaking of the Higg's scalars gauge symmetry leads to the Weinberg–Salam gauge theory, i.e. (1) SU(4) → O(5) → SU(2), the breaking of SU(2) with U(1) giving the WS theory; (2) SU(4) → O(4) = SU(2) ⊗ SU(2) → WS with U(1) breaking; (3) SU(4) → U(1) ⊗ SU(2) → SU(2) → WS.

Of course, all these decompositions imply, starting from the WS situation, the excitation of successive internal degrees of freedom so that the WS model is absolute for low-energy leptons, the Cabbibo angle appearing only among heavier particles, i.e. mixing of d^y and c^y (or c^b and u^b), so that we recover Yang's result (1977, 1978) for the Cabbibo angle.

C. The model is falsifiable in the sense that our internal motions in space–time imply the existence of *only* eight quarks q and eight leptons l and yield the prediction $R = \sigma_{e\bar{e}\to\text{hadrons}}/\sigma_{e\bar{e}\to\mu\bar{\mu}} = \Sigma_i Q_i^2 = 6$, which can be compared with the observed value 5, 5 at $\sqrt{s} \simeq 5$ GeV and represents a maximum possible value in this scheme. The model also predicts (as will be discussed elsewhere) that compound $qq\bar{q}$ and $\bar{q}q\bar{q}$ systems could have a different lifetime, so that the faster decay of a fermionic compound antiparticle could explain why we live in a particle world, as suggested by recent astrophysical evidence (Demaret *et al* 1978).

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