Open Questions in Quantum Physics

Invited Papers on the Foundations of Microphysics

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A CAUSAL FLUIDODYNAMICAL MODEL FOR THE RELATIVISTIC QUANTUM MECHANICS

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ABSTRACT

The physical basis for a fluidodynamical model of quantum mechanics and the subsequent possibility of a causal, non-local explanation of the EPR paradox are shortly discussed.

1. INTRODUCTION

This paper constitutes the physical basis for the deterministic explanation with action at a distance of the Einstein-Podolsky-Rosen paradox, which is discussed by Prof. Vigier in his paper.

The starting point of our work is the causal fluidodynamical interpretation of quantum mechanics given by Bohm.(1,2) So we effectively regard the wave function of an individual electron as a mathematical representation of an objectively real field. This field exerts a force on the particle by means of the well-known quantum potential.

However this interpretation was criticized by Takabayasi,(3) Pauli (4) and others. (5) We will list here only the problems that we consider connected with our research:

1. the assumption that $|\psi|^2$ coincides with the probability density is not appropriate in a theory aimed at giving a causal explanation of quantum mechanics;
2. the ψ field was so different from other objectively real fields that it does not provide a satisfactory model for quan-

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tum phenomena; in a word: what is a Madelung fluid?
3. this interpretation did not include spin;
4. the quantum potential for the N-body equation is non-local, so that the quantum mechanical forces may be said to transmit uncontrollable disturbance instantaneously from a particle to another through the medium of the ψ field.

We think that this set of questions still constitutes an open field of research, and we therefore shall try to give an outline of an organic answer: a stochastic hydrodynamical model of the relativistic quantum equations in a material aether and in the presence of a relativistic action at a distance.

2. "IS THERE AN AETHER?"

First of all we expose our point of view on the material subquantum medium in which particles are imbedded. We adopt the idea, first presented by Dirac,(6) that in the light of quantum mechanics "the aether is no longer ruled out by relativity". In fact, the velocity of the aether in each point (which would destroy the Lorentz isotropy of the vacuum) is subject to the uncertainty relations. So, the velocity of the aether at a certain point will be distributed over various values, and we may set up a wave function which makes all value of the velocity equally probable. Of course (see Fig.1) this wave function must be constant on the hyperboloid

$$v_0^2 - v_1^2 - v_2^2 - v_3^2 = 1. \tag{1}$$

In other words, Dirac has bypassed all former relativistic objections to the aether's existence by introducing for it a chaotic subquantal behaviour.

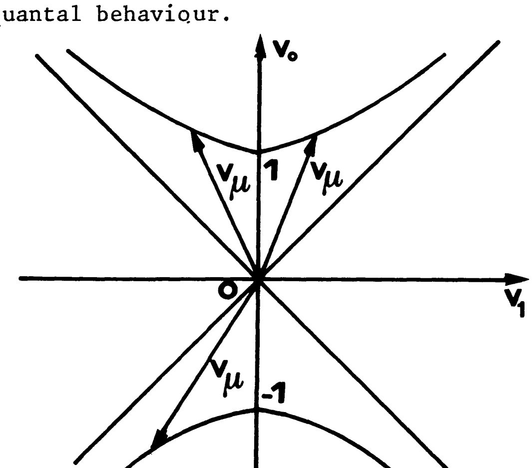

Fig.1. The velocity of the aether at a point is uniformly distributed on the hyperboloid(1).

A complete and satisfactory theory of Dirac's aether still doesn't exist. We list here only some questions and proposals about this problem.

1. To meet the question of the viscous drag in the aether we can build a superfluid model as Sudarshan et al. (7) did.

2. We can try (8) to connect the aether with the "negative energy sea", which still remains an essential basis for the second-quantization formalism. In this case, as shown in Fig. 2, the four-momenta of the aether will be distributed on the lower mass hyperboloid in an uniform way, so that

$$dN = K\sqrt{|ds^2|} \quad , \tag{2}$$

where K is a constant and dN is the number of states in a section ds of the hyperboloid.

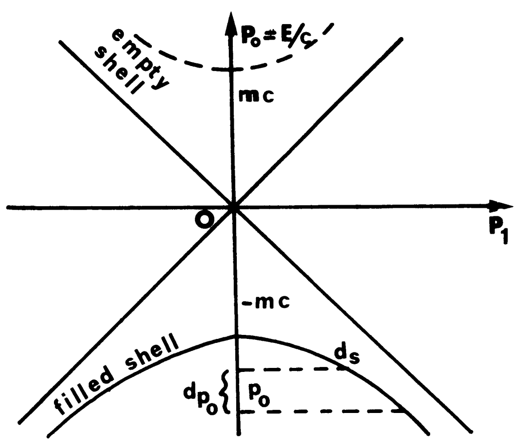

Fig. 2. Dirac's aether and negative energy sea

In this case the distribution in energy is not a constant:

$$\rho(p_0) = \frac{Kmc}{\sqrt{p_0^2 - m^2c^2}} \tag{3}$$

Of course, the number of the almost light-like four-momenta is predominant.

3. We can introduce the spin. (9)

4. We need a deeper analysis of the interactions between the aether and the particles.

3. HYDRODYNAMICAL ANALYSIS OF A SPINOR FIELD

In our model the quantum waves are perturbations on the aether produced by the presence of a particle. From that standpoint, the particles are "corpuscles" imbedded in the aether and in interaction with it by means of the quantum potential. Here $\hbar$ is a sort of coupling constant. In other words, an electron is a particle *IN* the wave, and ψ is a *REAL* wave that pilots particle-like concentrations of energy-momentum corresponding to electrons.

Coherently with this standpoint, we can now:

1. make a hydrodynamical analysis of quantum waves and attribute to them the usual quantities (like momentum, energy angular momentum) on the basis of the usual Lagrangian formalism;

2. design new experiences (10) to directly detect the existence of the de Broglie quantum waves,which are for us not only mathematical tools.

The second possibility being analyzed in other papers for this conference, we shall confine ourselves to quoting some hydrodynamical results (11) about the spinor field.

Starting from the scalar bilinear Lagrangian (with $\hbar = c = 1$)

$$L = m\bar{\psi}\psi - \overline{(i\partial\!\!\!/ - eA\!\!\!/)\psi}\,(i\partial\!\!\!/ - eA\!\!\!/)\psi \tag{4}$$

we get as Euler-Lagrange equation the well-known Feynman and Gell-Mann (12) second order equation

$$[(i\partial_\mu - eA_\mu)(i\partial^\mu - eA^\mu) - \tfrac{1}{2}\sigma_{\mu\nu} F^{\mu\nu}]\,\psi = m^2\psi \tag{5}$$

with

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu] \text{ and } F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \,. \tag{6}$$

Of course, the eq. (5) is not equivalent to the ordinary Dirac equation, which in this formalism plays the role of a constraint on the solutions of (5) that select spinors with positive-definite conserved density.

The current is now

$$J^\mu = \frac{i}{2m}[\bar{\psi}\partial^\mu\psi - \partial^\mu\bar{\psi}\psi - i\partial_\nu(\bar{\psi}\sigma^{\mu\nu}\psi)] - \frac{e}{m}A\bar{\psi}\psi, \tag{7}$$

so that the free part coincides with the Gordon form of the usual spinor current and is equivalent to Dirac's usual expression only for solutions that obey to the Dirac constraint.

If now we posit

$$\psi = Qw, \tag{8}$$

where $Q^2 = |\bar{\psi}\psi|$ is a real positive function and w an unitary spinor with $\bar{w}w = \pm 1$, the imaginary part of (5) furnishes

$$\partial_\mu J^\mu = 0, \tag{9}$$

where the current is given by (7) and the real part corresponds to a generalized Hamilton-Jacobi equation

$$\bar{w}(i\partial_\mu - eA_\mu)w\ \bar{w}(i\partial^\mu - eA^\mu)w - \frac{\Box Q}{Q} - \frac{e}{2}\bar{w}F^{\mu\nu}w\ \bar{w}\sigma_{\mu\nu}w - m^2 = 0 \tag{10}$$

with quantum potential

$$U = \frac{\Box Q}{Q} \ . \tag{11}$$

From the same Lagrangian (4) we can also calculate all the usual physical quantities associated with our wave (energy-momentum tensor, angular momentum, spin, etc;). We follow in this calculation the general method given by Halbwachs (12) in his book on the spinning fluids.

We stop here this argument by recalling that along these lines we can:

1. elaborate a two-particle case and explore in a direct way, for example, and in term of forces and torques, what the word "correlation" means in the instance of correlated electrons;

2. employ the usual formalism of stochastic derivation of quantum equation.

4. CONNECTION WITH STOCHASTIC PROCESSES

We do not recall here the results of the derivation of the relativistic quantum equations starting from Nelson's equations. (13) We remark only that, from the relativistic second order Nelson's equation, we can get only second-order quantum equations: It is because we adopted the second-order Feymann and Gell-Mann equation for our spinor field in the preceding section.

In order to preserve the quantum equations in our model, we present here (8,14) another derivation suggested to us by a seminar of Prof.A.Avez (15) on the probabilistic interpretation of the hyperbolic partial differential equations. We are able at present to derive the relativistic quantum equation for spinless particles.

To simplify our demonstration, we limit ourselves to the case in which it is assumed that random walks occur on a square lattice in a two dimensional space-time (see Fig.3). To analyze our random walks we describe our two dimensional space -time with the coordinates x^0, x^1 and consider a limiting process where in each step it is supposed that our particle, starting from an arbitrary point $P_0(x^0, x^1)$, can make only jumps of fixed length and always at the velocity of light. Of course, this prescription completely determines the lattice of all the possible positions of the particle.

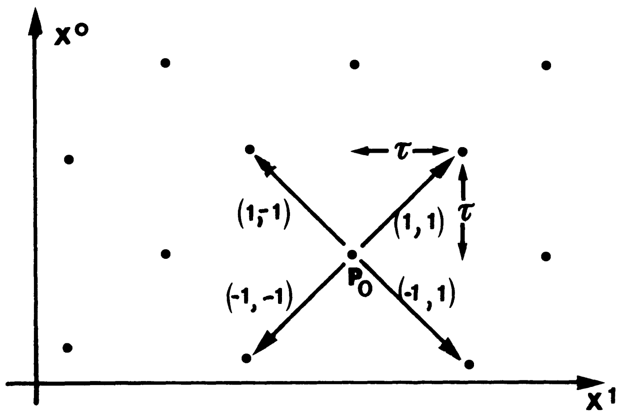

Fig. 3. Space-time lattice of dimension τ and starting point P_0 ; for each possible direction of the first jump we marked the corresponding value of the couple (t,s).

On this lattice the particle can follow an infinity of possible trajectories. In our calculation we will consider first a lattice with fixed dimensions. Indeed, for each jump we posit:

$$\Delta x^0 = t\tau \quad \text{and} \quad \Delta x^1 = s\tau, \ (t,s = \pm 1), \tag{12}$$

so that for the velocity we always have

$$v = \Delta x^1/\Delta x^0 = s/t = \pm 1 \tag{13}$$

Here τ is the parameter that fixes the lattice dimensions; of course, in order to recover the quantum equations, we will consider later the limit $\tau \to 0$. Moreover, it is clear that we also consider the possibility of trajectories running backward in time: We will interpret them as trajectories of antiparticles running forward in time, following the usual Feynman's interpretation.(16)

In order to describe random walks on this lattice, let we consider the following Markov process on the set of the four possible directions of the velocity. We define two sets of stochastic variables $\{\varepsilon_j\}$, $\{\eta_j\}$, in such a way that the only possible values of each ε_j and η_j are ± 1, following this prescription:

$$\varepsilon_j = \begin{cases} +1 \\ -1 \end{cases} \text{if in the (j+1)-th jump the sign of velocity} \begin{cases} \text{doesn't ch.} \\ \text{changes} \end{cases}$$

$$\eta_j = \begin{cases} +1 \\ -1 \end{cases} \text{if in the (j+1)-th jump the direction of time} \begin{cases} \text{doesn't ch.} \\ \text{changes} \end{cases}$$

with respect to the preceding j-th jump. It means that the realization of the signs of ε_j, η_j determines one of the four possible directions of the (j+1)-th jump on the basis of the direction of the j-th jump, as can be seen in Fig.4.

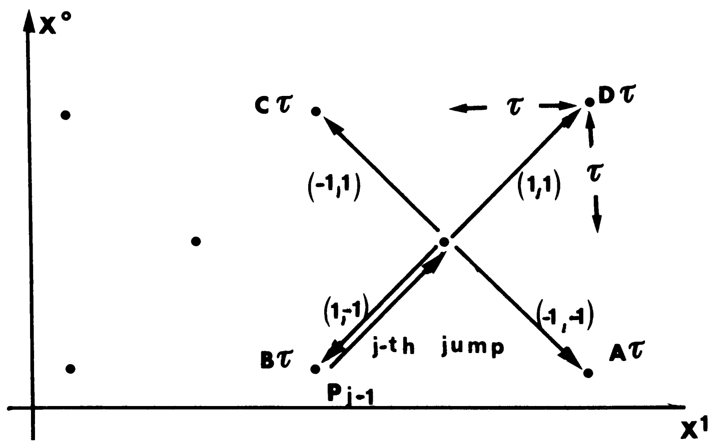

Fig.4. An example of the four possible successions of two jumps. For each possible (j+1)-th jump we marked the value of the couple (ε_j, η_j) and the corresponding probability.

Of course a sequence $\{\varepsilon_j, \eta_j\}$, with $j \in N$, of values of these stochastic variables completely determines one of the infinite possible trajectories, except for the first jump, because there is no "preceding" jump for it. Thus, starting from $P_0(x^0, x^1)$, in the first jump we can get one of the four possible points

$P_0(x^0+t\tau,\ x^1+s\tau)$ and, after N jumps, one of the points $P_N(x^0+tT_N,\ x^1+sD_N)$ where

$$T_N = \tau(1+\eta_1+\eta_1\eta_2+\ldots+\eta_1\eta_2\cdots\eta_{N-1}),$$
$$D_N = \tau(1+\varepsilon_1\eta_1+\varepsilon_1\varepsilon_2\eta_1\eta_2+\ldots+\varepsilon_1\varepsilon_2\cdots\varepsilon_{N-1}\eta_1\eta_2\cdots\eta_{N-1}). \qquad (14)$$

We come now to the problem of the assignment of a statistical weight to each trajectory. in order to do this e introduce for each júmp a probability for each realization of the signs of the corresponding couple ε_j, η_j. We listed these probabilities in Table 1. Moreover we, suppose that A,B,C,D are constant and positive over all of space-time.

Among these four constants we can also posit a relation that can be justified as a principle of mass-flux conservation. If we consider, e.g., as in Fig. 4, a particle arriving at P_j after its j-th jump, we must remember, for the (j+1)-th jump, that the

ε_j	η_j	Probability
-1	-1	$A\tau$
+1	-1	$B\tau$
-1	+1	$C\tau$
+1	+1	$D\tau$

Table 1. Probabilities for the four possible successions of two jumps.

particles proceeding backward in time must be consider as antiparticles going forward in time. From this perspective, if we want to conserve the flux of particles across the point P_j between the j-th and the (j+1)-th jumps, we must remark that:

(a) in the j-th jump we have only a particle going to the right;

(b) in the (j+1)-th jump we have a "fraction" $D\tau$ of particles and $B\tau$ of antiparticles going to the right and a "fraction" $C\tau$ of particles and $A\tau$ of antiparticles going to the left.

If we remember that particles and antiparticles have the same mass, the mass flux conservation across P_j finally gives

$$(-A + B - C + D)\tau = 1. \qquad (15)$$

We now consider a function $f(x^0,x^1)$ defined over all of space-time and, generall speaking, having complex values and then we introduce the following set of functions:

$$F_N^{t,s}(x^0,x^1) = \langle f(P_N)\rangle \quad , \tag{16}$$

where $\langle\cdot\rangle$ indicates an average over all the possible points P_N that are reached along trajectories consisting of N jumps, starting from P_0, with a first jump made in the direction fixed by (t,s). Of course, because of the arbitrariness of the starting point P_0 , the functions $F_N^{t,s}$ are defined over all of space-time.

We now make our average for the first jump, so that, from (15) and passing to the limit $N \to \infty$(for fixed τ), we have

$$\begin{aligned} F^{t,s}(x^0,x^1) &= F^{t,s}(x^0+t\tau,x^1+s\tau) \\ &A\tau\,[F^{-t,s}(x^0+t\tau,x^1+s\tau) + F^{t,s}(x^0+t\tau,x^1+s\tau)] \\ &B\tau\,[F^{-t,-s}(x^0+t\tau,x^1+s\tau) - F^{t,s}(x^0+t\tau,x^1+s\tau)] \\ &C\tau\,[F^{t,-s}(x^0+t\tau,x^1+s\tau) + F^{t,s}(x^0+t\tau,x^1+s\tau)] \quad , \end{aligned} \tag{17}$$

where $F^{t,s}$ are our functions for $N \to \infty$.

In the limit $\tau \to 0$, when our lattice tends to recover all of space-time, we get the following set of four partial differential equations:

$$\begin{aligned} -\partial_0\, F^{t,s} = \frac{s}{t}\,\partial_1 F^{t,s} + \frac{A}{t}(F^{-t,s}+F^{t,s}) + \frac{B}{t}(F^{-t,-s}-F^{t,s}) + \\ + \frac{C}{t}\,(F^{t,-s}+F^{t,s}). \end{aligned} \tag{18}$$

If we define now the following four linear combinations of $F^{t,s}$:

$$\begin{aligned} \phi &= F^{++} + F^{--} + F^{+-} + F^{-+}, \\ \chi &= F^{++} + F^{--} - F^{+-} - F^{-+}, \\ \psi &= -F^{++} + F^{--} - F^{+-} + F^{-+}, \\ \omega &= -F^{++} + F^{--} + F^{+-} - F^{-+}, \end{aligned} \tag{19}$$

we can build a new set of equations equivalent to (18):

$$\begin{aligned} \partial_0\phi + \partial_1\chi &= 2(C - B)\psi \ , \\ \partial_0\chi + \partial_1\phi &= 2(A - B)\omega \ , \\ \partial_0\psi + \partial_1\omega &= 2(A + C)\phi \ , \\ \partial_0\omega + \partial_1\psi &= 0. \end{aligned} \tag{20}$$

By differentiation and successive linear combination one gets

$$\begin{aligned}
\Box\phi &= 2(A-2B+C)\,\partial_0\psi - 4(A-B)(A+C)\,\phi\ ,\\
\Box\chi &= 2(A-2B+C)\,\partial_0\omega\ ,\\
\Box\psi &= -\,2(A+C)\,\partial_1\chi + 4(A+C)(C-B)\,\psi\ ,\\
\Box\omega &= 2(A+C)\,\partial_0\chi - 4(A+C)(A-B)\,\omega\ ,
\end{aligned}\tag{21}$$

(where $\Box$ is a two-dimensional d'Alembert operator).
Setting:

$$B = \frac{A+C}{2}\ , \qquad 2(A^2-C^2) = m^2, \tag{22}$$

one is finally led to

$$\begin{aligned}
(\Box + m^2)\phi &= 0,\\
\Box\chi &= 0,\\
(\Box + m^2)\psi &= -\,2(A+C)\,\partial_1\psi\ ,\\
(\Box + m^2)\omega &= 2(A+C)\,\partial_0\chi\ .
\end{aligned}\tag{23}$$

We interpret the first equation as a Klein-Gordon equation for a function ϕ which is the average of an arbitrary function f over all the final points that an reached along all possible trajectories realized in an infinite number of jumps; in this average we consider also the first jump by supposing that the four possibilities for the signs of t,s are equiprobable.

In the previous derivation we proved that each solution (ϕ,χ,ψ,ω) of (20) is a solution of (23), but it is possible to show that not all the solutions of (23) are solutions of (20). In other words, we proved the statement "the function ϕ defined as the average (19) always is a solution of a Klein-Gordon equation";now, what about the inverse statement, "all the solutions of a Klein-Gordon equation are interpretable as averages satisfying a system like (20)"? It is easy to show that, if ϕ is an arbitrary solution of the Klein-Gordon equation, we always can determine the functions χ,ψ,ω in such a way that (ϕ,χ,ψ,ω) is a solution of (20). In fact, if ϕ is an arbitrary solution of the Klein-Gordon equation, we choose χ as an arbitrary solution of $\Box\chi = 0$ and then postulate

$$\begin{aligned}
\psi &= \frac{1}{C-A}\,[\partial_0\phi + \partial_1\chi]\ ,\\
\omega &= \frac{1}{A-C}\,[\partial_0\chi + \partial_1\phi]\ .
\end{aligned}\tag{24}$$

It is only a question of calculation to show that this (ϕ,χ,ψ,ω) is a solution both of (20) and (23).

5. TWO-PARTICLE SYSTEM AND NON-LOCAL INTERACTIONS

The stochastic derivations of relativistic quantum equations can be generalized (at least starting from Nelson's equation (13)) to the case of two particles. The method (17) is simply based on the introduction of an eight-dimensional configuration space, so that the pair position is defined by an eight-component vector.

The same results were also obtained by Namsrai (18) in a recent paper.

We now remark that in this case, as first observed by Bohm, the quantum potential will be non local. Moreover, this result is in agreement with

(a) Ghirardi's observation (19) that the usual stochastic models of quantum mechanics can not eliminate the quantum non-locality;
(b) the consequences of the hydrodynamical analysis (11) of the two particle quantum field which led to instantaneous interactions.

Here we claim that this non-locality is based on the real action of a real quantum potential in a real material aether and that this non-local interaction solves the EPR paradox. (20) We shall devote here only a few words to explaining how we see the problem of the causal anomalies.

In our opinion, in a completely deterministic world there is no place for what we mean by the word "signal". With "completely deterministic world" we mean that we can uniquely predict all the word lines of all the particles by solving a Cauchy (with or without action at a distance) and starting with initial conditions given on a space-like surface. In such a world we claim that, even if we have non-local potentials or action at a distance, we cannot have causal anomalies because we can not have superluminal signals.

In fact, let us suppose that we are in such a completely deterministic world where particles interact even by means of non-local potentials. In that world we can solve the Cauchy problem for relativistic equations and then we can uniquely determine the world lines of our interacting particles by means of non-local potentials. Let us now consider two such particles (see Fig.5a), and let us explore the possibility for using action at a distance (i.e., the non-local potential connecting our two particles) to send superluminal signals from (1) to (2). How can we do it? Of course by disturbing the world line of (1) at a given time: This disturbance will propagate faster than light, or even instantaneously (see Fig.5b). Moreover, how can

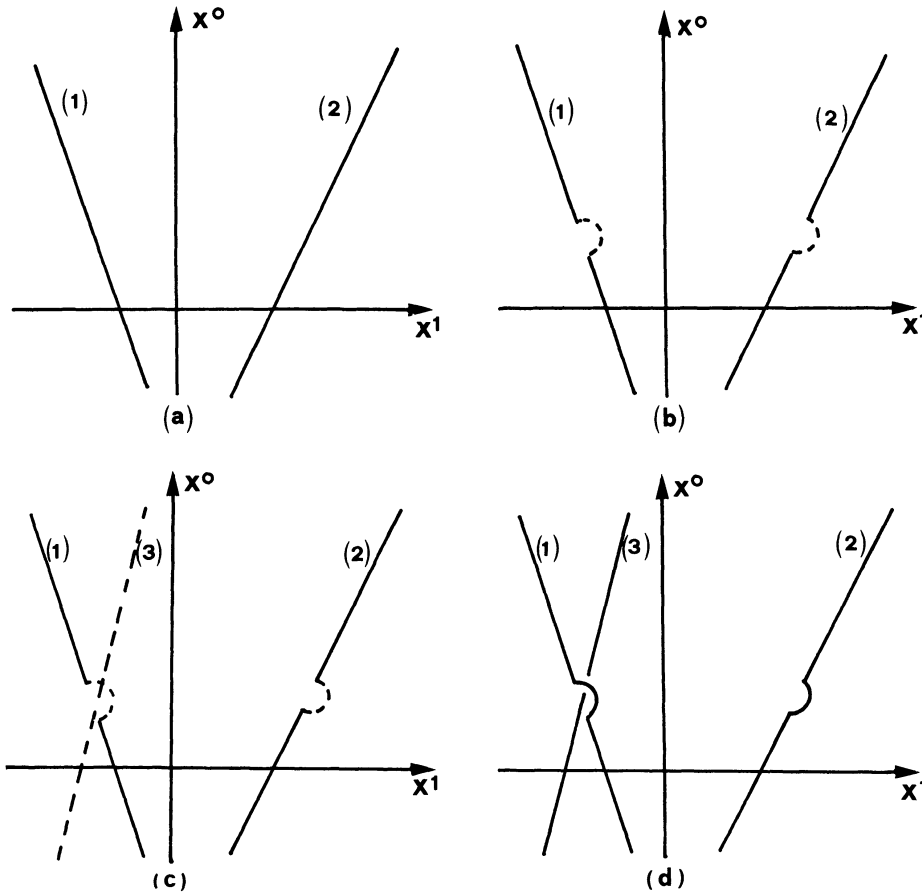

Fig. 5. What the word "signal" means in a completely deterministic world.

we disturb the particle (1)? Of course, by another particle (3) (see Fig.5c). But, if the world is uniquely determined, the world line of (3) was at its place in space-time before I decided to disturb (1), so that Fig. 5c can not be a real modification of Fig.5a. In a completely deterministic world there is no possible "modification": The world *IS* and we can not intervene from exterior in its tissue in order to modify it because we are *IN* the world. If the particle (3) exists, it produces no disturbance of (1) or (2): We have only to solve our Cauchy problem for three particles and we would have three world lines without disturbance, and thus without "signal" (see Fig. 5d).

In fact, it is clear that a "signal" always need a free will that is external to the physical world and that, at a given time, decides to modify a regular behaviour in order to send a message. Indeed, a regular behaviour never constitutes a signal. For example, we can always arrange a line of lamps so that they all light up in a very short time interval, independently of one another. In that case, if we look at them without knowing the arrangement, we could have the illusion of something traveling faster than light, but in reality it cannot constitute a signal at all.

From that standpoint, if, as we believe, the particles of the human brain are in the physical world and behaves like other particles, there are no "signals" at all. In other words, the world is an unique configuration of events in space-time, and our problem is not to find how to "produce" events but to find the general laws that express the disposition of the events. We can predict or remember, hope or regret, because we have a mind and a memory, and then we explore the space-time in a "historical" way and have the illusion of the "production" of the events by other events because we have a partial (in space and time) knowledge of the world. In a certain sense, our idea of causality seems to us to be the awareness of determinism together with the illusion of an event production , given by our lack of information. So, I cannot kill my grandfather before I was born; but I will say more: I cannot kill my grandchild, if that it is not already written in the space-time. In the words of a rationalist philosopher of the XVII century, "Men are wrong in considering themselves free, and this opinion consists only in the fact that they are conscious of their actions and unaware of the causes which determines them ".(21)

In conclusion, if our world is completely deterministic, we have no problems about signals. We shall not discuss here the philosophical problems of a deterministic standpoint, but we will limit ourselves to posing the main physical question: Is a relativistic and completely deterministic picture of the world compatible with action at a distance? concerning this problem, Currie, Jordan and Sudarshan (22) demonstrated in 1963 the well-known "non-interaction theorem". Briefly this theorem states that, if in a classical phase space one wishes to describe a system of particles in such a manner that (1) the dynamics is given by a Hamiltonian, (2) the theory is relativistically covariant, (3) the coordinate variables of the individual particles transform correctly under Lorentz transformations, then the only such system is a collection of free particles. Of course, under these condition it is completely impossible to build a covariant deterministic world with non-local potentials.

A recent advance in analyical mechanics is the result that a predictive mechanics is possible if some particular conditions of compatibility are satisfied by the non local potentials. (23) In this way the door to construct a completely deterministic and covariant world with action at a distance is reopened; and, from the standpoint of a deterministic interpretation of the quantum mechanics, the main question becomes: Does the quantum potential for two correlated particles satisfy the compatibility conditions of predictive mechanics? The answer is yes (20); the detail of this demonstration is discussed in the paper of Prof. Vigier.

REFERENCES

1. E.Madelung, *Z.Phys.* *40*, 332 (1926).
2. D.Bohm, *Phys.Rev.* *85*, 166,180 (1952); *Phys.Rev.* *89*, 458(1953); *Prog.Theor.Phys.9*, 273 (1953).
3. T.Takabayasi, *Prog. Theor.Phys.* *8*, 143 (1952); *Prog.Theor.Phys.* *9*, 187 (1953).
4. W.Pauli, *Louis de Broglie, physicien et penseur* (Albin Michel, Paris, 1953).
5. J.B.Keller, *Phys.Rev.* *89*, 1040 (1953).
6. P.A.M. Dirac, *Nature* *168*, 906 (1951) ; *Nature* *169*, 702(1952).
7. K.P.Sinha, C.Sivaram, and E.C.G. Sudarshan, *Found.Phys.6*, 65, 717 (1976).
   K.P.Sinha and E.C.G.Sudarshan, *Found.Phys.8*, 823 (1978).
8. N.Cufaro Petroni and J.P.Vigier, *Found.Phys.13*, 253 (1983).
9. J.P.Vigier, *Lett.N.Cim.* *29*, 467 (1980); *Astron.Nachr.303*, 55 (1982).
10. F.Selleri, *Lett.N.Cim.* *1*, 908 (1969).
    A.Garuccio, K.Popper and J.P.Viger, *Phys.Lett.* *86A*, 397 (1981).
    J.et M.Andrade e Silva, *C.R.Acad.Sc.Paris* *290*, 50 (1980).
11. N.Cufaro Petroni, Ph.Gueret and J.P.Vigier, "Hydrodynamical Analysis of Spinor Fields" (Bari University preprint, 1983).
12. F.Halbwachs, *Theorie relativiste des fluides à spin* (Gauthier-Villars, Paris, 1960).
13. E.Nelson, *Phys.Rev.* *150*, 1079 (1966).
    L.de la Peña Auerbach, E.Braun and L.S.Garcia Colin, *J.Math. Phys.* *9*, 668 (1968).
    L.de la Peña Auerbach, *J.Math.Phys.* *10*, 1620 (1969).
    L.de la Peña Auerbach and A.M. Cetto, *Found.Phys.5*, 355(1975).
    W.Lehr and J.Park, *J.Math.Phys.18*, 1235 (1977).
    F.Guerra and P.Ruggiero, *Lett.N.Cim.* *23*, 529 (1978).
    J.P.Vigier, *Lett.N.Cim.* *24*, 265 (1979).
    N.Cufaro Petroni and J.P.Vigier, *Phys.Lett.* *73A*, 289 (1979); *Phys.Lett.* *81A*, 12 (1981).
14. N.Cufaro Petroni and J.P.Vigier, "Random Motions at the Velocity of Light and Relativistic Quantum Mechanics", in press on *J.Phys. A*.
15. A.Avez, "Interpretation Probabiliste d'Equations aux Derivées Partielles Hyperboliques Normales" (preprint Paris).
16. R.P.Feynman, *Phys.Rev.76*, 749,769 (1949).
17. N.Cufaro Petroni and J.P.Vigier, *Lett.N.Cim.* *26*, 149 (1979).

18. Kh.Namsrai, *Sov.J.Part.Nucl.* *12*, 449 (1981); *J.Phys.* *14A*,1307 (1981).
19. G.C.Ghirardi, C.Omero, A.Rimini and T.Weber, *Riv.N.Cim.* *1*,1 (1978).
20. N.Cufaro Petroni, Ph.Droz Vincent and J.P.Vigier,*Lett.N.Cim.* *31*, 415 (1981).
    N.Cufaro Petroni and J.P.Vigier, *Phys.Lett.* *88A*, 272 (1982); *Phys.Lett.93A*, 383 (1983).
21. B.Spinoza, *Ethica*, Part 1, Appendix; Part 2, prop. XXXV(scolium). (UTET, Torino, 1972).
22. D.G.Currie,T.F.Jordan and E.C.G.Sudarshan, *Rev.Mod.Phys.35*, 350 (1963).
23. Ph. Droz Vincent, *Phys.Scripta 2*, 129 (1970).
    L.Bel, *Ann. Inst. H.Poincaré 3*, 307 (1970).
    Ph. Droz Vincent, *Ann. Inst. H.Poincaré 27*, 407 (1977).
    A.Komar, *Phys.Rev. 18D*, 1881, 1887, 3617 (1978).
    L.Bel,*Phys.Rev. 18D*, 4770 (1978).
    Ph. Droz Vincent, *Phys.Rev. 19D*, 702 (1979);*Ann. Inst. H.Poincaré 32*, 377 (1980).
    R.P.Gaida, *Sov.J.Part.Nucl. 13*, 179 (1982).