Nuclear matrix elements for the neutrinoless Double Beta Decay: A critical review

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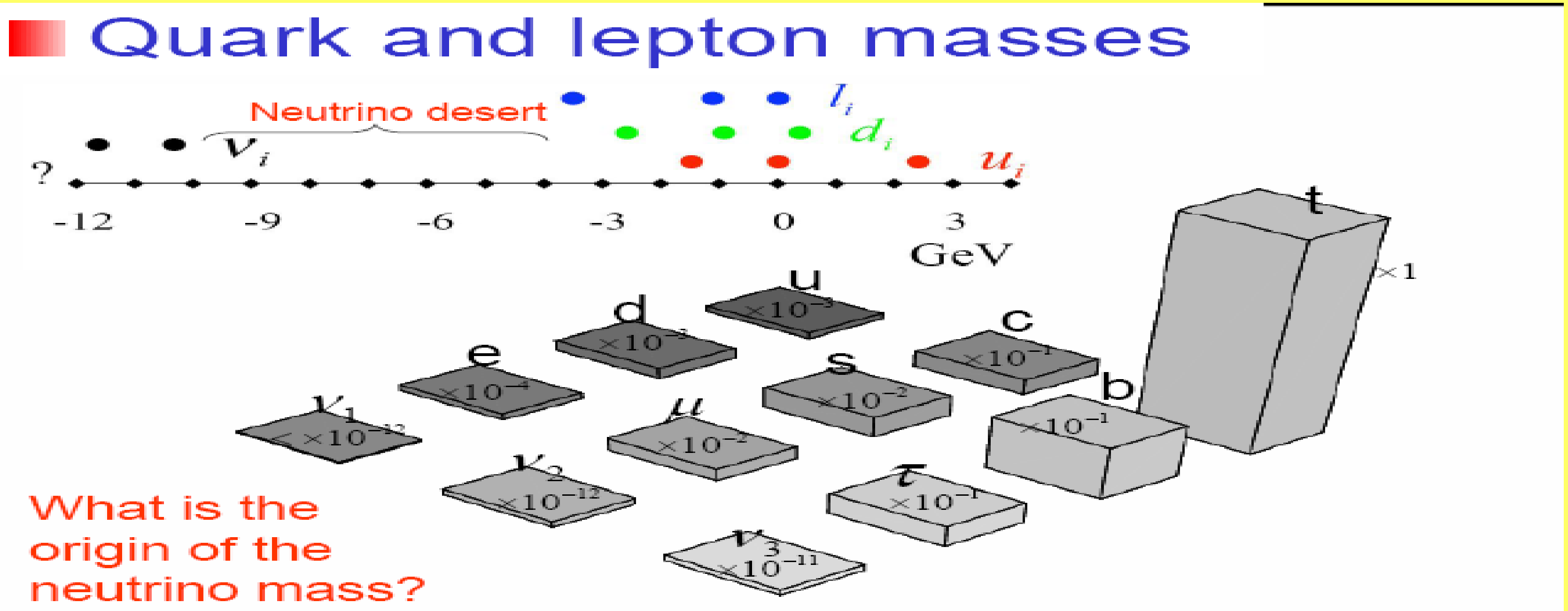
- Why is the Neutrino Mass so small ?
- What is the Origin of the Neutrino Mass?
- ν Masses(KATRIN; 0νββ matrix elements; astrophysics)
- Majorana or Dirac Neutrinos?



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Neutrino Masses

- Single Beta Decay (Mainz, Troisk)

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$ < 2.2 [eV]

- Double Beta Decay Majorana Mass (Tübingen):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$ < 0.27 [eV]

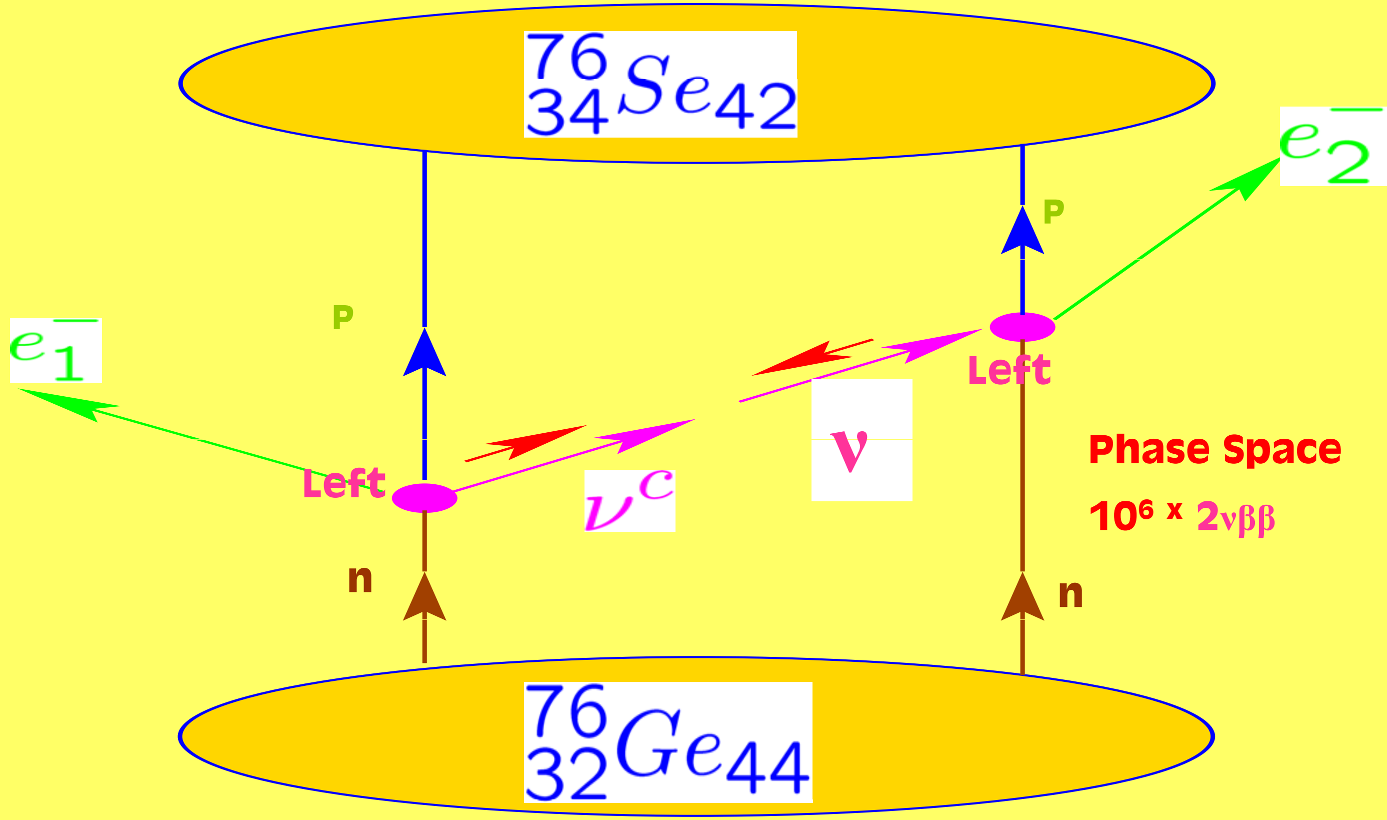
- Astophysics: $\Sigma = m_1 + m_2 + m_3 < 0.17$ to 2.0 [eV]

  Depends on Cosmological models (Hannestad)

- The neutrinoless Double Beta Decay forbidden in the Standard Model allows to determine the mass of massive Majorana Neutrinos, (if it yields the biggest term).
- Matrix elements as important as the data.
- Practically all Grand Unified Theories and Supersymmetry request massive Majorana Neutrinos

$$\nu = \bar{\nu}; \quad m_\nu > 0$$

0νββ-Decay (forbidden in Standard Model)



only for massive Majorana Neutrinos

$\nu = \nu^c$

GRAND UNIFICATION

Left-right Symmetric Models SO(10)

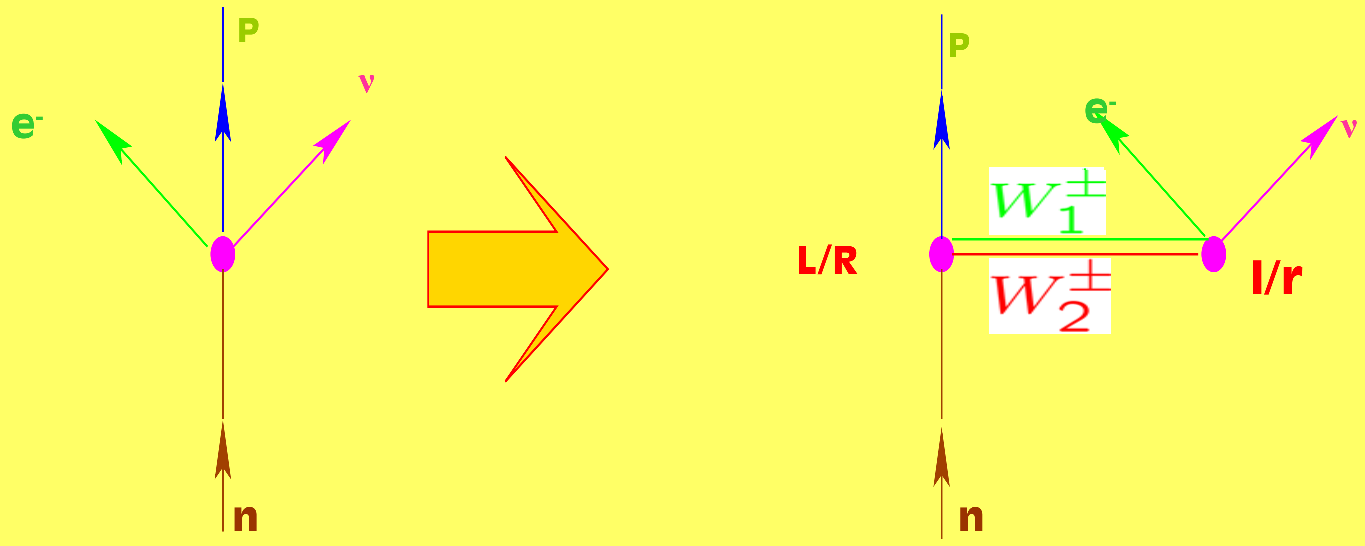
$$W_L^{\pm}; W_R^{\pm}; \quad \nu = \nu^c$$

Majorana Mass:

$$\mathcal{L} = \ldots + m_\nu \bar{\nu}^c \nu + m_\nu^* \bar{\nu} \nu^c$$

$$W_1^{\pm}(81GeV) = cos\vartheta W_L^{\pm} + sin\vartheta W_R^{\pm}$$

$$W_2^{\pm}(?GeV) = -sin\vartheta W_L^{\pm} + cos\vartheta W_R^{\pm}$$



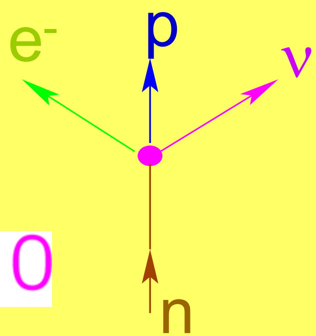
$$l/r = \bar{e}\gamma^{\mu}(1 \mp \gamma^5)\nu$$

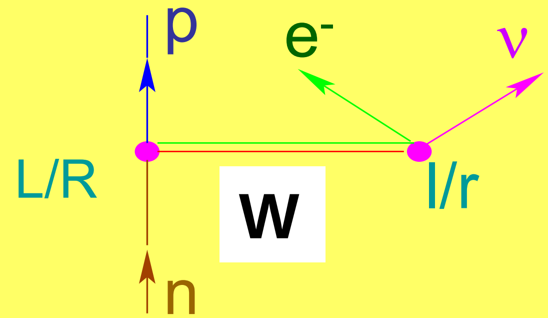
$$L/R = p^{+}(g_V \mp g_A \vec{\sigma})n$$

$g_V = 1;$ $g_A = 1.25$

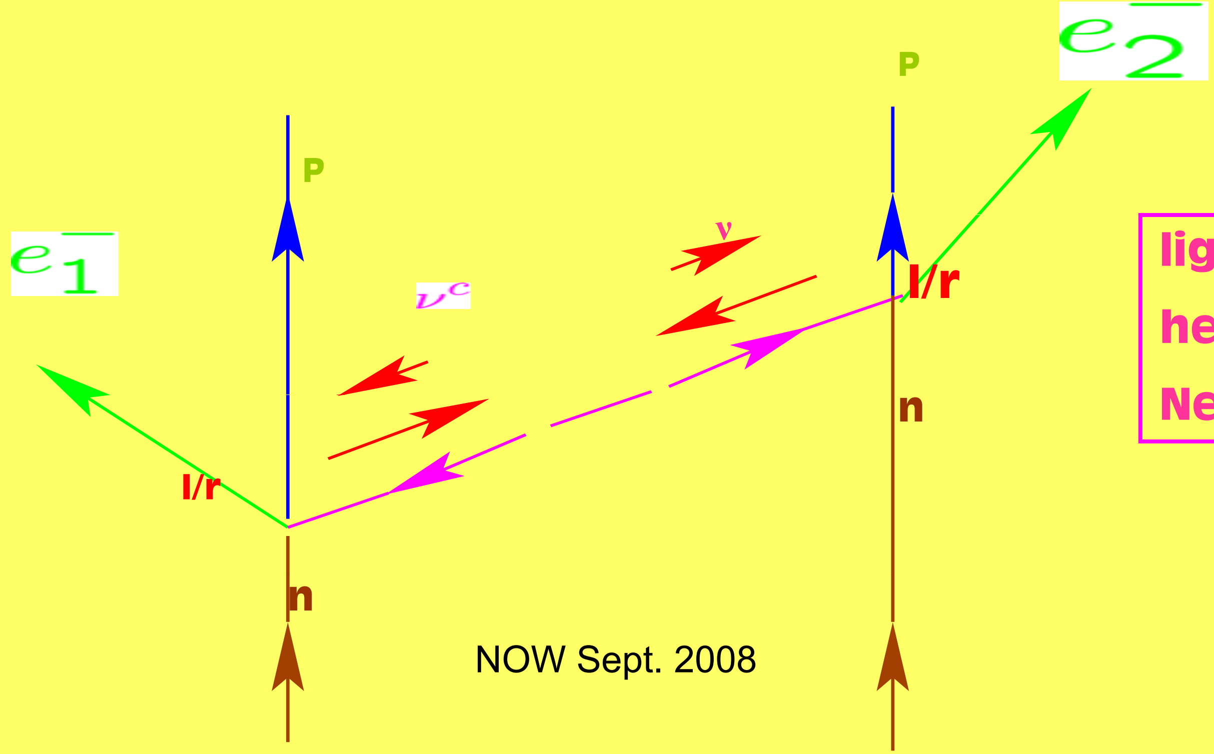
$$\hat{H}_{weak} =$$

$m_\nu \neq 0$





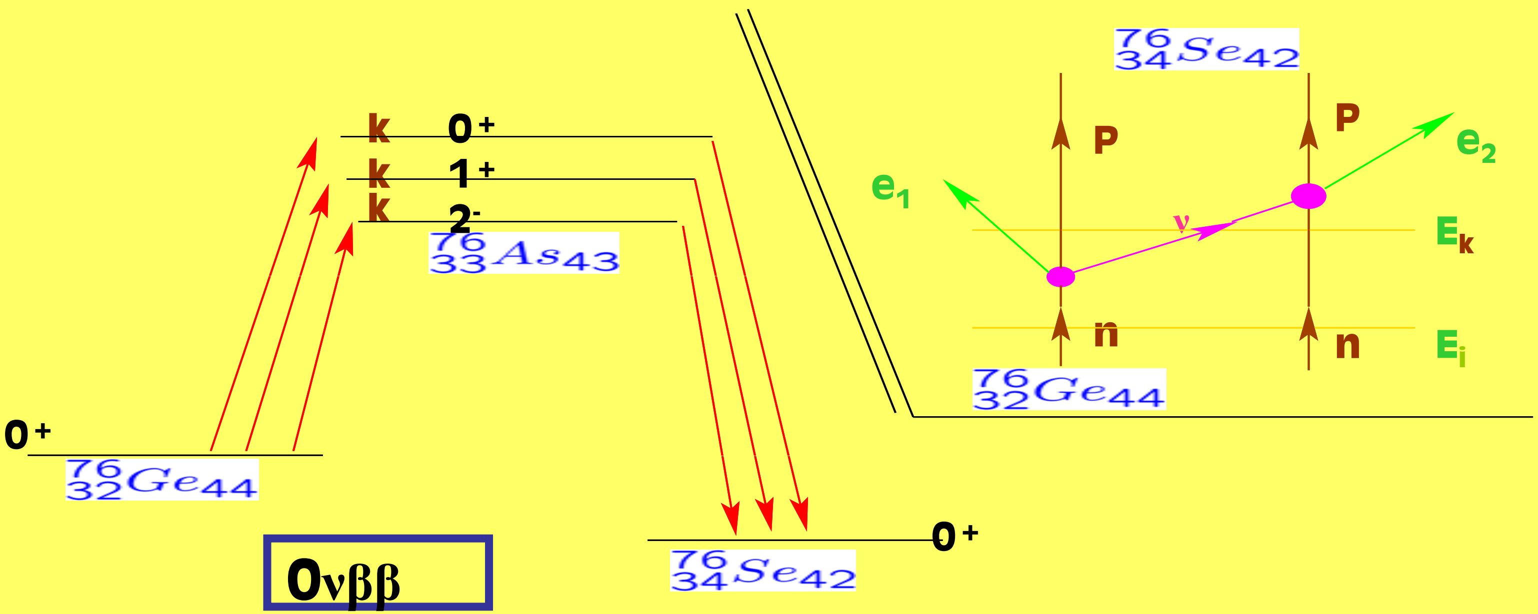
$$\hat{H}_{Weak}{}^{GUT} = \frac{G_F cos\vartheta_c}{\sqrt{2}}[1 \cdot l \cdot L + tg\vartheta r \cdot L + tg\vartheta l \cdot R + \frac{M_1^2}{M_2^2} r \cdot R + \ldots]$$



light ν
heavy N
Neutrinos

Theoretical Description of Nuclei:

Vadim Rodin, Fedor Simkovic,
Amand Faessler, Saleh Yousef



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The best choice: Quasi-Particle Random Phase Approximation (QRPA) and Shell Model

QRPA starts with Pairing: $a_i^+ = u_i c_i^+ - v_i c_{\bar{i}}$

$$A_\alpha^+ = [a_i^+ a_k^+]_{J\alpha}$$

$$Q_m^+ = \Sigma_\alpha [X_\alpha^m A_\alpha^+ - Y_\alpha^m A_\alpha]$$

$$|m> = Q_m^+|g>$$

$$[A_\alpha, A_\beta^+] = \delta_{\alpha,\beta} + \hat{X}$$

$$\hat{H} Q_m^+|g> = E_m Q_m^+|g> \hookrightarrow X_m^\alpha; Y_m^\alpha; E_m$$

Neutrinoless Double Beta-Decay Probability

$$T = \int dE_k(\nu)\Sigma_k \frac{< f|\hat{H}_W|k >< k|\hat{H}_W|i >}{E_{0^+}(^{76}Ge) - [E_k(e_1^-) + E_k(\nu) + E_{Kern}(^{76}As;k)]}$$

$$T = M_m < m_\nu > + M_\theta < tg\vartheta > + M_{WR} < \left(\frac{M_1}{M_2}\right)^2 >$$

$$+ M_{SUSY}\lambda'^2_{111} + M_{VR} < \frac{m_p}{M_{VR}} > + \ldots$$

$$w = \frac{2\pi}{\hbar}|T|^2\rho_f \leq 4.4 \cdot 10^{-33}[sec^{-1}] \quad ^{128}Te$$

Single Beta Decay
Electron Capture

$$m^2(\nu_e) = \sum_{i=1,2,3} |U_{ei}|^2 m_i^2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$|\langle m_{\beta\beta} \rangle| = |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}|$$

$$= |m_1 |U_{e1}|^2 \pm m_2 |U_{e2}|^2 \pm m_3 |U_{e3}|^2|$$

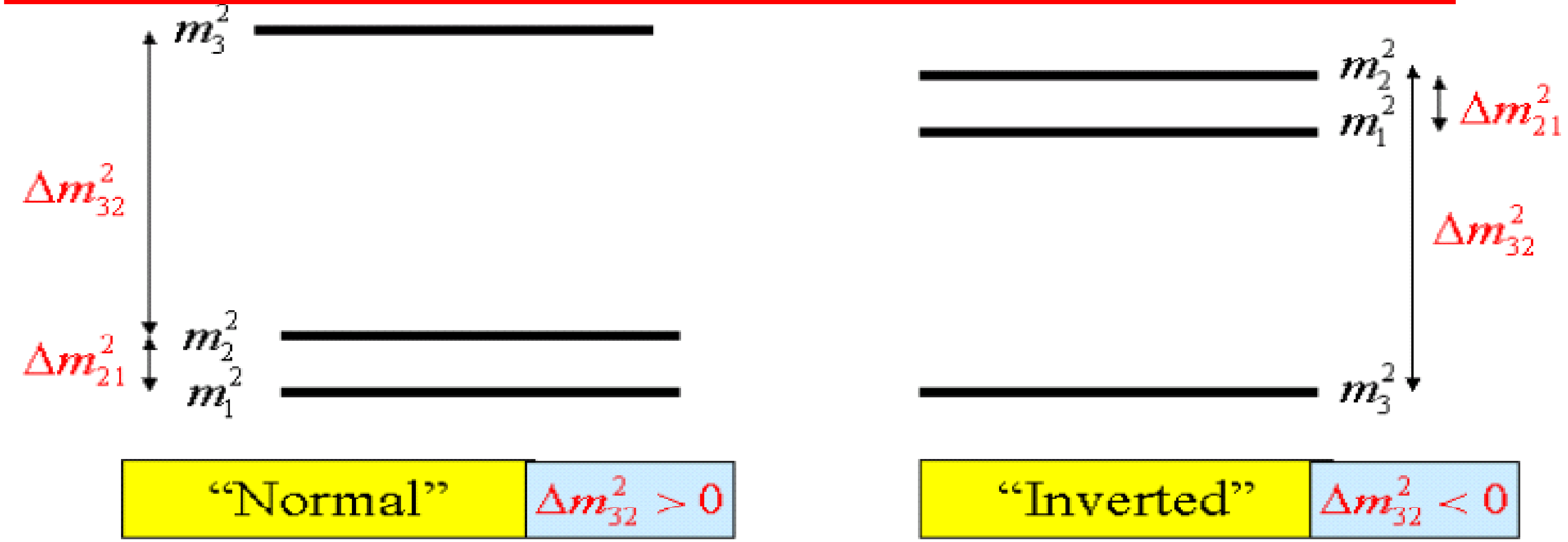
CP

Time reversal

CPT

Results from Oscillations:
No Hierarchy, no absolute Mass Scale

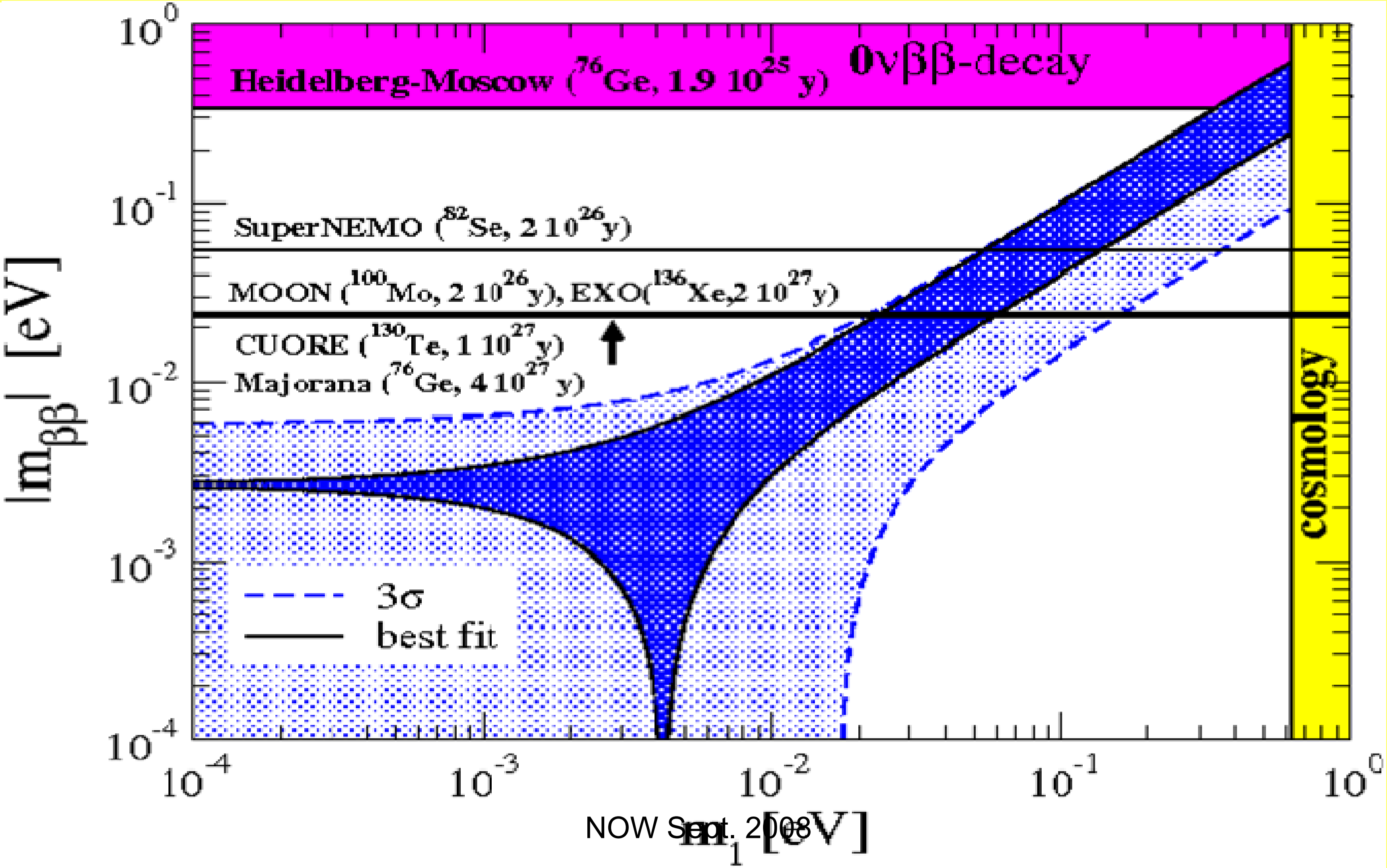
Fogli, Lisi, Marrone, Palazzo: Prog. Part. Nucl. Phys. 57(2006)742



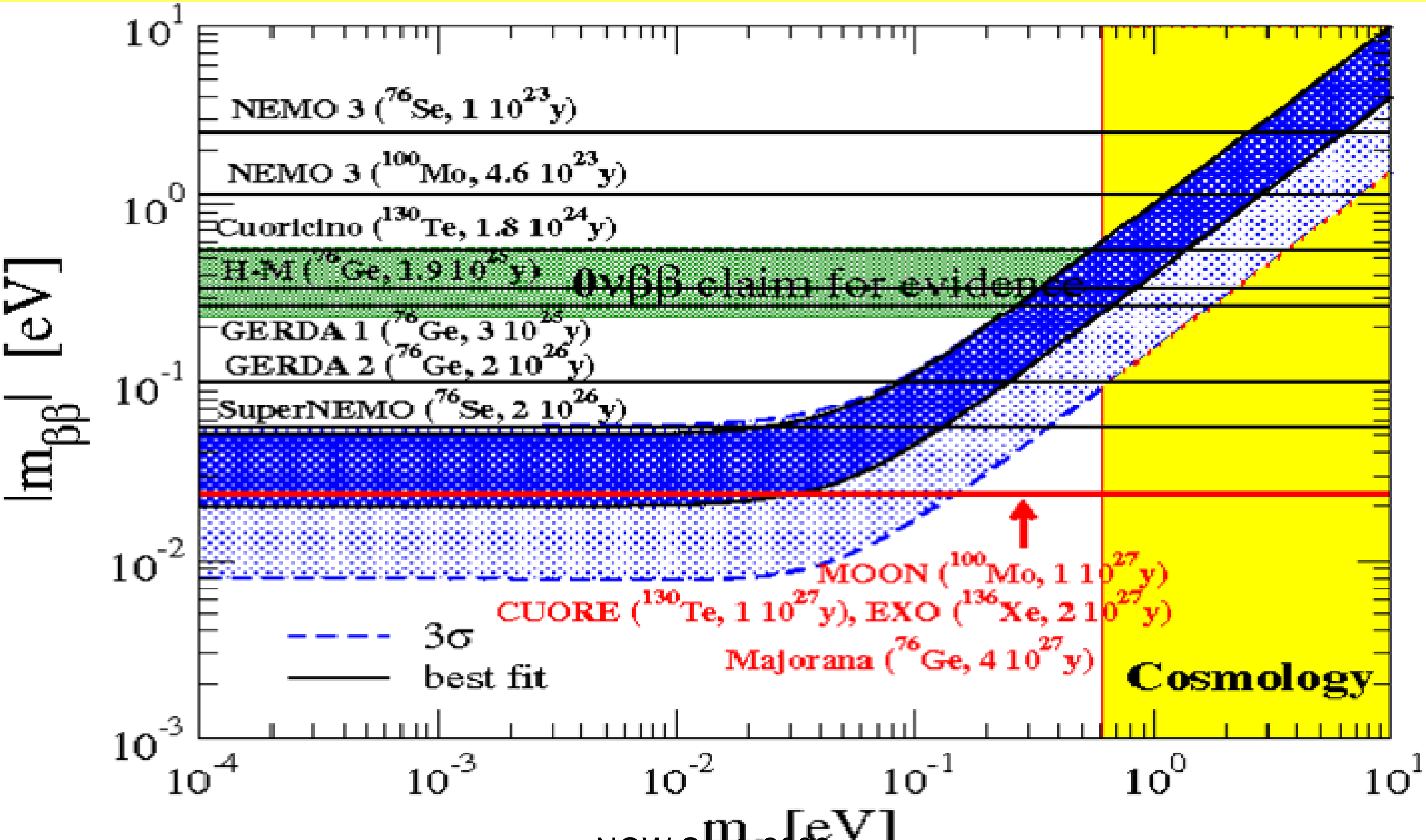
$$\Delta m_{21}^2 = 7.92 \cdot 10^{-5}[eV^2];\ \Delta m_{32}^2 = 2.4 \cdot 10^{-3}[eV^2]$$

$$\vartheta_{23} \approx 41.6^\circ \quad \vartheta_{12} \approx 34.1^\circ \quad |\vartheta_{13}| \leq 9^\circ$$

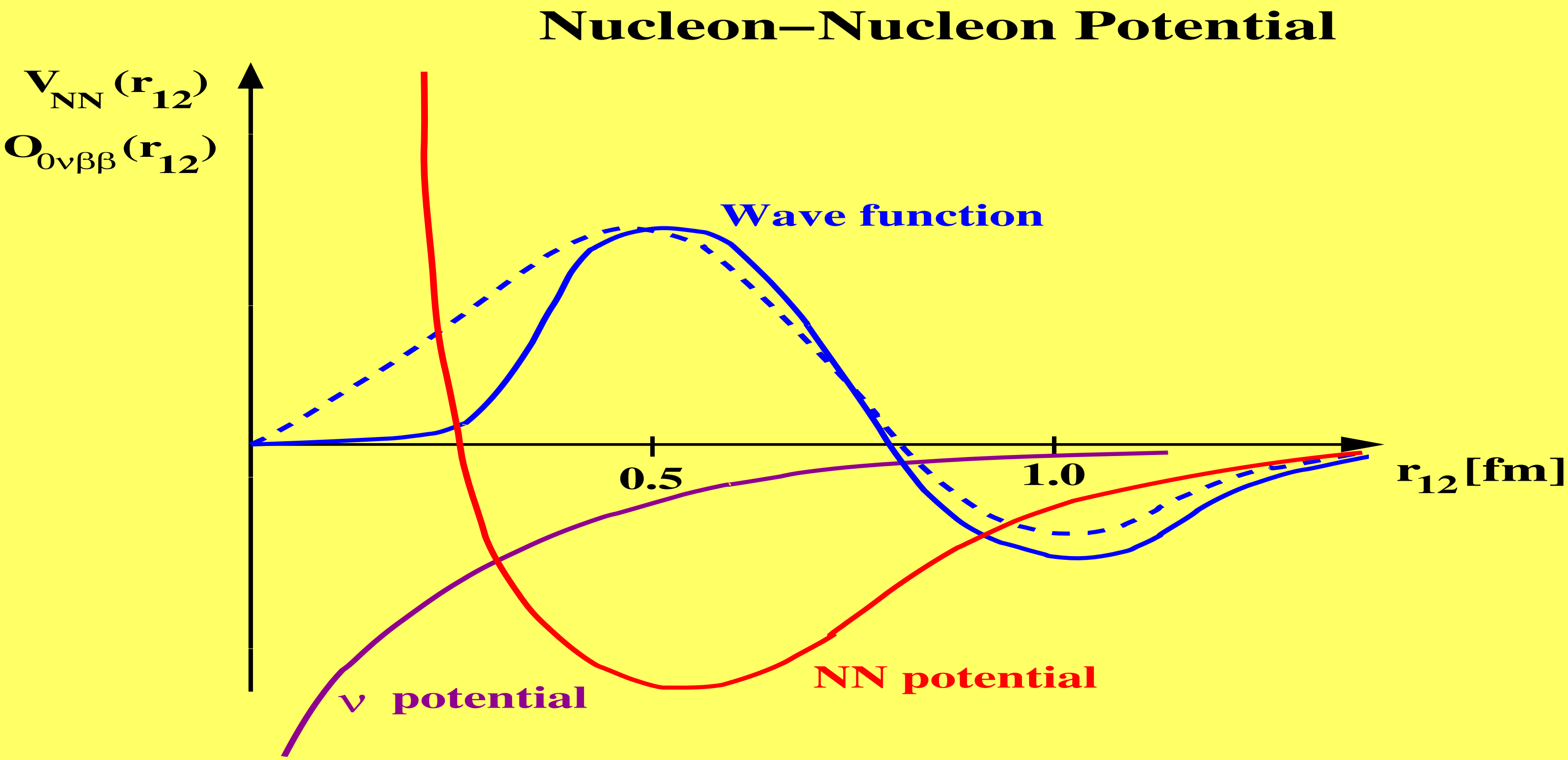
Normal Hierarchy: Double Beta Decay
Majorana Mass $m_{\beta\beta}$ versus lowest mass m_1



Inverted Hierarchy: Double Beta Decay
Majorana Mass $m_{\beta\beta}$ versus lowest mass m_3



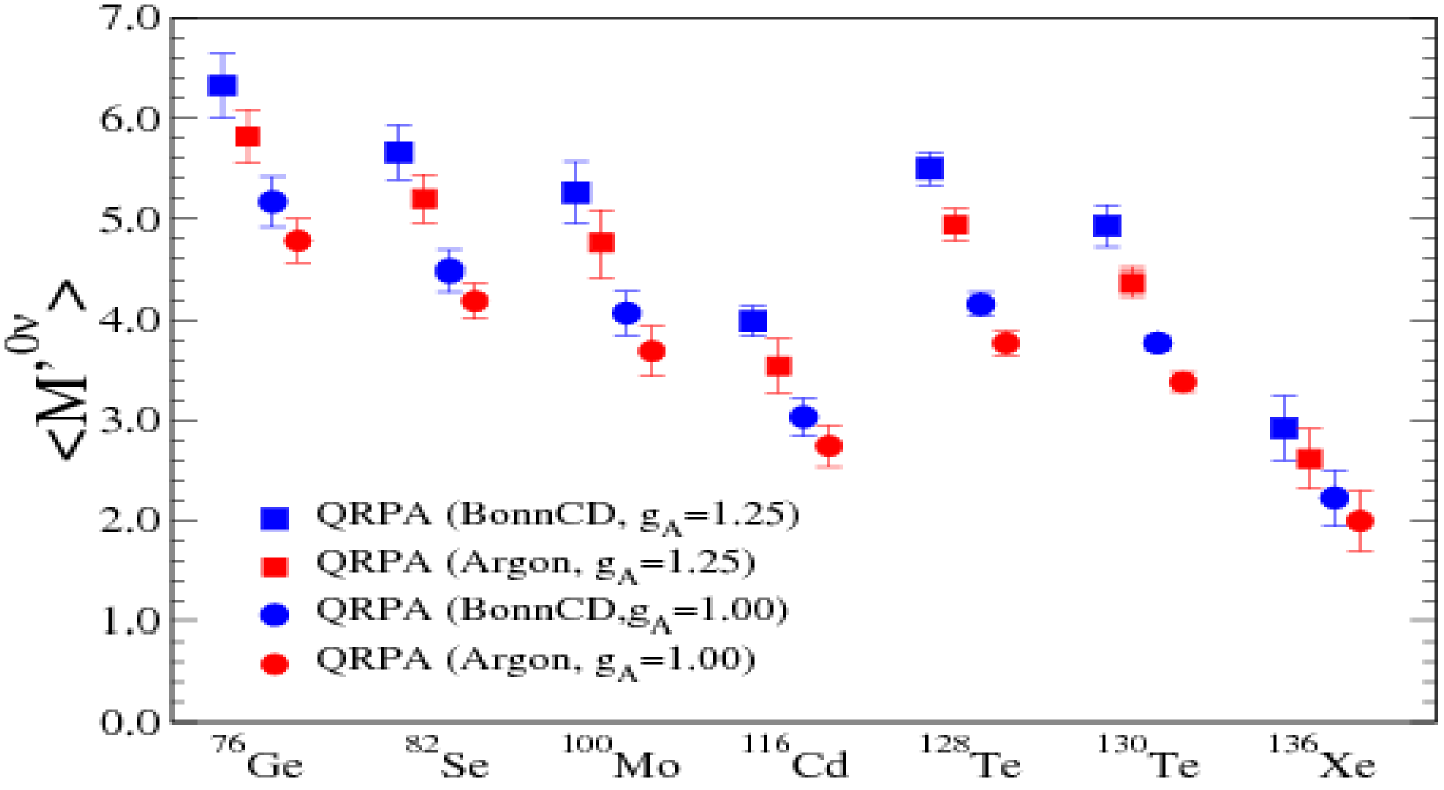
Uncorrelated and Correlated Relative N-N- (two neutrons → two protons) Wavefunction in the N-N-Potential



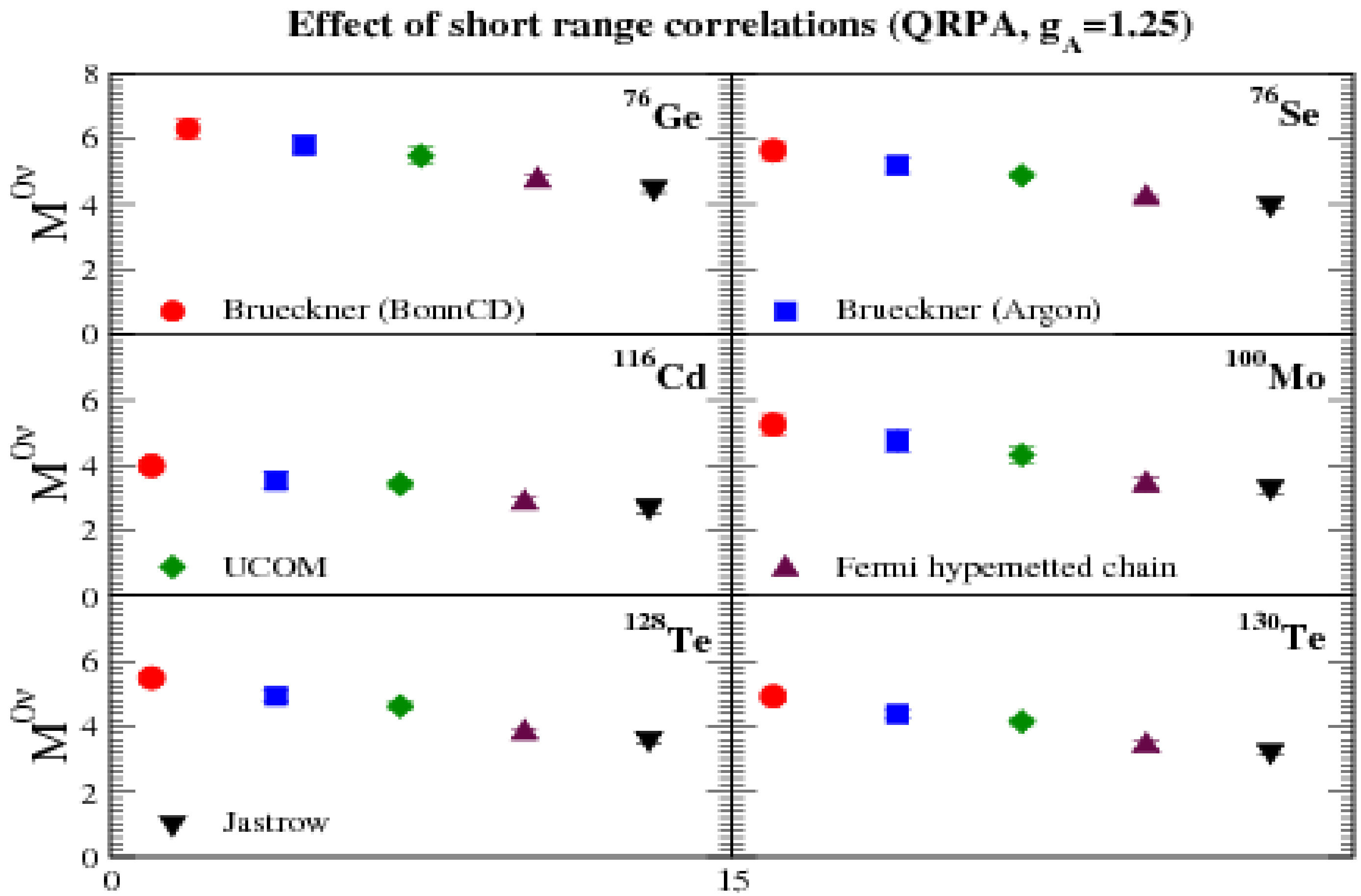
Short Range Correlations

First consistent treatment of pairing, nucleon-nucleon G-matrix, nuclear structure and short range correlation by the same realistic force by Brückner theory (Relativist. BG-eq.)

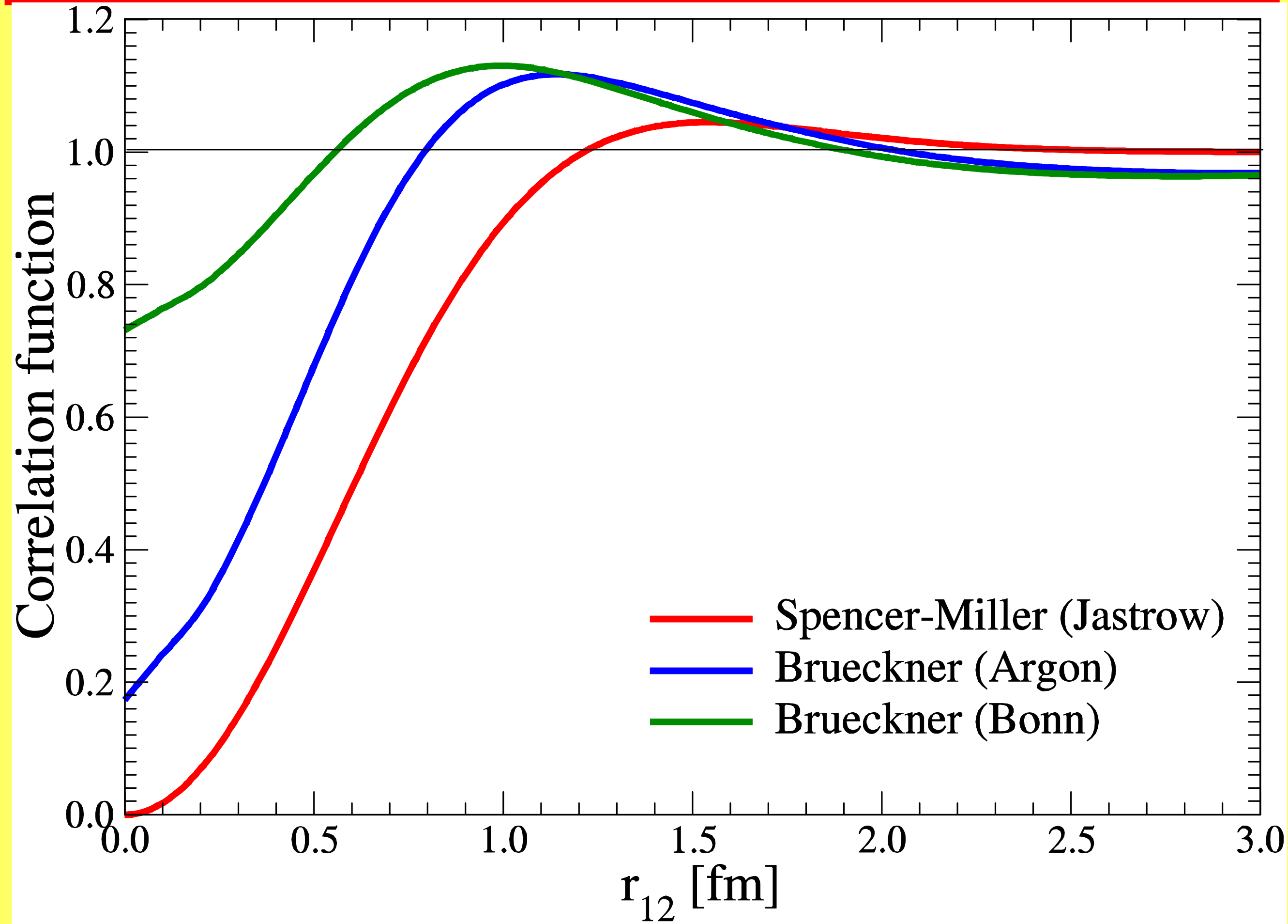
Rodin, Simkovic, Stauf, Muether and Faessler.



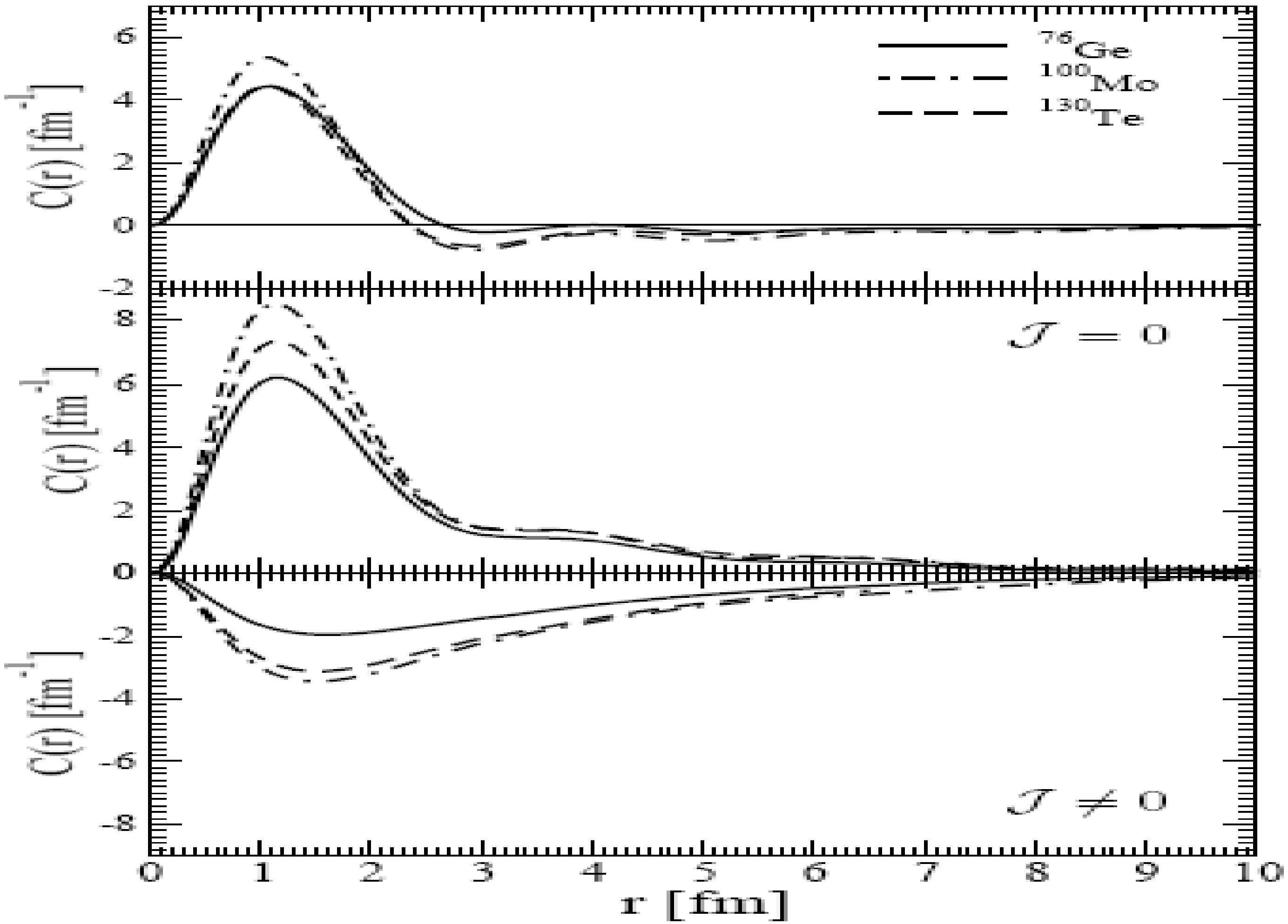
Different treatments of short range correlations: Brückner Bonn CD; Brückner Argonne; UCOM; Hypernetted chain; Jastrow



Different Short Range Correlations



Radial Contributions to the 0νββ Matrix Element



Different treatment of Short Range Correlations (SRC)

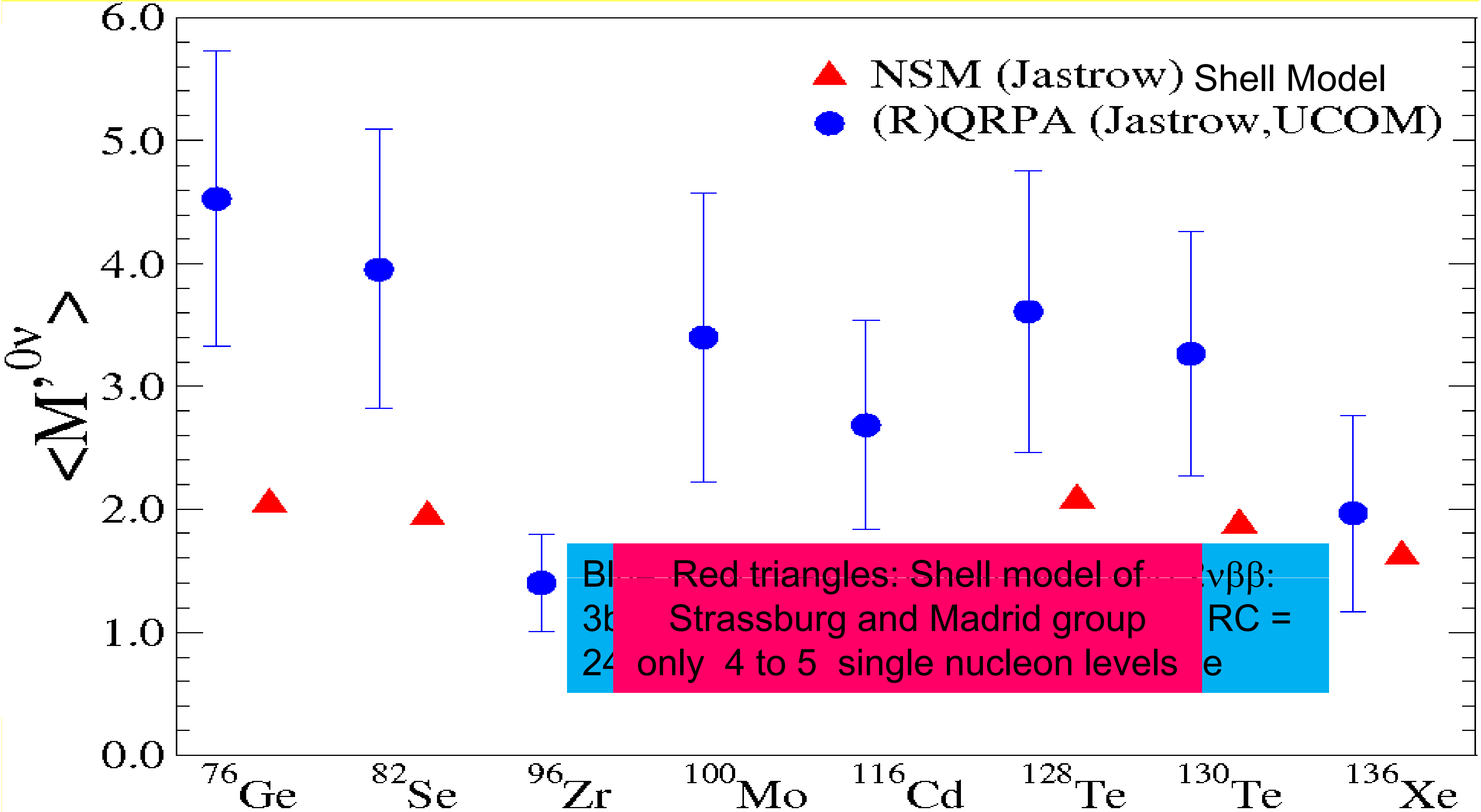
- Brückner Correlations: The same realistic force for Pairing, G-matrix elements, for the solution of the Many Body Problem and Short Range Correlations. All as solutions of the same relativistic Bethe-Goldstone Eq.. First fully consistent calculation.

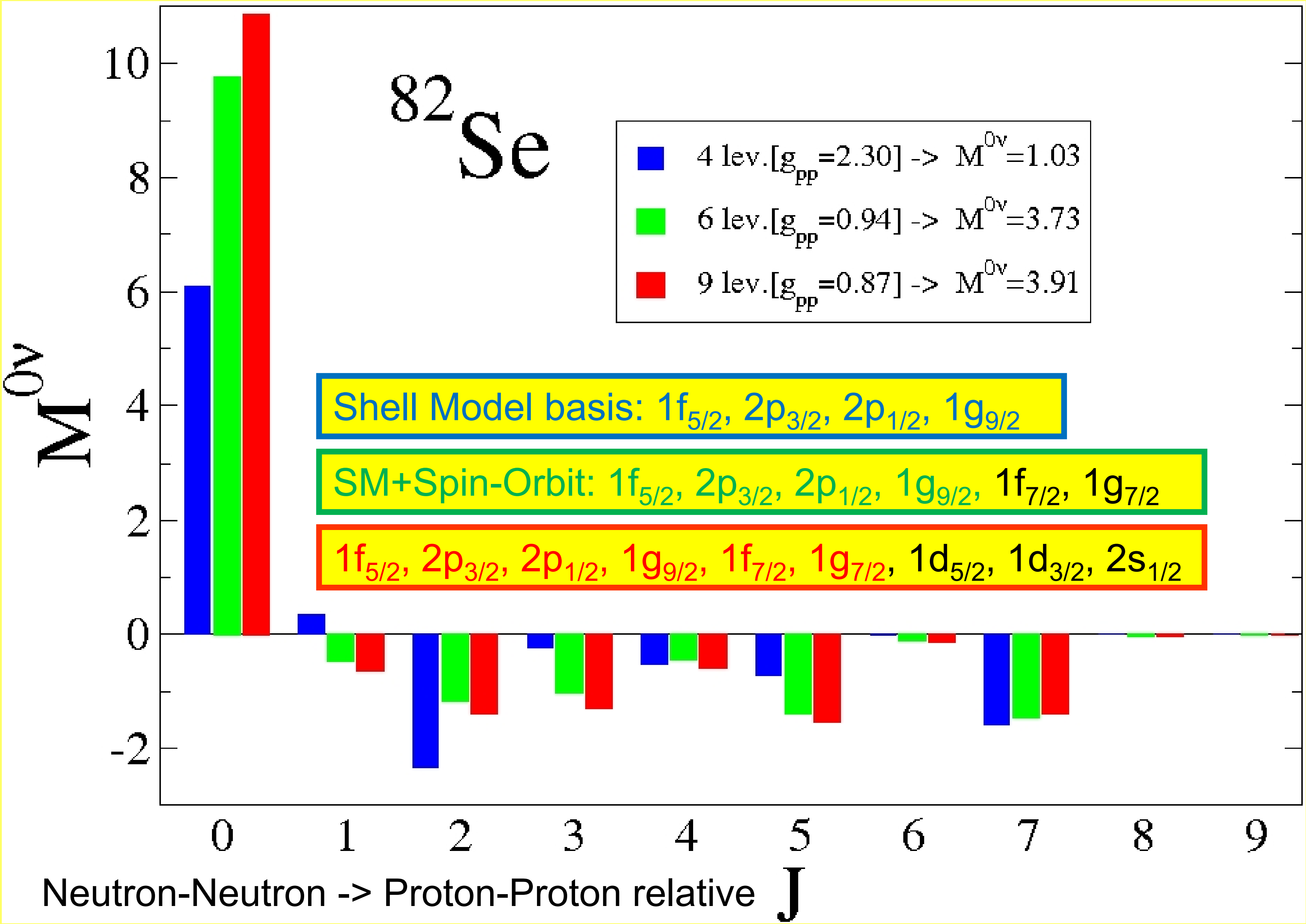
- UCOM: Different forces and treatment for Nuclear Structure and SRC. SRC (AV18) fitted to an analytic function with about 3(plus) parameters to the Deuteron and suppression of the Long Range Changes.

- Fermi Hypernetted chain: SRC parametrized by 3(plus) parameters fixed by the many body variational approach (Argonne).

- Jastrow: Fit of Gerry Miller and Spencer to Brückner results with two parameters to S-waves.

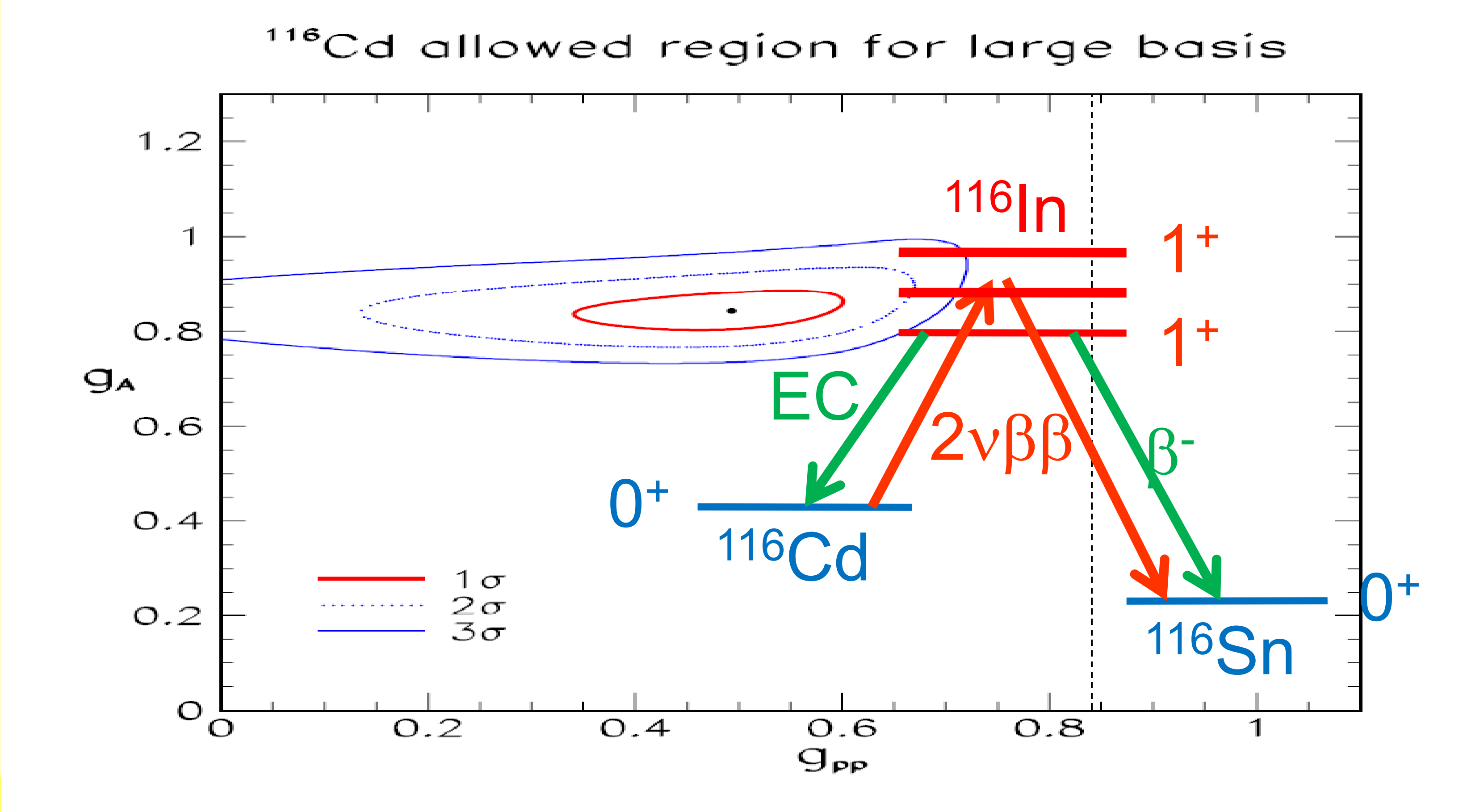
QRPA and RQRPA with errors from Basis Size, exp. 2-Neutrino Decay, axial Coupling Constant g_A = 1.25 and 1.00 and Short Range Correlations (UCOM + Jastrow)

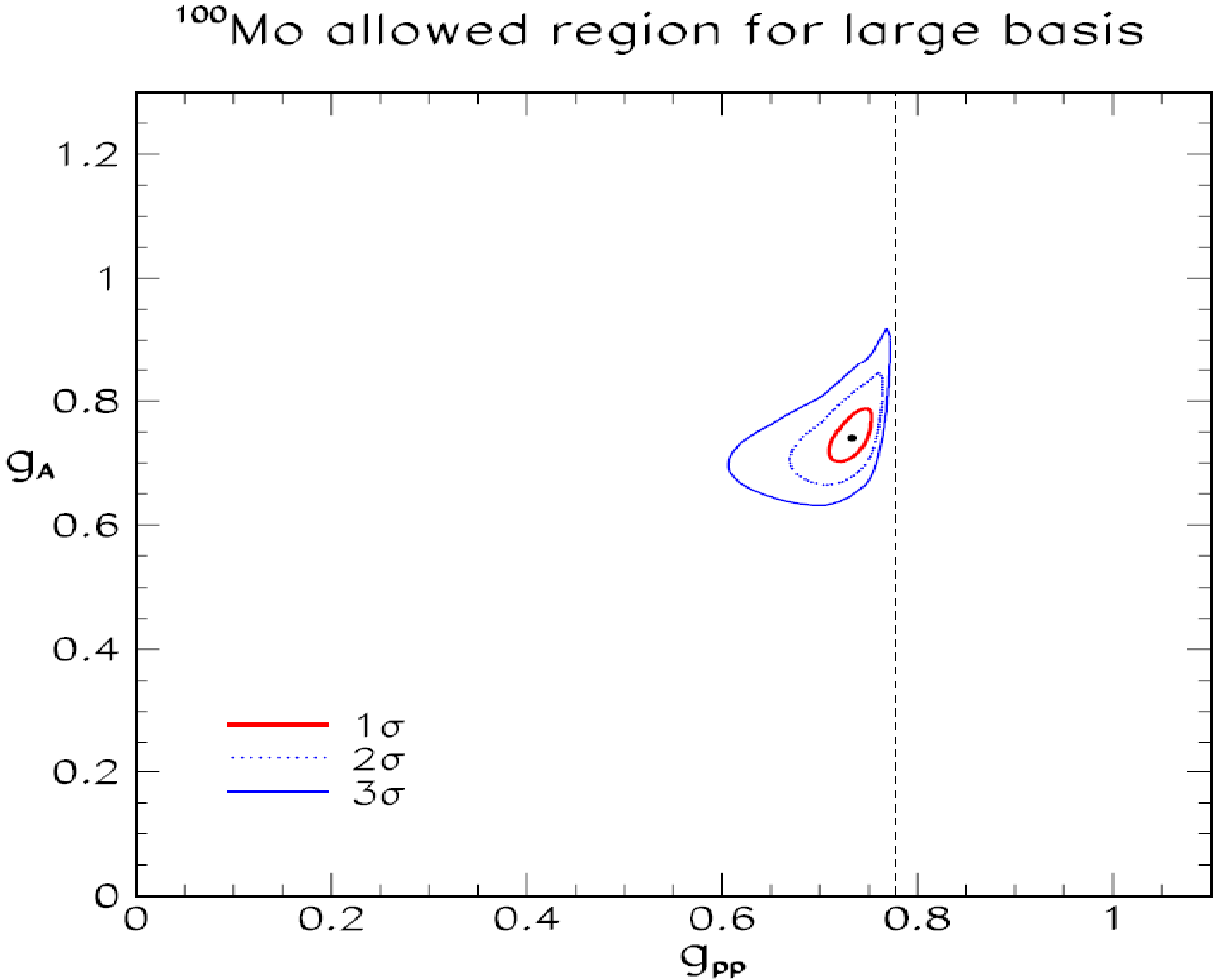




^{82}Se
4 lev.[g_{pp}=2.30] -> $M^{0\nu}$=1.03
6 lev.[g_{pp}=0.94] -> $M^{0\nu}$=3.73
9 lev.[g_{pp}=0.87] -> $M^{0\nu}$=3.91
Shell Model basis: $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$
SM+Spin-Orbit: $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1f_{7/2}$, $1g_{7/2}$
$1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $1g_{9/2}$, $1f_{7/2}$, $1g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$
$M^{0\nu}$
10
8
6
4
2
0
-2
0 1 2 3 4 5 6 7 8 9
Neutron-Neutron -> Proton-Proton relative J

Overconstraining the 0νββ Matrix Element
Faessler, Rodin, Fogli, Lisi, Rodin Rotunno, Simkovic; ArXiv: 0711.3996[nucl-th]; ^{116}Cd, ^{100} Mo, ^{128}Te





^{100}Mo allowed region for large basis
1.2
1
0.8
0.6
0.4
0.2
0
g_A
0
0.2
0.4
0.6
0.8
1
g_{pp}
1σ
2σ
3σ

Quenched Axial Charges g_A in Medium Mass Nuclei

Skouras, Manakos; J. Phys. G 19(1993) 731

^{98}Cd, ^{94}Ru: $g_A = 0.47 \rightarrow 0.62$

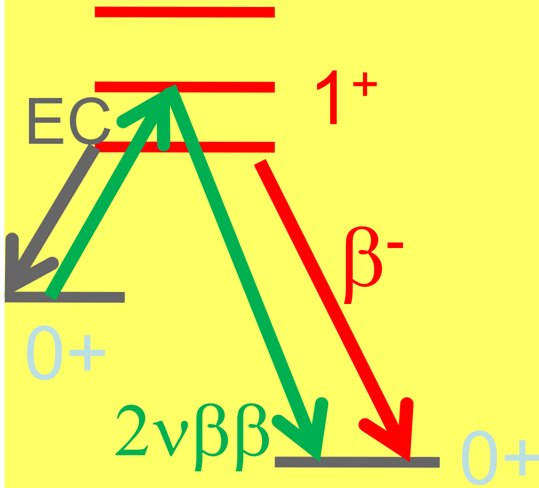
Brown, Rykaczewsky;Phys. Rev. C 50(1994)R2270

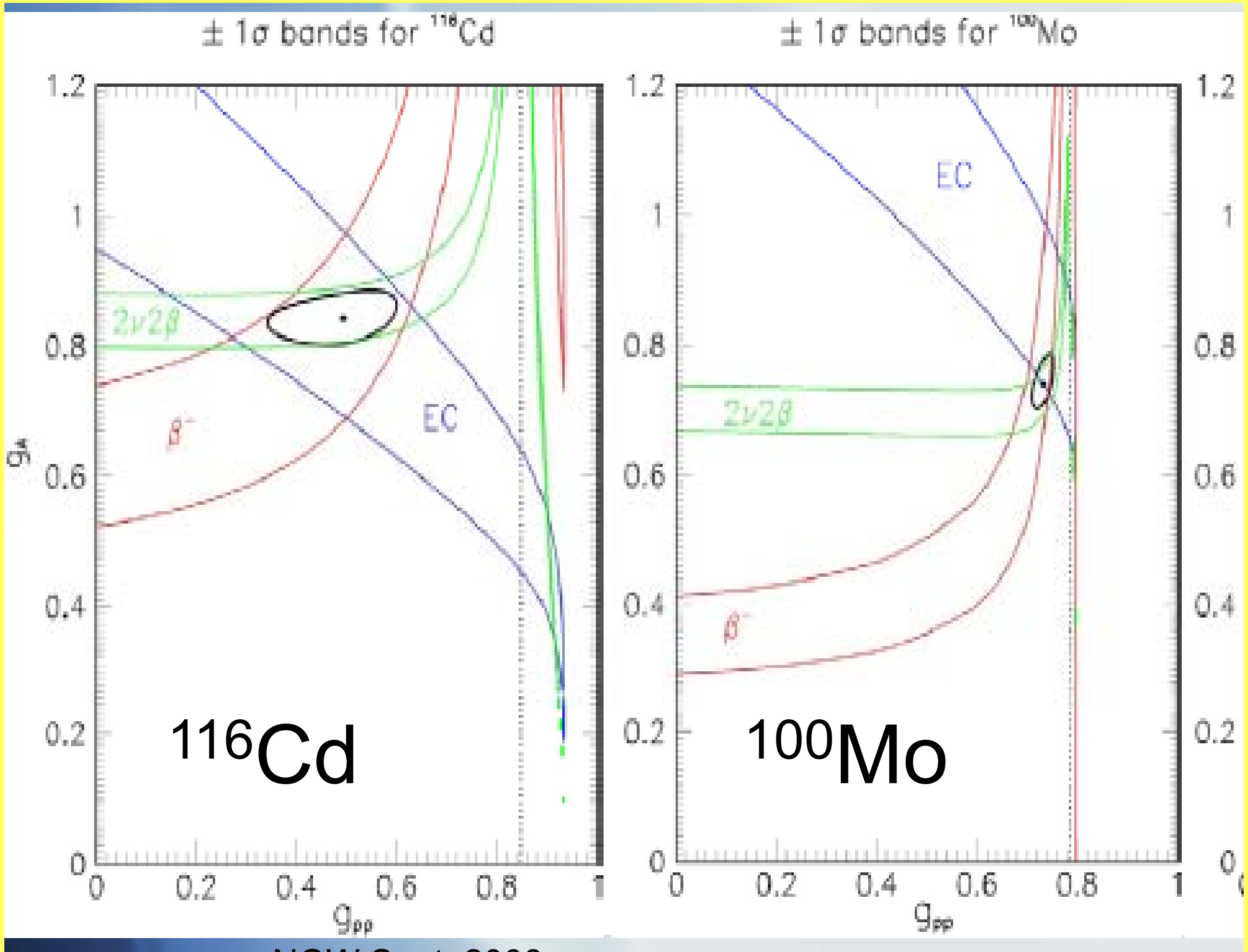
~^{100}Sn: $g_A = 0.64$

Juodagalvis, Dean; Phys. Rev. C 72(2005)024306

A= 90→97: $g_A = 0.62$

Overconstraining the $2\nu\beta\beta$ by adjusting g_{pp} und g_A to β^-, EC and $2\nu\beta\beta$ for ^{116}Cd and ^{100}Mo (Bari+Tuebingen)





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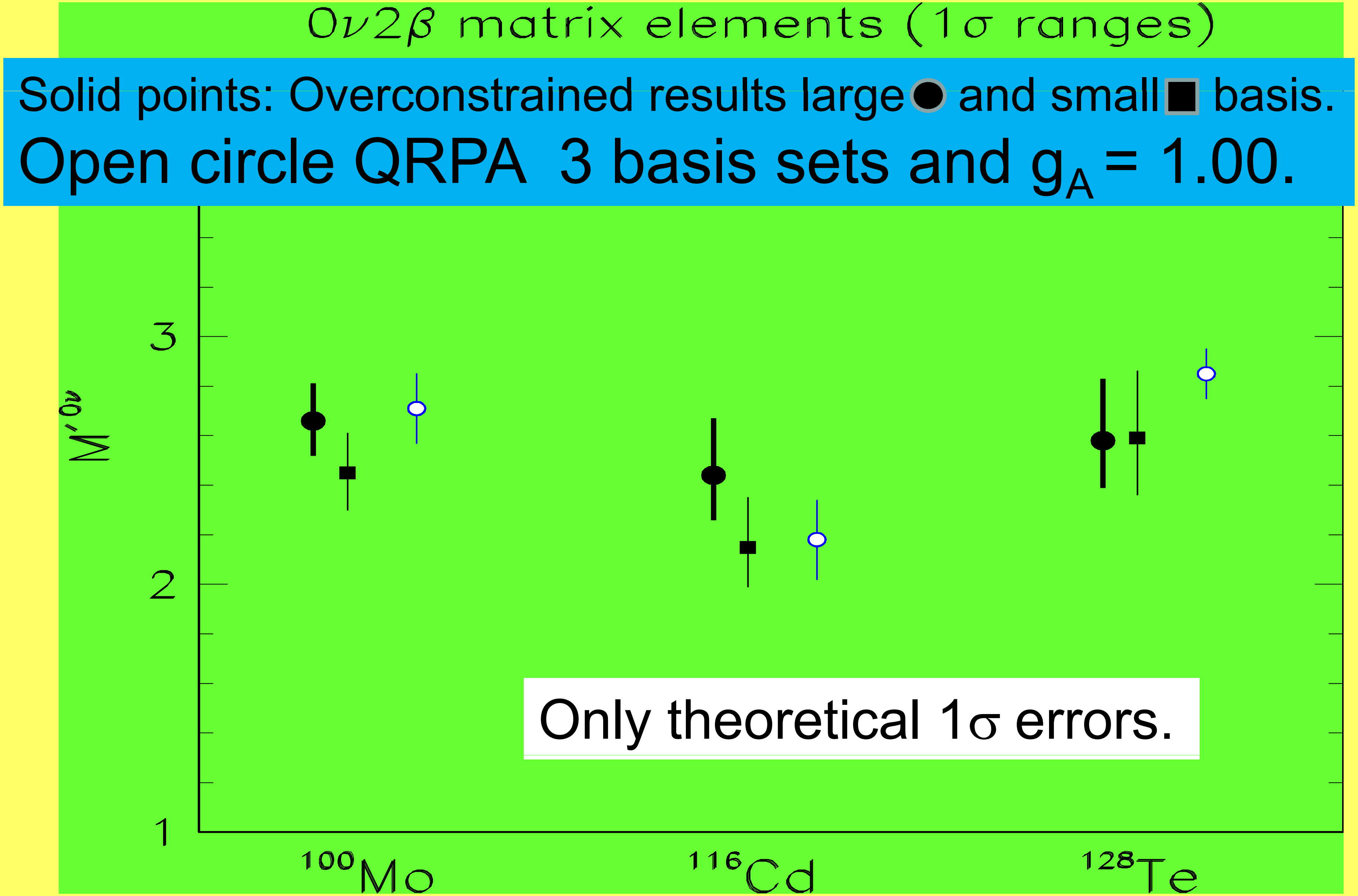

0ν2β matrix elements (1σ ranges)
Solid points: Overconstrained results large ● and small ■ basis.
Open circle QRPA 3 basis sets and g_A = 1.00.
M'^{0ν}
3
2
1
Only theoretical 1σ errors.
100Mo
116Cd
128Te

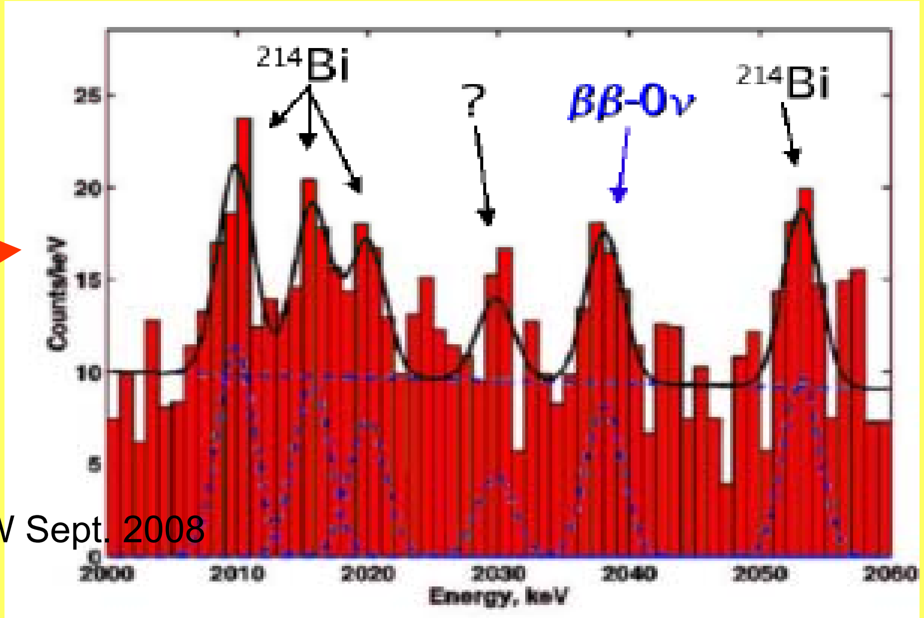
HD claim for Detection of 0ν DBD

hep-ph/0512263 HM collaboration claim the 0νDBD of ^{76}Ge

Source = Detector

- 10.9 kg - (86% from 8% nat.) ^{76}Ge
- Gran Sasso Laboratory (Italy)

Spectrum with 71.7 kg•y



Summary: Neutrino Mass from $0\nu\beta\beta$

Theory with R-QRPA and g_A =1.25

Exp. Klapdor et al. Mod. Phys. Lett. A21,1547(2006) ; ^{76}Ge

$T(1/2; 0\nu\beta\beta)$ = (2.23 +0.44 -0.31) x 10^{25} years; 6σ

- **$\langle m(\nu)\rangle$ = 0.24 [eV] (exp+-0.02; theor+-0.01) [eV]**
  Bonn CD, no short range correlations

- **$\langle m(\nu)\rangle$ = 0.22 [eV] (exp+-0.02;theor+-0.01) [eV]**
  Bonn CD, Consistent Brückner Correlations

- **$\langle m(\nu)\rangle$ = 0.24 [eV] (exp+-0.02; theor+-0.01)**
  Argonne, Consistent Brückner Correlations

THE END

- **$\langle m(\nu)\rangle$ = 0.30 [eV] (exp+-0.03;theor+-0.01) [eV]**
  Bonn CD, Fermi Hypernetted Chain (Argonne in nuclei)

- **$\langle m(\nu)\rangle$ = 0.26 [eV] (exp+-0.02;thero+-0.01) Bonn CD, UCOM (AV18 in D)**

- **$\langle m(\nu)\rangle$ = 0.31 [eV] (exp +-0.03;theor+-0.02) [eV] Bonn CD, Jastrow**