Deviations from Tri-bimaximal Mixing

MAX-PLANCK-GESELLSCHAFT

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NOW, 08/09/08

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1958 2008
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$m_\nu = m_L - m_D^T \, M_R^{-1} \, m_D$

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

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- Albright, W.R., Phys. Lett. B **665**, 378 (2008)
- Pakvasa, W.R., Weiler, Phys. Rev. Lett. **100**, 111801 (2008)
- Plentinger and W.R., Phys. Lett. B **625**, 264 (2005)

Outline

- Tri-bimaximal Mixing: $U_{\rm TBM}$ and $(m_\nu)_{\rm TBM}$
- Deviations from TBM due to m_ν
- Deviations from TBM due to m_ℓ
- Non-zero U_{e3}?
- Parameterizing Deviations from TBM

Tri-bimaximal Mixing

Data are pretty much compatible with (Harrison, Perkins, Scott)

$$U \simeq U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P$$

Sometimes one writes

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P$$

which is $\theta_{23} = -\pi/4$ in PDG description

The Mass Matrix

Special case of μ–τ symmetry

$$(m_\nu)_{\rm TBM} = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

Correlations between mass matrix elements $\leftrightarrow$ flavor symmetries:

A_4, $\Delta(27)$, $\Sigma(81)$, T', $\mathcal{PSL}_2(7)$, $SU(3)$,...

My Favorite Mass Matrix for TBM

$$\frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}$$

- state m_2 is flavor democratic
- if $m_1 = 0$ and type II see-saw ($m_\nu = m_L - m_D^T\, M_R^{-1}\, m_D$):

$$m_\nu = \underbrace{\frac{\sqrt{\Delta m_\odot^2}}{3}\begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}}_{m_L} + \underbrace{\frac{\sqrt{\Delta m_\mathrm{A}^2}}{2}\begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}}_{-m_D\, M_R^{-1}\, m_D^T}$$

`Lindner, W.R., JHEP` **`0705`**`, 089 (2007)`

Perturbing the TBM mass matrix

$$m_\nu = \begin{pmatrix} A\,(1+\epsilon_1) & B\,(1+\epsilon_2) & B\,(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_4) & \frac{1}{2}(A+B-D)\,(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_6) \end{pmatrix}$$

TBM perturbed by modifying mass matrix entries:

small complex parameters $\epsilon_i = |\epsilon_i|\, e^{i\phi_1}$ with $|\epsilon_i| \leq 0.2$

`Albright, W.R., Phys. Lett. B` **`665`**`, 378 (2008)`

How do the results depend on neutrino mass scale/ordering?

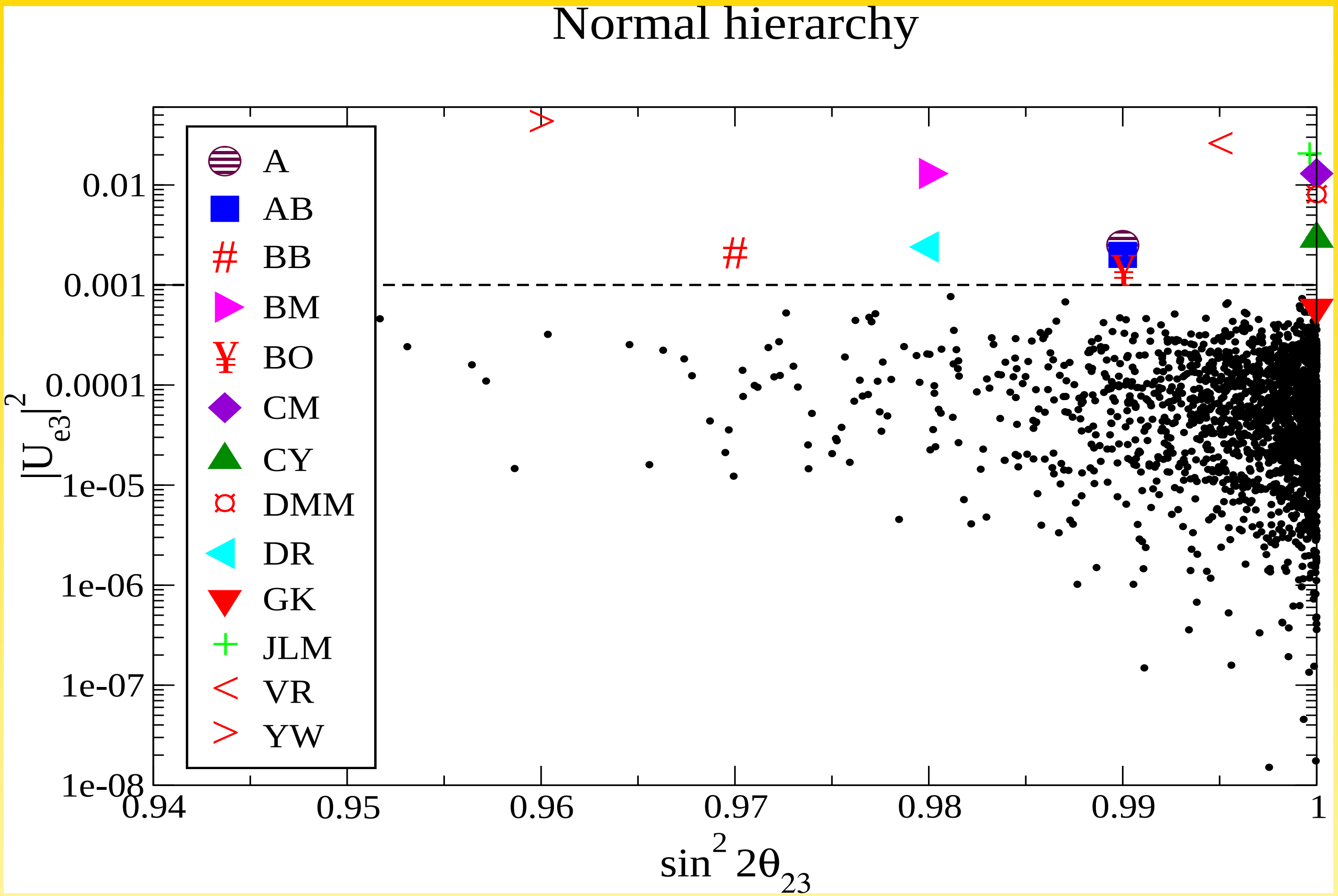
Normal Hierarchy

$$m_\nu = \begin{pmatrix} A & B\,(1+|\epsilon|) & B\,(1-|\epsilon|) \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$\Rightarrow \quad \boxed{|U_{e3}|^2 \simeq \frac{4}{9}\,\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}\,|\epsilon|^2 \lesssim 7 \cdot 10^{-4}}$$

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D)\,(1+|\epsilon|) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1-|\epsilon|) \end{pmatrix}$$

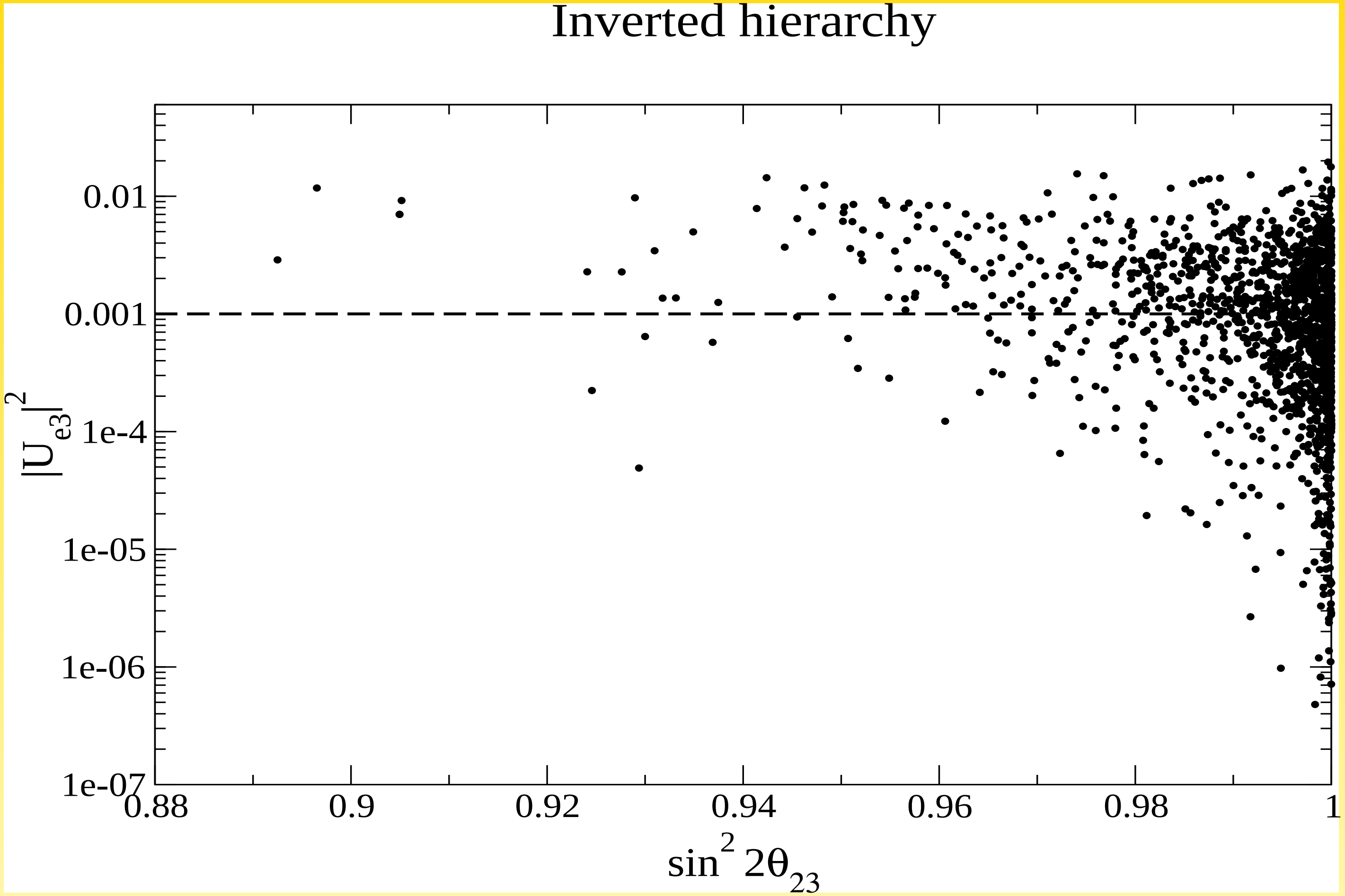
$$\Rightarrow \quad \boxed{\sin^2 2\theta_{23} \simeq 1 - |\epsilon|^2 \gtrsim 0.96}$$



Compared with 13 $SO(10)$ GUTs:

<u>$SO(10)$ predicts NH and $|U_{e3}|^2 \geq 10^{-3}$</u>

GUTs don't usually fit with IH:



in case of IH and broken TBM possible that $|U_{e3}|^2 \geq 10^{-3}$

Inverted Hierarchy

suppose $\alpha = \pi/2$ which means that $|m_{ee}| \simeq \sqrt{\Delta m^2_{\rm A}}\, \cos 2\theta_{12}$

$$m_\nu = \left(\begin{array}{ccc} A & B\,(1 + |\epsilon|) & B\,(1 - |\epsilon|) \\ \cdot & \frac{1}{2}(A + B + D) & \frac{1}{2}(A + B - D) \\ \cdot & \cdot & \frac{1}{2}(A + B + D) \end{array} \right)$$

$$|U_{e3}|^2 \simeq |\epsilon|^2 \left(\frac{8}{81} + \frac{16}{27} \, \frac{m_3}{\sqrt{\Delta m^2_{\rm A}}} \right) \lesssim 10^{-2}$$

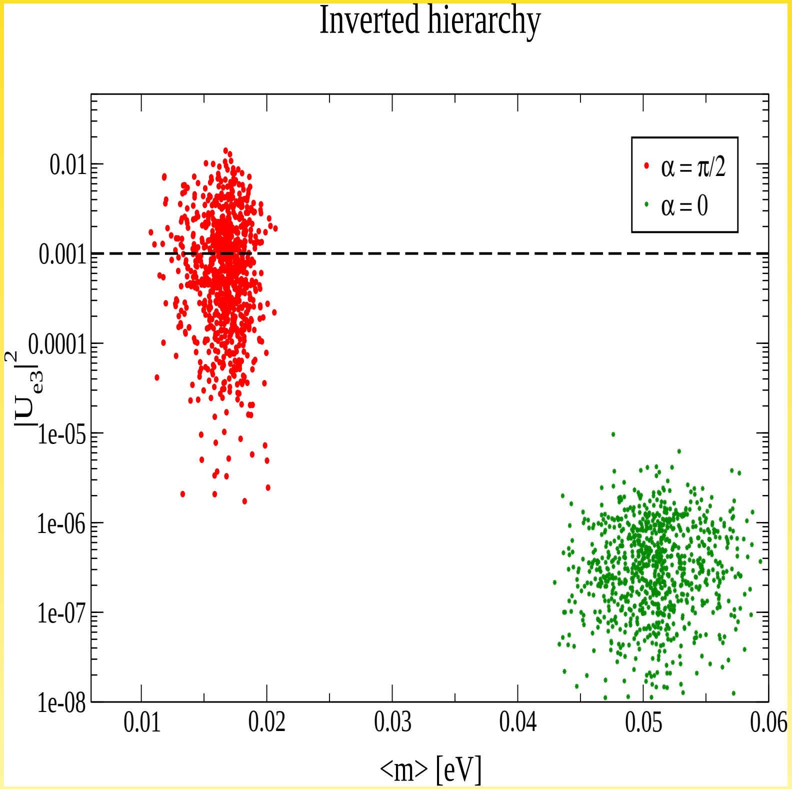
$$\sin^2 2\theta_{23} \simeq 1 - \left(\frac{16}{9}\right)^2 \, |\epsilon|^2 \gtrsim 0.87$$

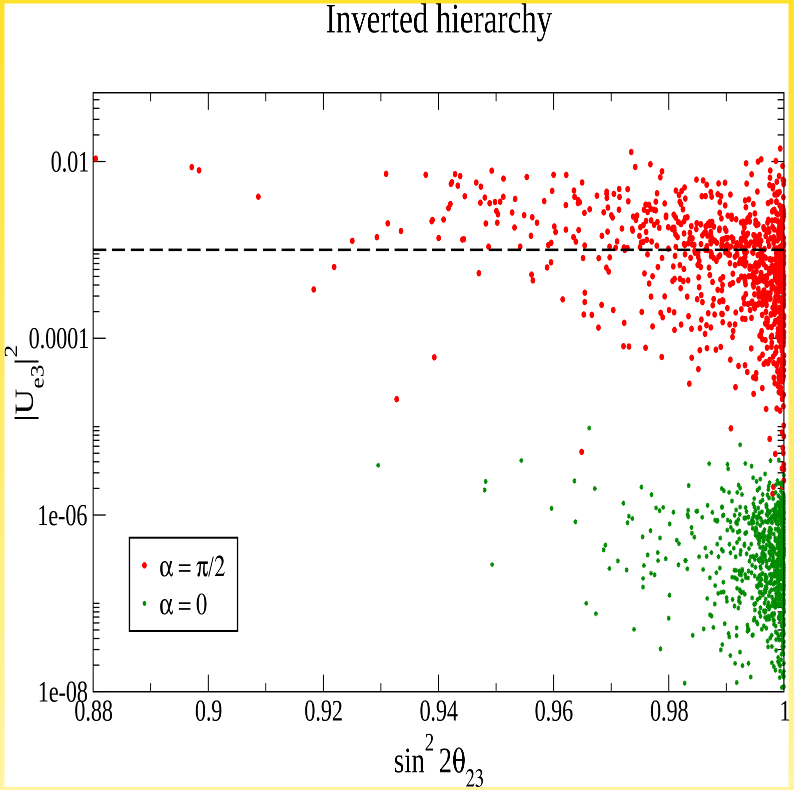
Inverted Hierarchy

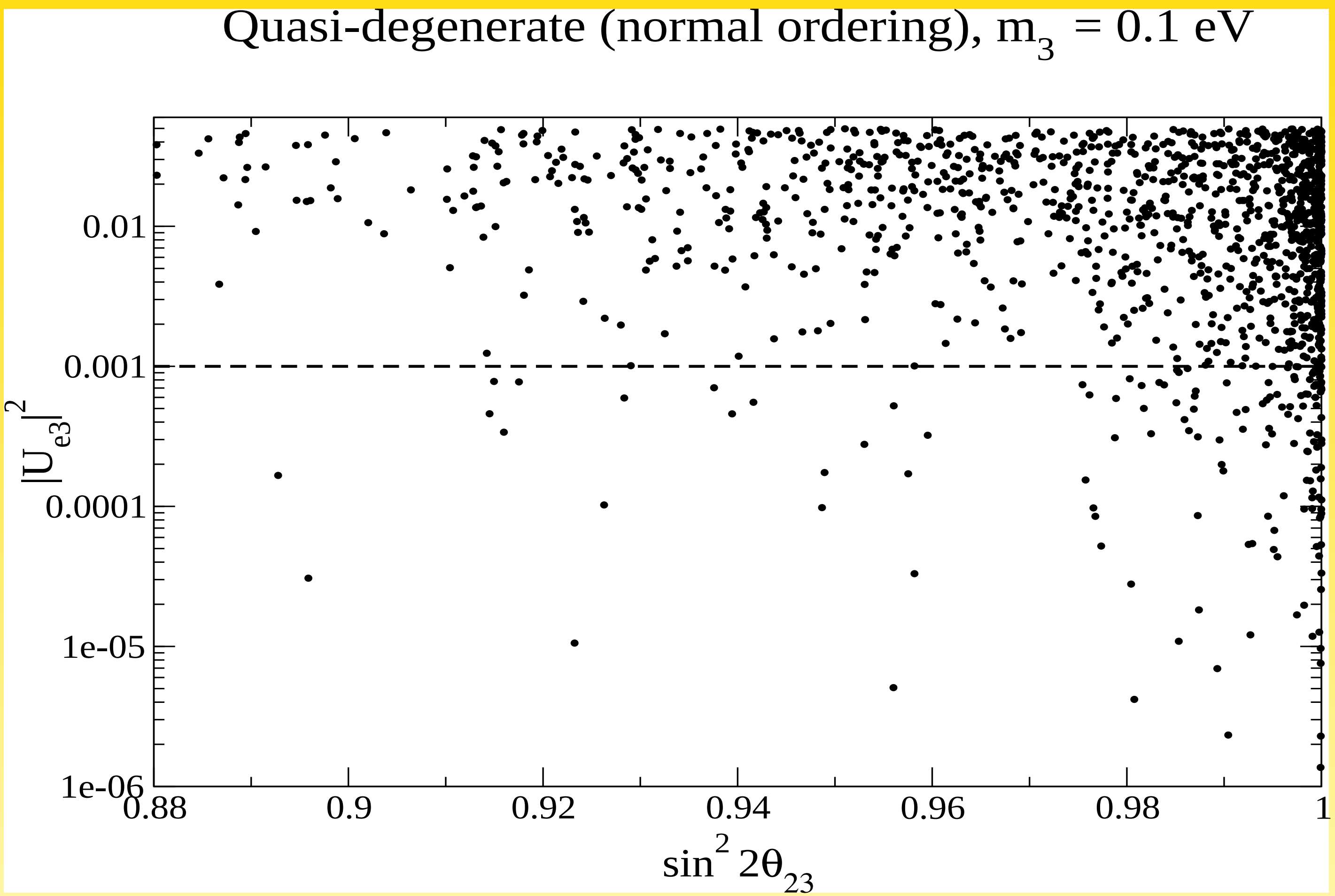
suppose $\alpha = 0$ which means that $|m_{ee}| \simeq \sqrt{\Delta m^2_{\rm A}}$

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D)\,(1-|\epsilon|) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+|\epsilon|) \end{pmatrix}$$

$$\Rightarrow \quad \boxed{|U_{e3}|^2 = \mathcal{O}(|\epsilon|^2\,\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}) \text{ and } \sin^2 2\theta_{23} \simeq 1 - |\epsilon|^2 \gtrsim 0.96}$$


Inverted hierarchy
α = π/2
α = 0
|U_e3|^2
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0.01
0.02
0.03
0.04
0.05
0.06
<m> [eV]


Inverted hierarchy
α = π/2
α = 0
|U_e3|^2
0.01
0.0001
1e-06
1e-08
0.88
0.9
0.92
0.94
0.96
0.98
1
sin^2 2θ_23



Results scale with $(m_{\rm QD}/\sqrt{\Delta m_{\rm A}^2})^2$

Model	Hierarchy	$\sin^2 2\theta_{23}$	$\lvert U_{e3}\rvert^2$	$\sin^2 \theta_{12}$
A	NH	0.99	0.0025	0.31
AB	NH	0.99	0.0020	0.28
BB	NH	0.97	0.0021	0.29
BM	NH	0.98	0.013	0.31
BO	NH	0.99	0.0014	0.27
CM	NH	1.00	0.013	0.27
CY	NH	1.00	0.0029	0.29
DMM	NH	1.00	0.0078	–
DR	NH	0.98	0.0024	0.30
GK	NH	1.00	0.00059	0.31
JLM	NH	1.0	0.0189	0.29
VR	NH	0.995	0.024	0.34
YW	NH	0.96	0.04	0.29
S-B TBM	NH	$\gtrsim 0.94$	$\lesssim 10^{-3}$	–
S-B TBM	IH	$\gtrsim 0.87$	$\lesssim 10^{-2}$	–
S-B TBM	QD	–	–	–

Radiative Corrections

$$m_\nu = \begin{pmatrix} A & B & B\,(1+\epsilon_{\rm RG}) \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D)\,(1+\epsilon_{\rm RG}) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_{\rm RG})^2 \end{pmatrix}$$

with

$$\epsilon_{\rm RG} = c\,\frac{m_\tau^2}{16\pi^2\,v^2}\ln\frac{M_X}{m_Z} \quad \text{and } c = -3/2 \text{ or } 1+\tan^2\beta$$

is included up to $\tan\beta \simeq 70$ for $M_X = 10^{15}$ GeV

In the SM: $\epsilon_{\rm RG} = -2.26 \cdot 10^{-5}$

In the MSSM: $\epsilon_{\rm RG} = 0.034$ for $\tan\beta = 15$

Analytical:

$$\theta_{ij} \simeq \theta_{ij}^{\rm TBM} + k_{ij}\, \epsilon_{\rm RG}$$

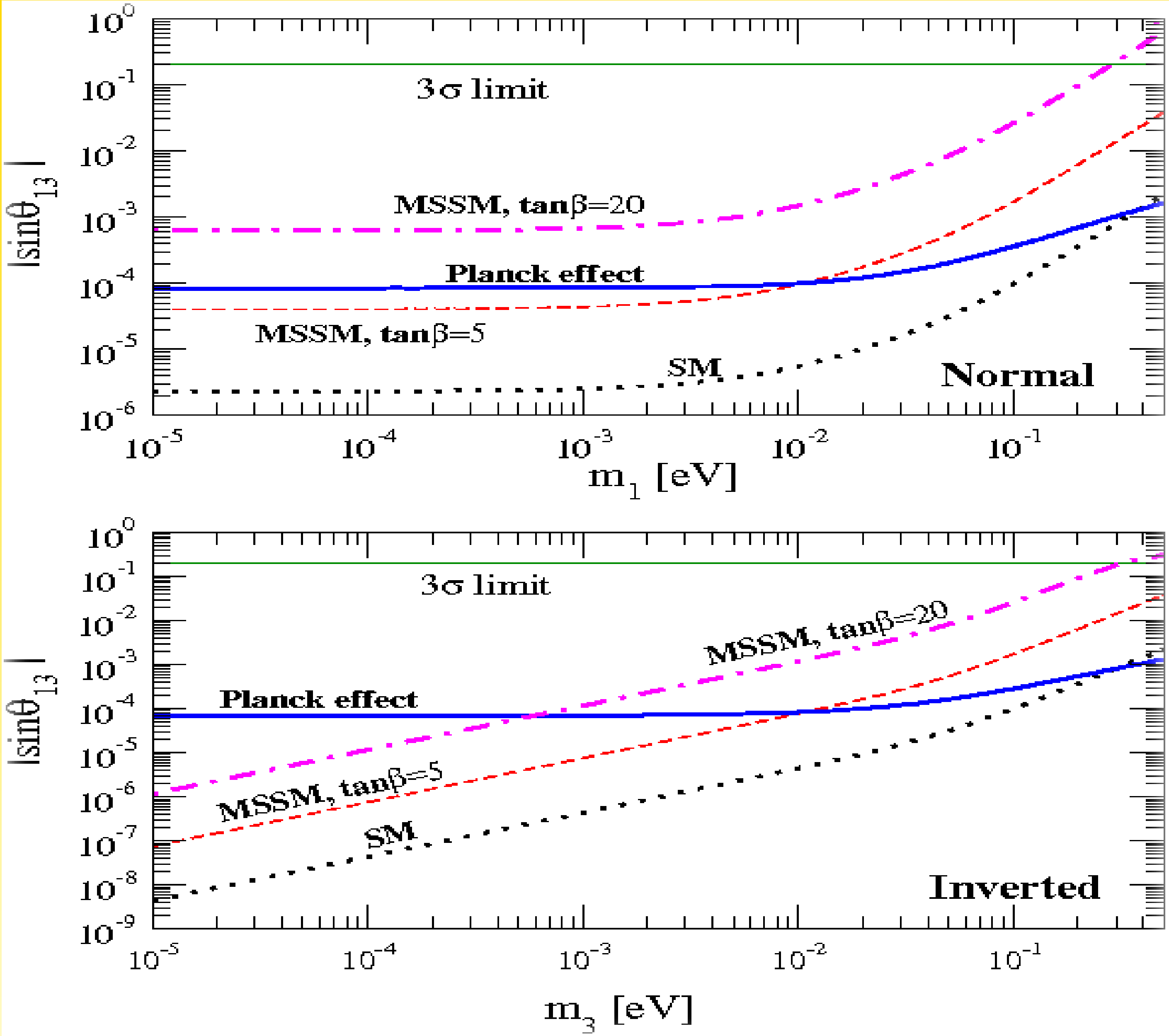
$$k_{12} = \frac{\sqrt{2}}{3} \frac{\left|m_1 + m_2\, e^{i\alpha_2}\right|^2}{\Delta m^2_\odot}$$

$$k_{23} = -\left(\frac{2}{3} \frac{\left|m_2 + m_3\, e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} + \frac{1}{3} \frac{\left|m_1 + m_3\, e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

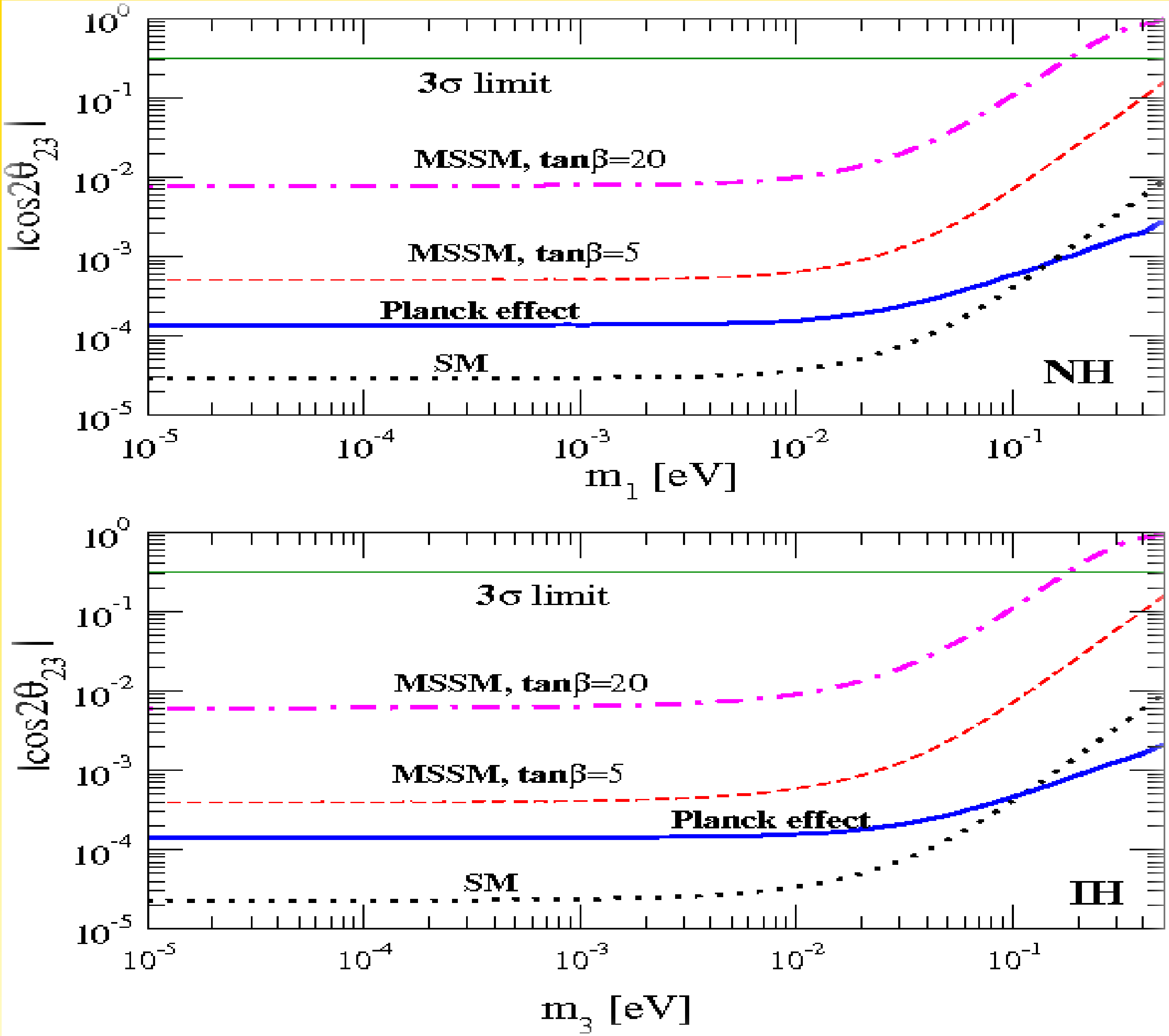
$$k_{13} = -\frac{\sqrt{2}}{3}\left(\frac{\left|m_2 + m_3\, e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} - \frac{\left|m_1 + m_3\, e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

Dighe, Goswami, W.R., Phys. Rev. D **75**, 073023 (2007);

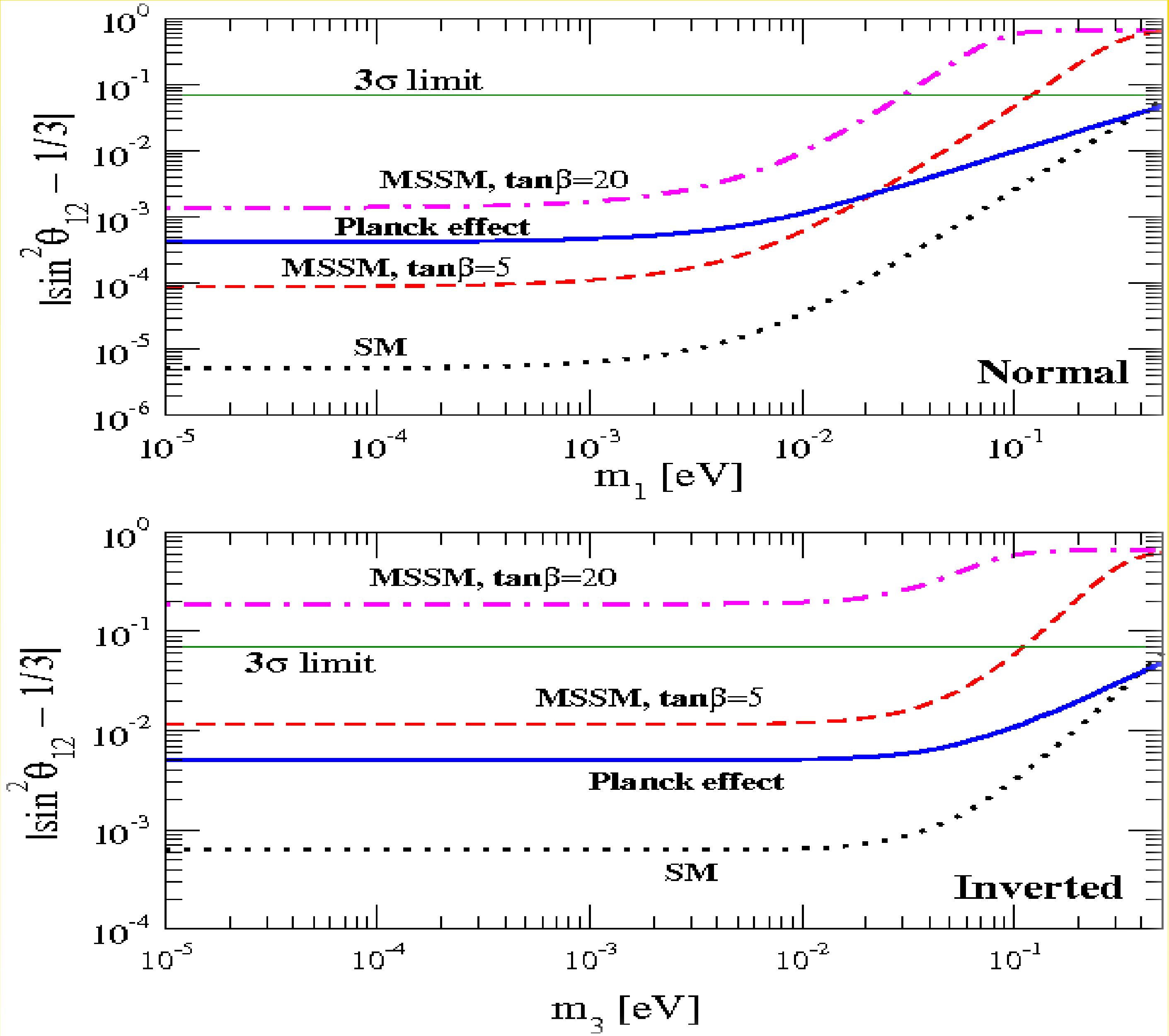
Dighe, Goswami, Roy, Phys. Rev. D **76**, 096005 (2007)



3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Normal
|sinθ13|
m1 [eV]
3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Inverted
|sinθ13|
m3 [eV]



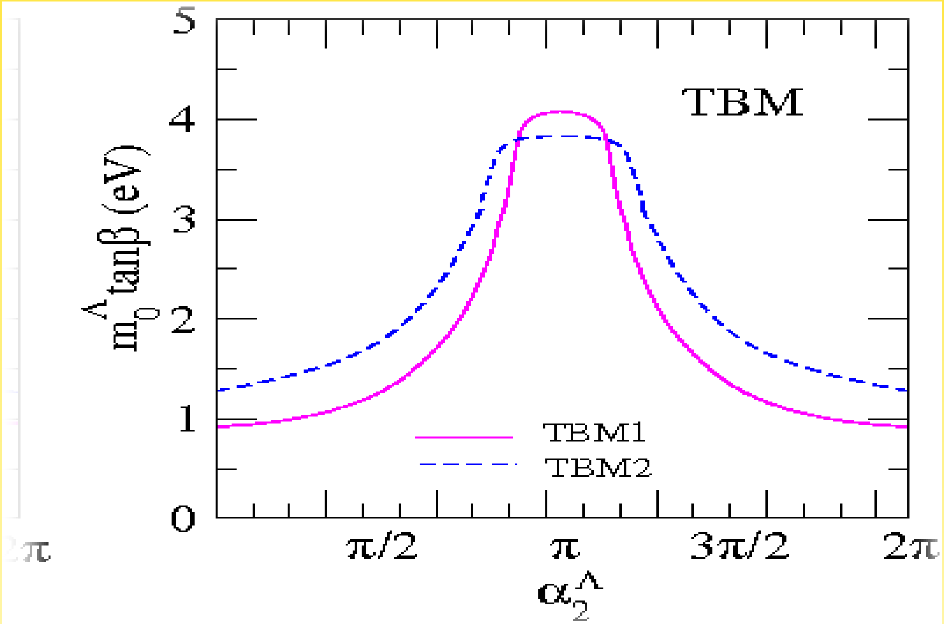
3σ limit
MSSM, tanβ=20
MSSM, tanβ=5
Planck effect
SM
NH
|cos2θ23 |
m1 [eV]
3σ limit
MSSM, tanβ=20
MSSM, tanβ=5
Planck effect
SM
IH
|cos2θ23 |
m3 [eV]



3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Normal
$|\sin^2\theta_{12} - 1/3|$
m_1 [eV]
MSSM, tanβ=20
3σ limit
MSSM, tanβ=5
Planck effect
SM
Inverted
$|\sin^2\theta_{12} - 1/3|$
m_3 [eV]

$$k_{12} = \frac{\sqrt{2}}{3} \frac{\left| m_1 + m_2 \, e^{i\alpha_2} \right|^2}{\Delta m^2_\odot} \Rightarrow \text{ strong effect for IH and QD}$$

$$|m_{ee}| \simeq m_0 \sqrt{1 - \sin^2 2\theta_{12} \, \sin^2 \alpha_2/2}$$



Dighe, Goswami, Roy, Phys. Rev. D **76**, 096005 (2007)

Charged Lepton Corrections

$$U = U_\ell^\dagger \, U_\nu$$

With $U_\nu = U_{\rm TBM}$ and $U_\ell =$ "CKM"

Counting of phases:

$$U = \tilde{U}_\ell^\dagger \, P_\nu \, \tilde{U}_\nu \, Q_\nu$$

- $\tilde{U}$ "PDG-like", i.e., 3 angles and one phase
- $P_\nu = \text{diag}(1, e^{i\phi}, e^{i\omega})$ and $Q_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$

Result

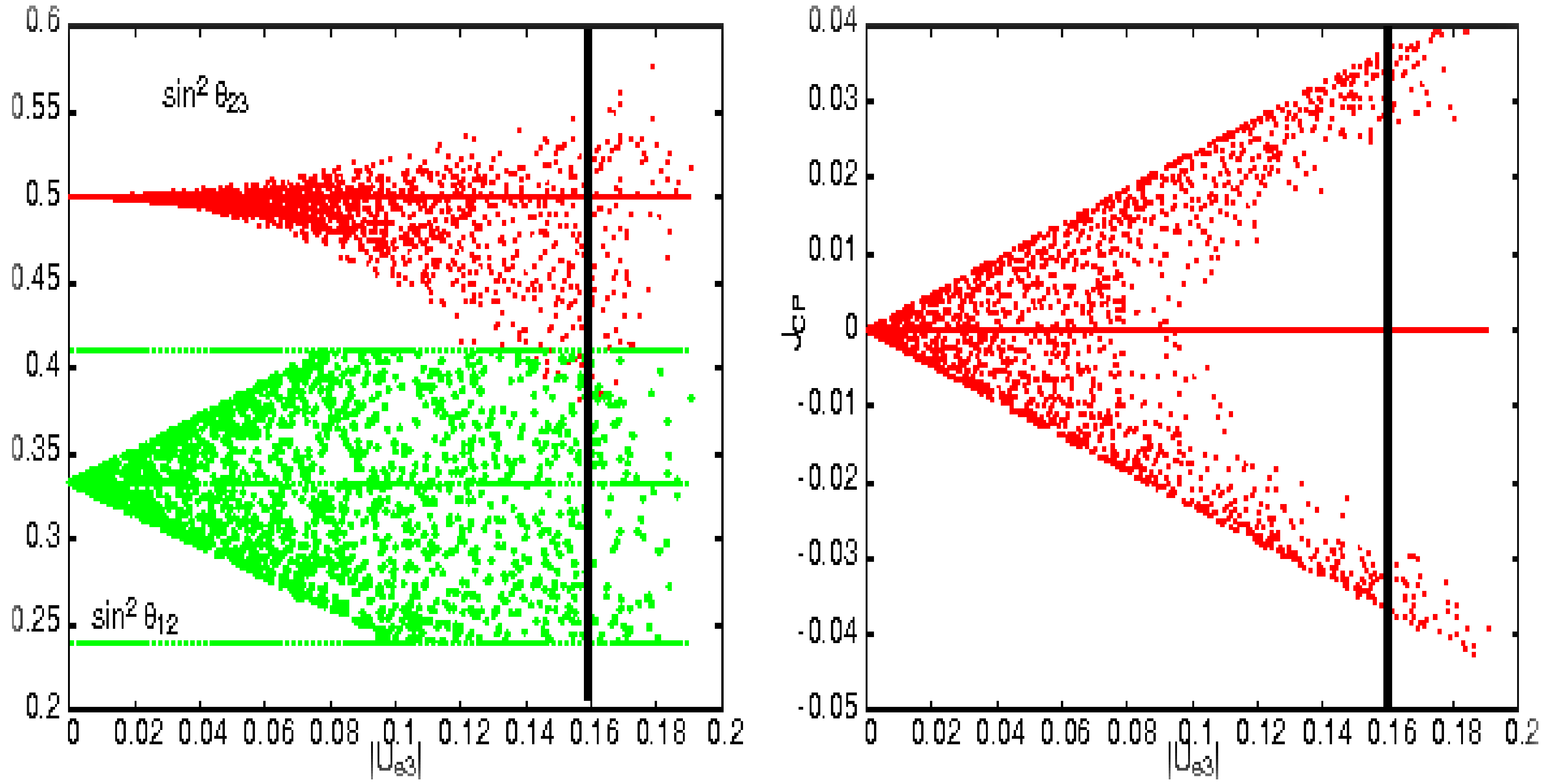
$$U_\ell = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$ gives

$$\sin^2\theta_{12} \simeq \frac{1}{3} - \frac{2}{3}\,\lambda\cos\phi$$
$$|U_{e3}| \simeq \frac{1}{\sqrt{2}}\,\lambda \quad , \quad J_{\rm CP} = \frac{\lambda}{6}\sin\phi$$
$$\sin^2\theta_{23} \simeq \frac{1}{2} - \mathcal{O}(\lambda^2)$$

Direct correlation between U_{e3}, $\sin^2\theta_{12}$ and CP violation!

King; Frampton, Petcov, W.R.; Plentinger, W.R.; Picariello; Koide; King, Boudjemaa,...

(if TBM from charged leptons: U_{e3}, $\sin^2\theta_{23}$ and CP violation (Hochmuth, Petcov, W.R.))



If U_ℓ is equal to CKM matrix: maximal CP violation!

Plentinger and W.R., Phys. Lett. B **625**, 264 (2005)

The (weak) hint for non-zero U_{e3} from Bari

$$|U_{e3}|^2 = 0.016 \pm 0.010$$

Origin from broken μ–τ symmetry or broken TBM:

- charged leptons: $|U_{e3}| = \lambda/\sqrt{2}$ and $\sin^2\theta_{12} = \sin^2\theta_{12}^\nu - \sin 2\theta_{12}^\nu \, |U_{e3}| \cos\phi$

$$\Rightarrow \lambda = 0.18^{+0.05}_{-0.07}$$

  and maximal (zero) CP violation if U_ν is tri-bimaximal (bimaximal)

- perturbed μ–τ symmetric mass matrix

$\Rightarrow$ must be IH or QD

- RG with $m_1 \gtrsim 0.1$ eV and $\tan\beta \gtrsim 15$
- *OR: it's a GUT*

Describing Deviations from TBM

Recall

$$U = R_{23}(\theta_{23})\, U_\delta^\dagger\, R_{13}(\theta_{13})\, U_\delta\, R_{12}(\theta_{12})$$

and

$$U_{\rm TBM} = R_{23}\left(\frac{\pi}{4}\right)\, R_{12}\left(\sin^{-1}\sqrt{\frac{1}{3}}\right)$$

Therefore, parameterize PMNS:

$$U = R_{23}\left(\frac{\pi}{4}\right)\, R_{23}(\epsilon_{23})\, U_\delta^\dagger\, R_{13}(\epsilon_{13})\, U_\delta\, R_{12}(\epsilon_{12})\, R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right)$$

"Triminimal Parametrization"

`Pakvasa, W.R., Weiler, Phys. Rev. Lett.` **100**, `111801 (2008)`

Triminimal Parametrization

$$U = R_{23}\left(\frac{\pi}{4}\right) R_{23}(\epsilon_{23})\, U_\delta^\dagger\, R_{13}(\epsilon_{13})\, U_\delta\, R_{12}(\epsilon_{12})\, R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right)$$

Only one small ϵ_{ij} responsible for deviation of θ_{ij} from $\theta_{ij}^{\mathrm{TBM}}$

$$\begin{aligned}
\sin^2\theta_{12} &\simeq \frac{1}{3} + \frac{2\sqrt{2}}{3}\,\epsilon_{12} \\
\sin^2\theta_{23} &\simeq \frac{1}{2} + \epsilon_{23} \\
U_{e3} &= \sin\epsilon_{13}\, e^{-i\delta}
\end{aligned}$$

Summary

Deviations from TBM are expected

- perturb m_ν: only IH and QD can get $|U_{e3}|^2 \geq 10^{-3}$
- perturb m_ℓ: $|U_{e3}| = \lambda/\sqrt{2}$, CP violation maximal if $\lambda = \sin\theta_C$
- RG: characteristic dependence on neutrino mass and $|m_{ee}|$
- potential to explain hint for non-zero U_{e3}