

# $2\nu\beta\beta$ decay of deformed nuclei with realistic NN forces

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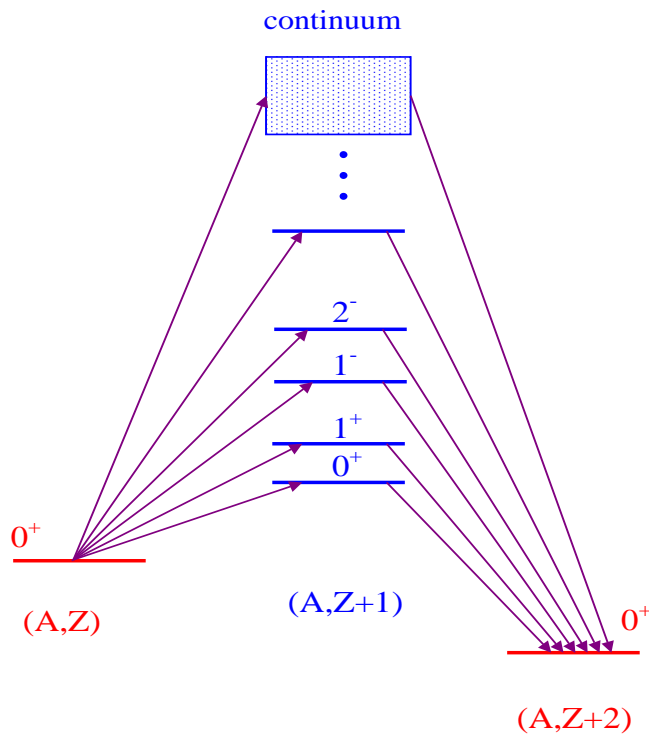
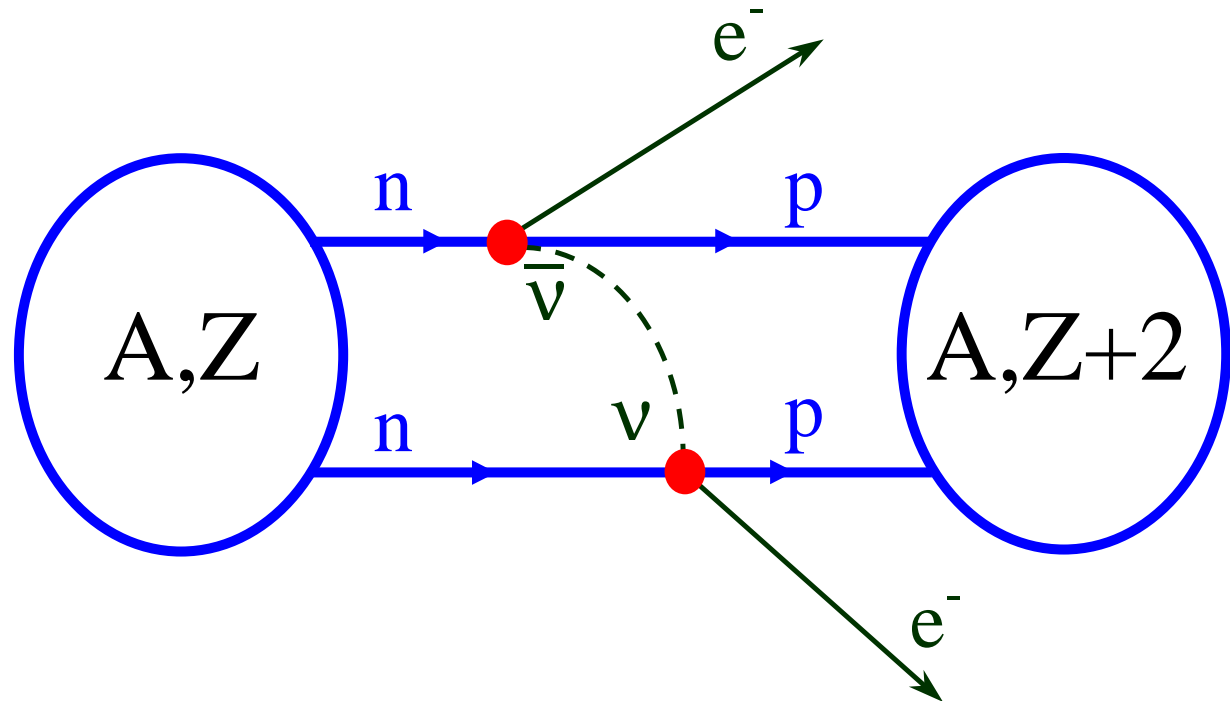
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*NOW 2008, Conca Specchiulla, 09/09/2008*

# Introduction

## Nuclear $0\nu\beta\beta$ -decay ( $\bar{\nu} = \nu$ )

Light neutrino  
exchange mechanism

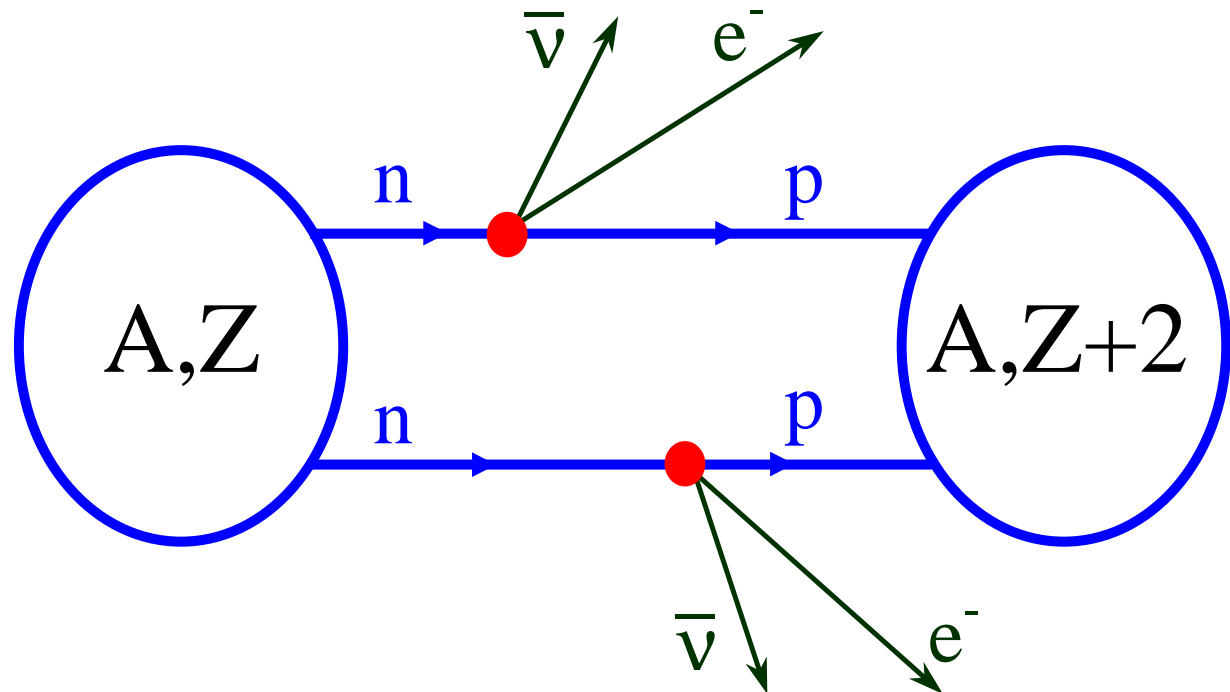


virtual excitation  
of states of all multiplicities  
in  $(A, Z+1)$  nucleus

# Introduction

## Nuclear $2\nu\beta\beta$ -decay

second order weak process  
within SM



# Introduction

measured  $T_{1/2}^{2\nu}$  (compilation of A. Barabash, 2005)

Isotope	$T_{1/2}^{2\nu}$ , in $10^{19}$ y
$^{48}\text{Ca}$	$4.2^{+2.1}_{-1.0}$
$^{76}\text{Ge}$	$150 \pm 10$
$^{82}\text{Se}$	$9.2 \pm 0.7$
$^{96}\text{Zr}$	$2.0 \pm 0.3$
$^{100}\text{Mo}$	$0.71 \pm 0.04$
$^{116}\text{Cd}$	$3.0 \pm 0.2$
$^{128}\text{Te}$	$(2.5 \pm 0.3) \times 10^5$
$^{130}\text{Te}$	$90 \pm 10$
$^{136}\text{Xe}$	$> 81$ (90% CL)
$^{150}\text{Nd}$	$0.78 \pm 0.07$
$^{238}\text{U}$	$200 \pm 60$

$2\nu\beta\beta$  $0\nu\beta\beta$ 

**Inverse Half-Lives**  $[T_{1/2}(0^+ \rightarrow 0^+)]^{-1}$

$$G^{2\nu}(Q, Z) |M_{GT}^{2\nu}|^2$$

$$m_{\beta\beta}^2 G^{0\nu}(Q, Z) \left| M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu} \right|^2$$

$$\text{Eff. neutrino mass } m_{\beta\beta} = \sum_j m_j U_{ej}^2$$

$U_{ej}$  — first row of the neutrino mixing matrix

$2\nu\beta\beta$  $0\nu\beta\beta$ 

## Nuclear Matrix Elements

$$M_{GT}^{2\nu} =$$

$$\sum_s \frac{\langle 0_f || \hat{\beta}^- || s \rangle \langle s || \hat{\beta}^- || 0_i \rangle}{E_s - (M_i + M_f)/2}$$

$$\hat{\beta}^- = \sum_k \sigma_k \tau_k^-$$

$$M_{GT}^{0\nu} =$$

$$\langle 0_f | \sum_{ik} P_\nu(r_{ik}, \bar{\omega}) \tau_i^- \tau_k^- \sigma_i \cdot \sigma_k | 0_i \rangle$$

Neutrino potential :  $P_\nu(r, \bar{\omega}) =$

$$\frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \bar{\omega})}$$
$$\approx \frac{R}{r} \phi(\bar{\omega}r)$$

# Introduction

V.R., A. Faessler, F. Simkovic, P. Vogel, PRC 68 (2003); NPA 766 (2006); NPA 793 (2007)

$g_{pp}$  fitted to  $2\nu\beta\beta$ -decay half-life  $\Rightarrow$  stable  $M^{0\nu}$

$0\nu\beta\beta$  half-lives  $T_{1/2}^{0\nu}$  (in years) assuming  $\langle m_{\beta\beta} \rangle = 50$  meV.

Transition	$M^{0\nu}$	$T_{1/2}^{0\nu}$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	3.92	$8.6 \cdot 10^{26}$
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	3.49	$2.4 \cdot 10^{26}$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.95	$2.2 \cdot 10^{26}$
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	4.16	$2.2 \cdot 10^{25}$

↑

strongly deformed  $\Leftarrow$  SM (Nowacki'04): “you can forget about it!”

SNO+ (& SuperNEMO): 0.1% of  $^{nat}\text{Nd} \Rightarrow 56$  kg of  $^{150}\text{Nd} \Rightarrow m_{\beta\beta} \simeq 0.1$  eV

**But:**  $M^{0\nu} = 1.57$  pseudo-SU(3) model of Hirsch *et al.* NPA582 (1995)

$M^{0\nu} = 1.61$  projected HFB appr. of K. Chaturvedi *et al.* arXiv:0805.4073 [nucl-th]

## QRPA in deformed nuclei

- $2\nu\beta\beta$  in deformed nuclei; QRPA with schematic separable forces  
F. Šimkovic, L. Pacearescu, A. Faessler, NPA **733** (2004)  
R. Alvarez-Rodriguez *et al.*, PRC **70** (2004)
- QRPA with realistic forces in deformed nuclei ( $0\nu\beta\beta$  of  $^{150}\text{Nd}$ )  
applied first to  $2\nu\beta\beta$  (PhD thesis of M. Saleh Yousef)  
M. Saleh Yousef, V.R., A. Faessler, F. Šimkovic – arXiv:0806.0964 [nucl-th]  
 $0\nu\beta\beta$  (PhD thesis of D. Fang, work in progress)



# QRPA in deformed nuclei

## Basic relationships

$$|1M(K), m\rangle = \sqrt{\frac{3}{16\pi^2}} [\mathcal{D}_{MK}^1(\phi, \theta, \psi) Q_{m,K}^\dagger + (-1)^{1+K} \mathcal{D}_{M-K}^1(\phi, \theta, \psi) Q_{m,-K}^\dagger] |0_{g.s.}^+\rangle \quad (K = \pm 1),$$

$$|1M(K), m\rangle = \sqrt{\frac{3}{8\pi^2}} \mathcal{D}_{MK}^1(\phi, \theta, \psi) Q_{m,K}^\dagger |0_{g.s.}^+\rangle \quad (K = 0)$$

$$M_{GT}^{2\nu} = \sum_{K=0,\pm 1} \sum_{m_i m_f} \frac{\langle 0_f^+ | \bar{\beta}_K^- | K^+, m_f \rangle \langle K^+, m_f | K^+, m_i \rangle \langle K^+, m_i | \beta_K^- | 0_i^+ \rangle}{\bar{\omega}_{K,m_i m_f}}$$

case I (shifted QRPA spectrum)

$$\bar{\omega}_{K,m_i m_f} = (\omega_{K,m_f} - \omega_{K,1_f} + \omega_{K,m_i} - \omega_{K,1_i})/2 + \bar{\omega}_{1_{1,exp}^+}$$

case II (unshifted QRPA spectrum)

$$\bar{\omega}_{K,m_i m_f} = (\omega_K^{m_f} + \omega_K^{m_i})/2$$

# QRPA in deformed nuclei

Deformed Woods-Saxon s.p. wave functions  $|\tau\Omega_\tau\rangle$   
decomposed over the spherical harmonic oscillator ones  $|\eta\Omega\rangle$

$$|\tau\Omega_\tau\rangle = \sum_{\eta} B_{\eta}^{\tau} |\eta\Omega_\tau\rangle$$

$|\eta\Omega\rangle = \sum_{\Sigma} C_{l\Omega-\Sigma, 1/2, \Sigma}^{j\Omega} |Nl\Lambda = \Omega_\tau - \Sigma\rangle |\Sigma\rangle$  is the spherical harmonic oscillator wave function in the  $j$ -coupled scheme

# QRPA in deformed nuclei

## Two-body deformed wave function

$$|p\bar{n}\rangle = \sum_{\eta_p \eta_n, J} F_{p\eta_p n\eta_n}^{JK} |\eta_p \eta_n, JK\rangle$$

$$|\eta_p \eta_n, JK\rangle = C_{j_p \Omega_p j_n \Omega_n}^{JK} |\eta_p \Omega_p\rangle |\eta_n \Omega_n\rangle \text{ and } F_{p\eta_p n\eta_n}^{JK} = B_{\eta_p}^p B_{\eta_n}^n (-1)^{j_n - \Omega_n} C_{j_p \Omega_p j_n - \Omega_n}^{JK}$$

## Two-body residual interaction m.e.

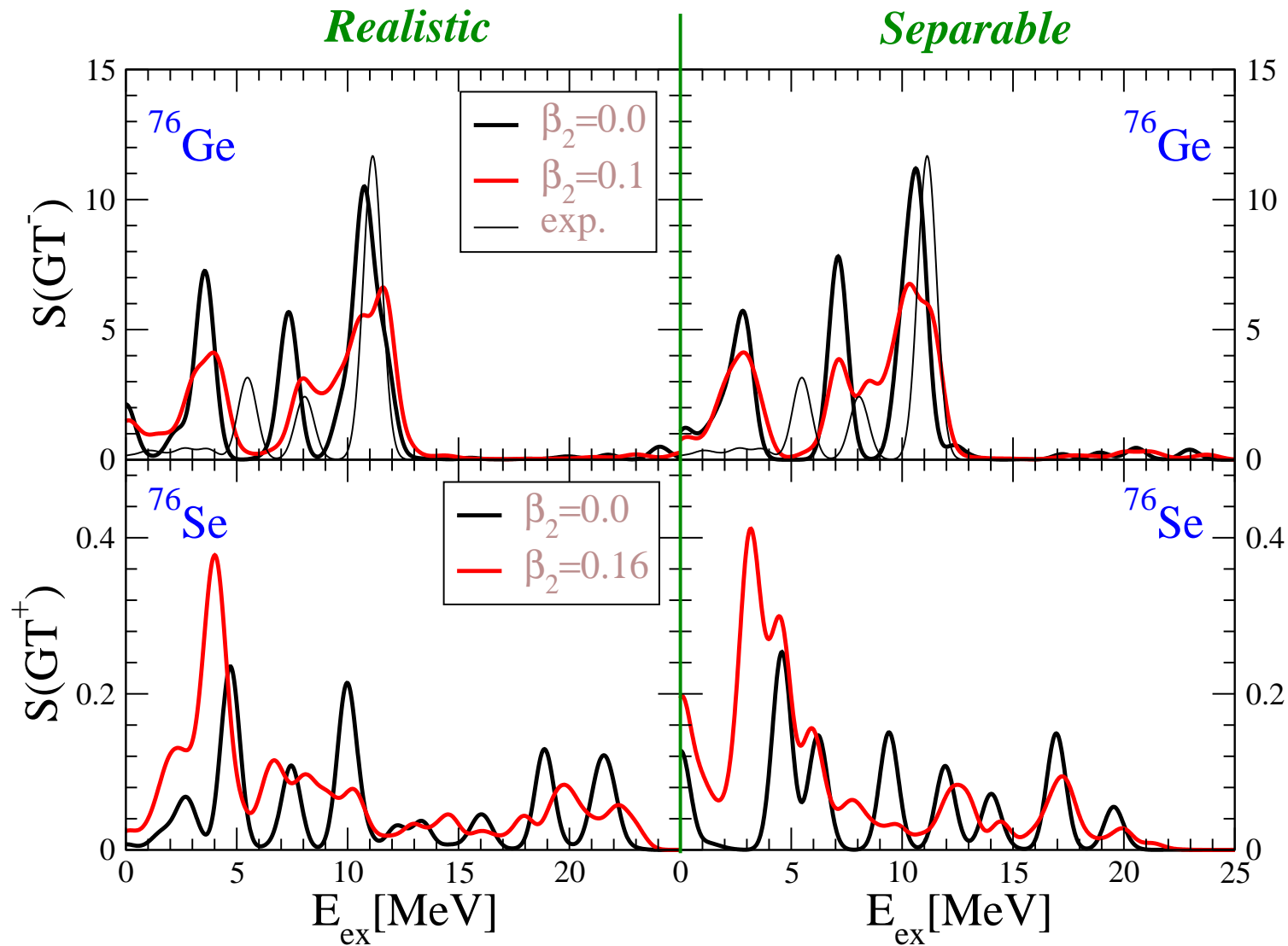
$$V_{p\bar{n}, p'\bar{n}'} = -2 \sum_J \sum_{\eta_p \eta_n} \sum_{\eta_{p'} \eta_{n'}} F_{p\eta_p n\eta_n}^{JK} F_{p'\eta_{p'} n'\eta_{n'}}^{JK} G(\eta_p \eta_n \eta_{p'} \eta_{n'}, J)$$

$$V_{pn', p'n} = 2 \sum_J \sum_{\eta_p \eta_n} \sum_{\eta_{p'} \eta_{n'}} F_{p\eta_p \bar{n}'\eta_{n'}}^{JK'} F_{p'\eta_{p'} \bar{n}\eta_n}^{JK'} G(\eta_p \eta_{n'} \eta_{p'} \eta_n, J)$$

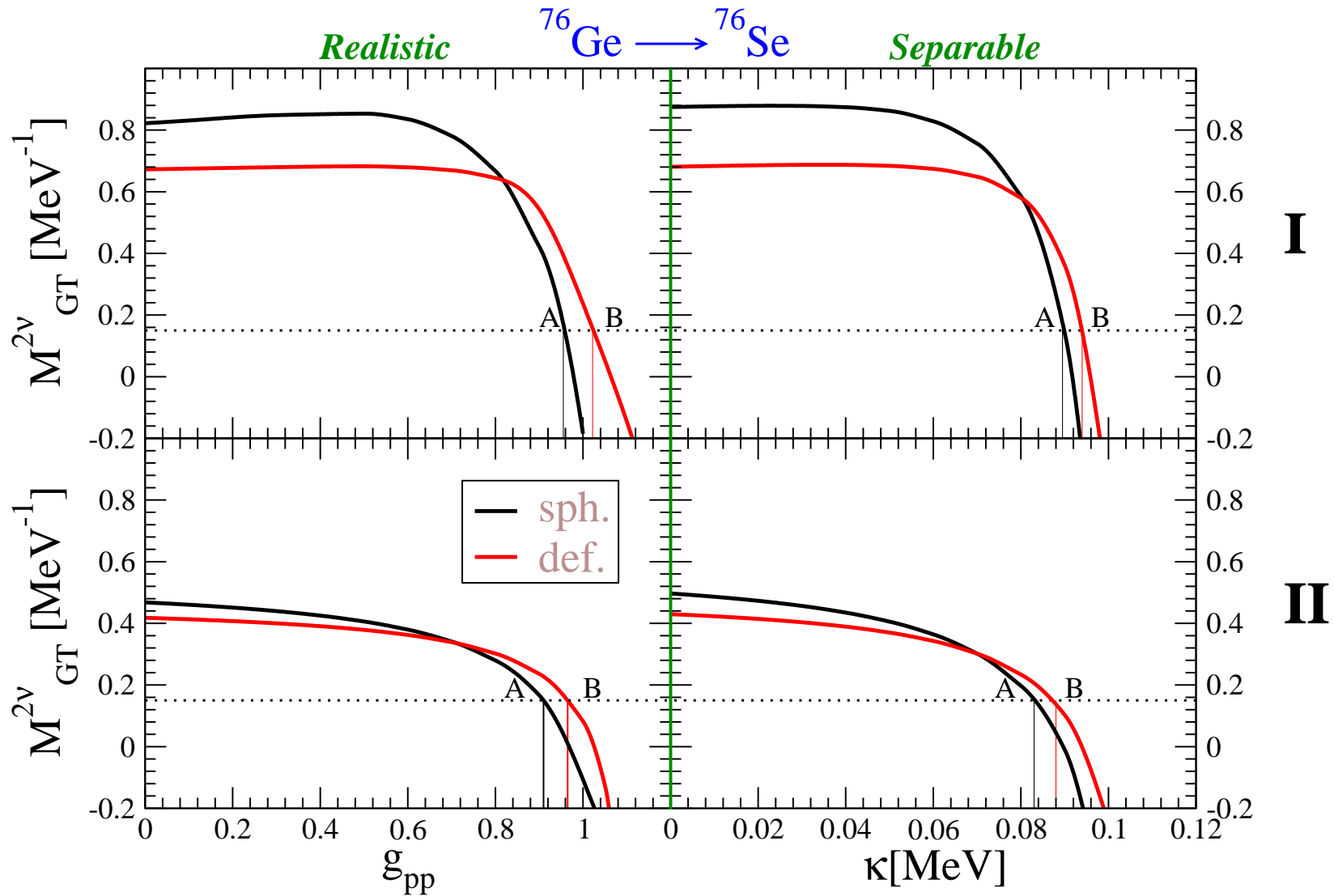
$$K'_{pn'} = \Omega_p + \Omega_{n'} = \Omega_{p'} + \Omega_n$$

# Results

GT strength functions ( $g_{ph} = 1.15$ ;  $\chi = 3.73/A^{0.7}$  MeV)



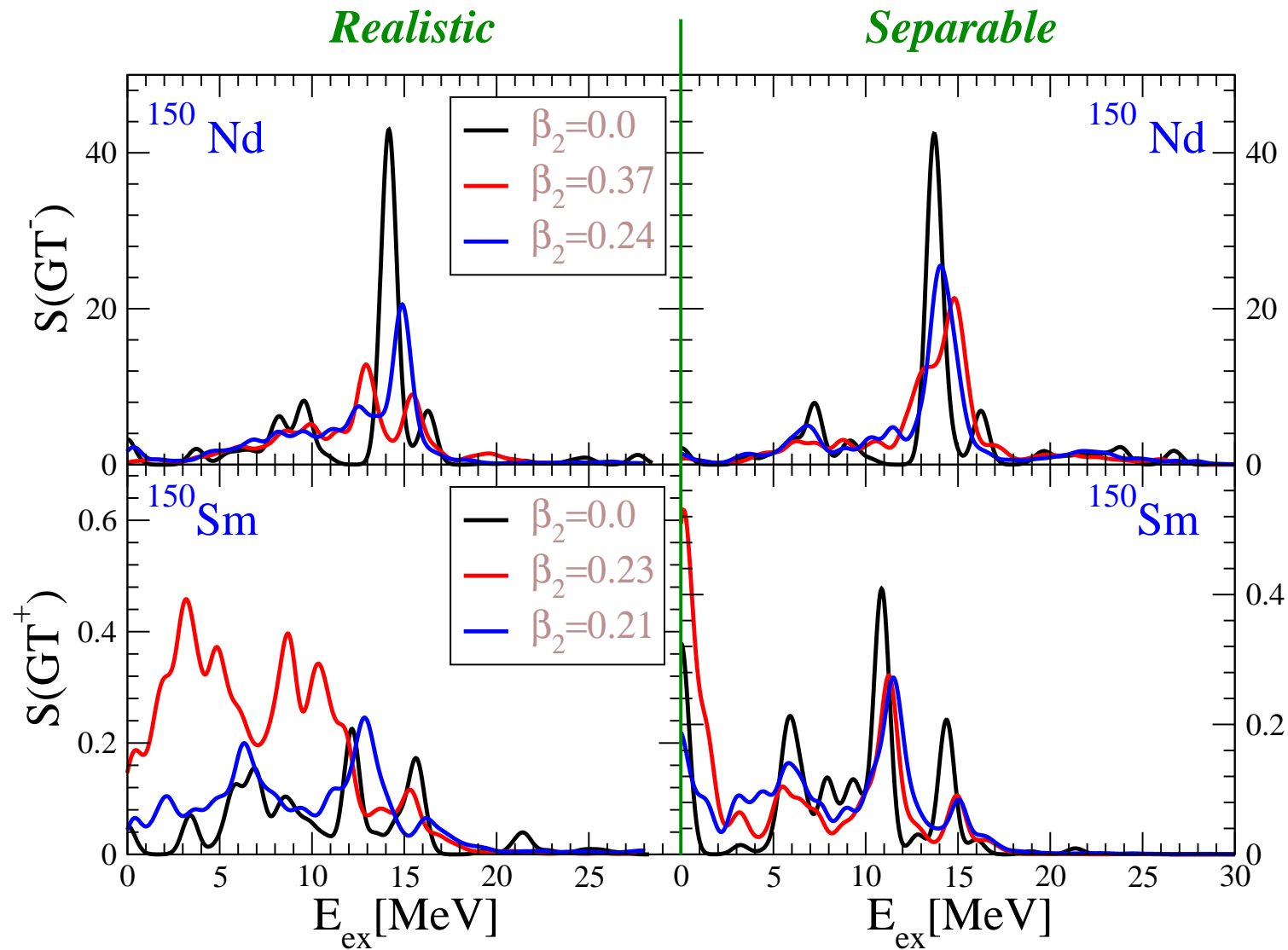
# Results

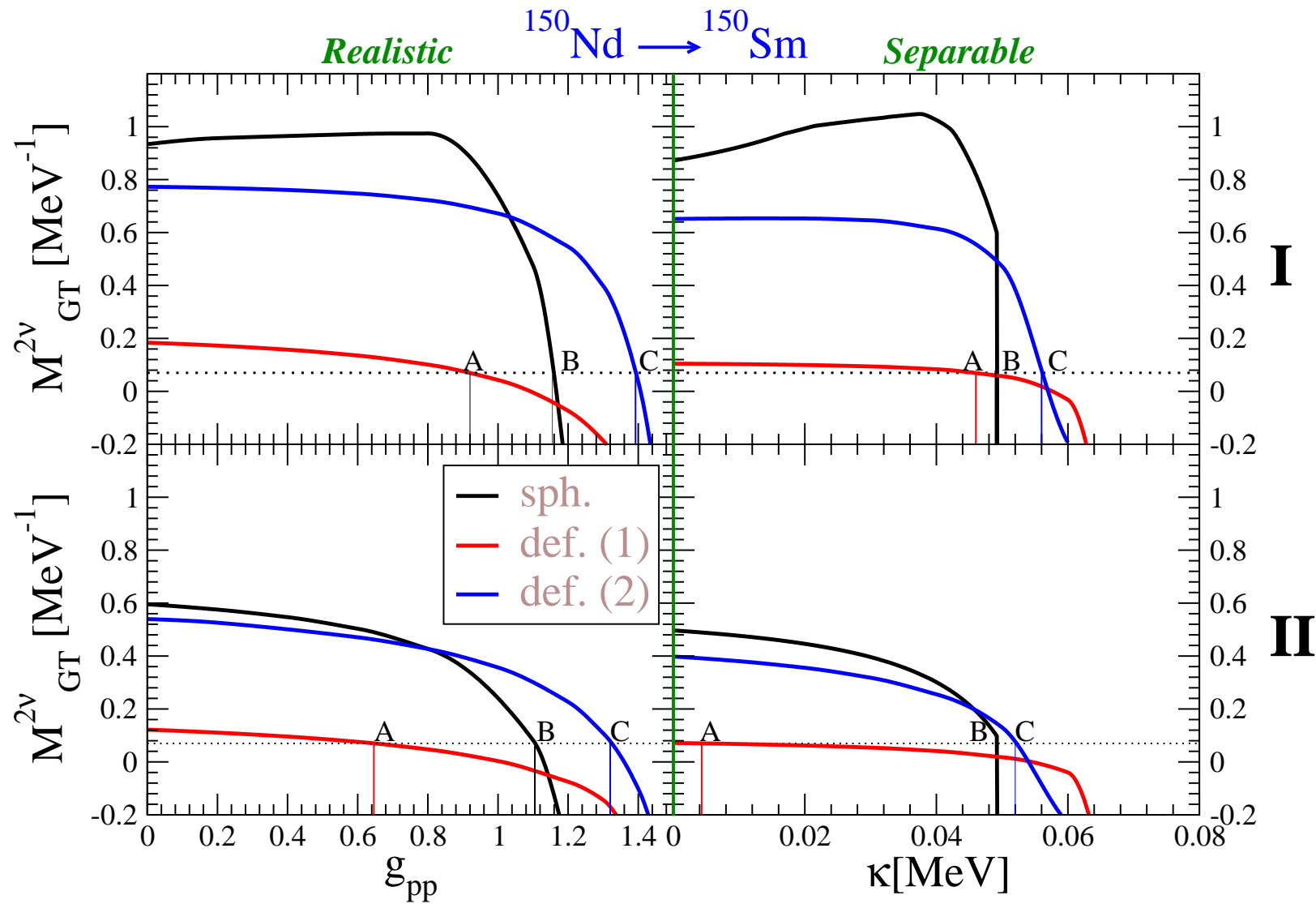


def.= exp. defor.:  $\beta_2(^{76}\text{Ge})= 0.1, \beta_2(^{76}\text{Se})= 0.16$  (P. Raghavan, At. Data Nucl. Data Tabl. **42** (1989))

# Results

## GT strength functions





**def. (1)** — exp. defor.:  $\beta_2(^{150}\text{Nd}) = 0.37 \pm 0.09$ ,  $\beta_2(^{150}\text{Sm}) = 0.23 \pm 0.03$

(P. Raghavan, At. Data Nucl. Data Tabl. **42** (1989))

**def. (2)** — calc. defor.:  $\beta_2(^{150}\text{Nd}) = 0.24$ ,  $\beta_2(^{150}\text{Sm}) = 0.21$

(P. Moeller et al., At. Data Nucl. Data Tabl. **59** (1995))

## Conclusions

- Realistic (G-matrix based) NN interaction is implemented in the QRPA equations for deformed nuclei
- GT strength functions and  $2\nu\beta\beta$  decay matrix element are calculated for  $^{76}\text{Ge}$  and  $^{150}\text{Nd}$ .
- Prospect for QRPA calculation of  $M^{0\nu}$  for  $^{150}\text{Nd}$  is opened