Exact partition functions for the effective confining string in Lattice Gauge Theories.

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Plan of the Talk

- Introduction to Lattice Gauge Theories
- Quark-antiquark potential: the Nambu-Goto effective string
- Polyakov loop correlators.
- Interfaces in the 3d Ising model.
- Conclusions

Main goal of this talk:

Show that the Nambu-Goto effective string is a poweful tool to study several non-perturbative observables in LGTs and in the 3d Ising model.

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References

M.C., M. Hasenbusch and M.Panero
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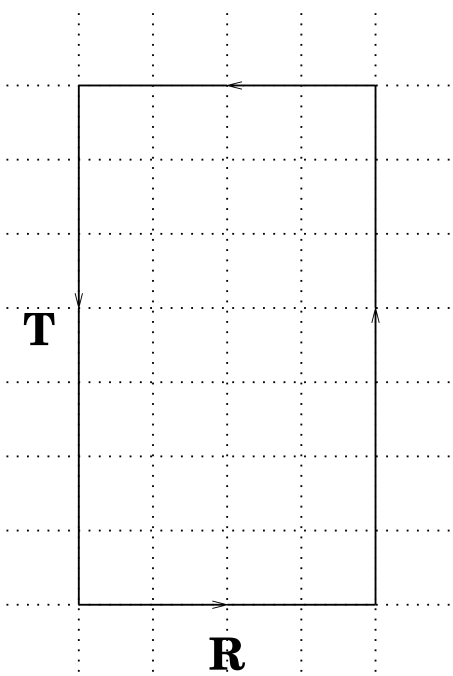
String theory and Lattice Gauge Theories

- Conjecture: Two color sources in a confining gauge theory are bound together by a thin flux tube, which can fluctuate like a massless string.

- Main consequence: Linearly rising potential.

- Wilson loop:



- Potential $V(R)$ between two external, massive quark and anti-quark sources from Wilson loops

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-LV(R)} \quad \text{(large } L\text{)}$$

$$V(R) = -\lim_{L\to\infty} \frac{1}{L} \log \langle W(L,R)\rangle$$

In the limit of infinite mass quarks (pure gauge theory) we find the famous "area law" for the Wilson loop

- Area law $\leftrightarrow$ linear potential

$$\langle W(T,R)\rangle \sim \mathrm{e}^{-\sigma RT}; \qquad V(R) = \sigma R + \ldots$$

σ is the string tension

Quantum corrections and effective models

The area law is exact in the strong coupling limit ($\beta \to 0$). As the continuum limit ($\beta \to \infty$) is approached quantum corrections become important.

- Leading correction for large R: The "Lüscher term"

$$V(R) = \sigma R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) \ . \qquad (1)$$

  from quantum fluctuations of $d-2$ massless modes: transverse fluctuations of the string [Lüscher, Symanzik and Weisz, (1980)]

- Can be derived assuming a (very naive!) "SOS picture" for the fluctuations of the surface bordered by the Wilson loop $\to$ two-dimensional conformal field theory of $d-2$ free bosons

- This is the first (and simplest) example of an effective description for the interquark potential.

- Consequence: Using known results of CFT's we can predict the behaviour of the Wilson loop for finite values of L.

$$< W(R,L) >= e^{-(\sigma RL)} Z_q(R,L)$$

Where $Z_q(R,L)$ is the partition function of $d-2$ free massless scalar fields living on the rectangle defined by the Wilson loop: $R \times L$

$$Z_q(R,L) \propto \left[\frac{\eta(\tau)}{\sqrt{R}}\right]^{-\frac{d-2}{2}}$$

where $\eta(\tau)$ is the Dedekind η function and $\tau = iL/R$.

- The $L \leftrightarrow R$ simmetry is ensured by this identity

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\;\eta(\tau)$$

known as the "modular" transformation of the η.

- Defining: $F(R,L) \equiv -\log < W(R,L) >$ and expanding the Dedekind function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n) \quad ; \quad q = e^{2\pi i \tau} \quad ,$$

one finds

$$F(R,L) = \sigma RL - (d-2)\left[\frac{\pi L}{24R} + \frac{1}{4}\log R\right] + \ldots$$

From which we find as anticipated:

$$V(R) = \sigma\, R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) \; .$$

The following ratio is particularly useful to single out the effective string contribution from a collection of Wilson loops (it requires a very precise knowledge of σ):

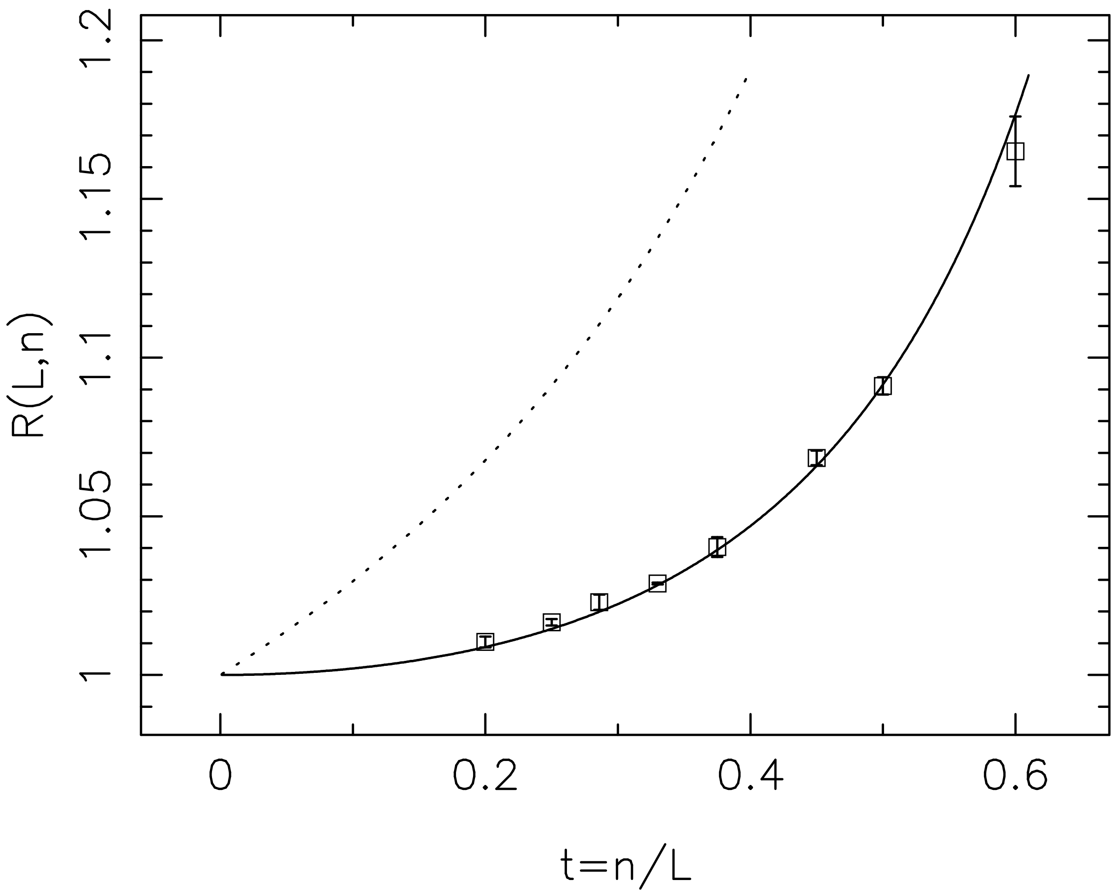
$$R(L,n) \equiv \frac{\langle W(L+n,L-n)\rangle}{\langle W(L,L)\rangle} \exp(-n^2\sigma)$$

It is easy to see that $R(L,n)$ depends only on $t = n/L$:

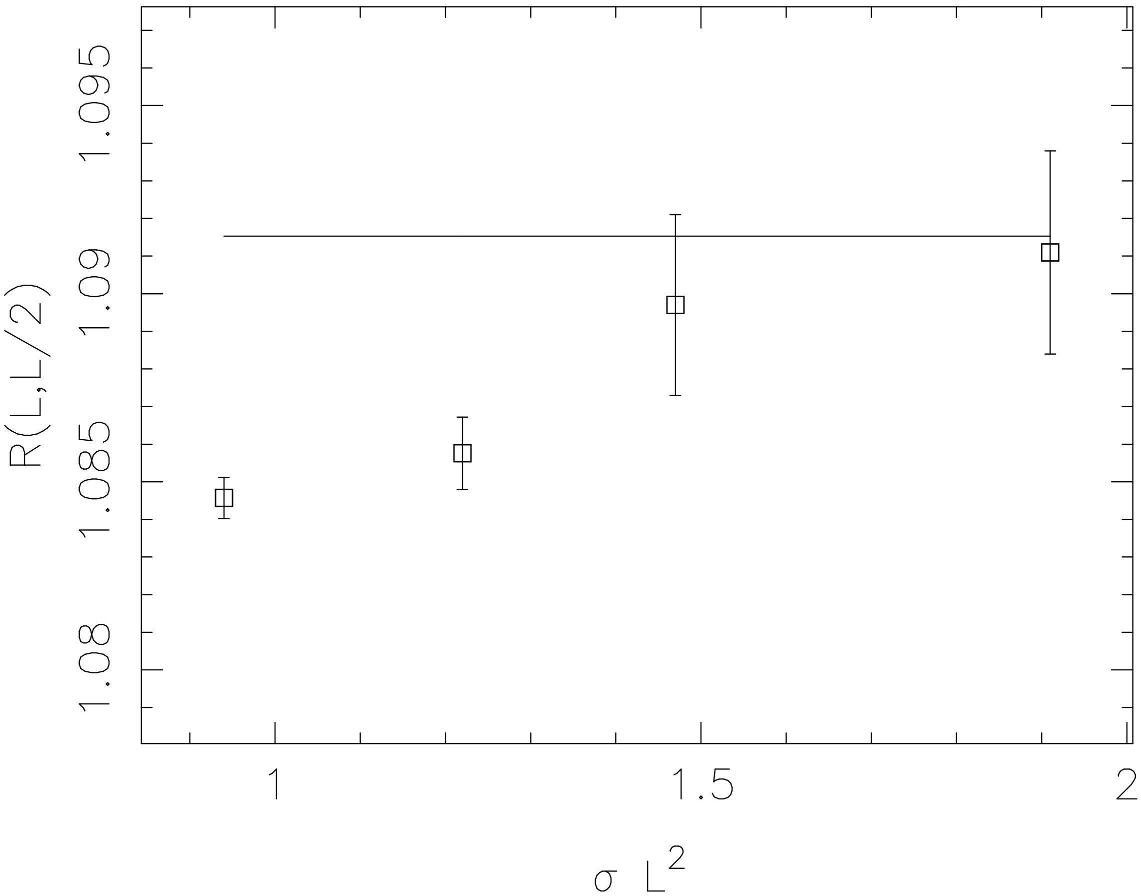
$$R(L,n) = F(t) = \left[\frac{\eta(i)\sqrt{1-t}}{\eta\left(i\frac{1+t}{1-t}\right)} \right]^{1/2}$$

and does not contain any adjustable parameters.

This behaviour can be tested with Montecarlo simulation. In the case of the 3d Ising gauge model very precise simulations can be performed and a perfect agreement in the large R regime is found



However corrections appear as R decreases and suggest that a more sophisticated effective description is needed.



Short R deviations: the prediction of the free string model is $R(L, L/2) = 1.09153\ldots$ (straight line).

The underlying string model should determine a specific form of the effective theory, and an expression of the potential $V(R)$ that extends to finite values of R.

Various models of effective strings

- "Free" theory: $d-2$ bosonic fields living on the surface spanned by the string, describing its transverse fluctuations

- Standard bosonic string theory: Nambu-Goto action $\propto$ area of the world-sheet surface
  - Possible first-order formulation á la Polyakov (we'll use this)
  - In $d \neq 26$, bosonic string is ill-defined (Weyl invariance broken by quantum effects).

- Attempts to a consistent string theory description: Polchinski-Strominger, Polyakov, AdS/CFT
  - This is the aim, of course. However, we'll not touch the subject in this talk...

The Nambu-Goto string

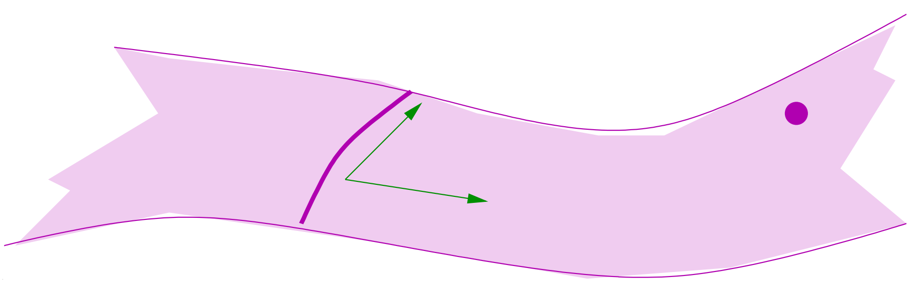
- Action $\sim$ area of the surface spanned by the string in its motion:

$$S = -\sigma \int d\xi^0 d\xi^1 \sqrt{\det g_{\alpha\beta}} \tag{2}$$

where $g_{\alpha\beta}$ is the metric "induced" on the w.s. by the embedding:

$$g_{\alpha\beta} = \frac{\partial X^M}{\partial \xi^\alpha} \frac{\partial X^N}{\partial \xi^\beta} G_{MN} \tag{3}$$

ξ^α = world-sheet coords. (ξ^0 = proper time, ξ^1 spans the extension of the string)



Nambu-Goto: perturbative approach

This is the "standard" approach to effective string calculation. It traces back to the seminal paper by Dietz and Filk in 1982.

- One can use the world-sheet re-parametrization invariance of the NG action to choose a "physical gauge":
  - The w.s. coordinates ξ^0, ξ^1 are identified with two target space coordinates x^0, x^1

- One can study the 2d QFT for the $d-2$ transverse bosonic fields with the gauge-fixed NG action

$$Z = \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \sqrt{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+(\partial_0 \vec{X}\wedge\partial_1 \vec{X})^2}}$$

$$= \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \left\{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+\text{int.s}\right\}}$$

perturbatively, the loop expansion parameter being $1/(\sigma A)$

- Problem: This gauge fixing is anomalous (unless we are in $d = 26$)

- "Effective" solution: It can be shown (Olesen, 1985) that the corrections due to the anomaly decay very rapidly with R thus one can hope to use this model as an "effective" large distance description. Indeed as $R \to \infty$ the NG action in the physical gauge reduces to a collection of (d-2) free bosonic fields (i.e. to the Lüscher term)

- Results: in 1982 Dietz-Filk were able to perform the calculation up to 2 loop for the 3 geometries (disk, cylinder, torus). Higher order terms are too difficult to be evaluated

Nambu-Goto: First order formulation and covariant quantization.

A way to avoid the complexity of the perturbative approach and to evaluate the EXACT partition function for the Nambu-Goto effective string is to resort to the covariant quantization of the string:

- The NG action can be written in a 1st order formulation (no awkward square roots)

$$S = -\sigma \int d\xi^0 d\xi^1 \sqrt{h} h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^M \tag{4}$$

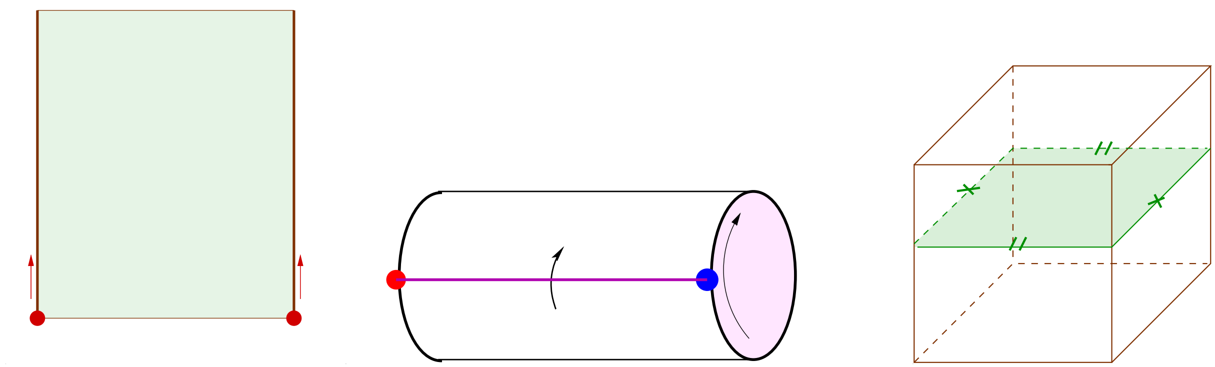
  with $h_{\alpha\beta}$ = independent w.s metric

- Re-parametrization and Weyl invariance can be used to set $h_{\alpha\beta} \to \eta_{\alpha\beta}$
  - Actually, Weyl invariance is broken by quantum effects in $d \neq 26$ → the "Liouville mode" does not decouple at the quantum level.

- As a result one ends up with a free action but with:
  - Virasoro constraints $T_{\alpha\beta} = 0$ from $h^{\alpha\beta}$ e.o.m.
  - a residual conformal invariance
- In a covariant quantization, the Virasoro constraints are imposed on physical states á la BRST
  - All d directions are treated on the same footing
  - Introduction of ghosts
- The quantum anomaly for $d \neq 26$ manifests itself with the appearance of a new field: the Liouville mode.
  - In this framework neglecting the anomaly corresponds to treat the Liouville mode as a classical field (and decouple it from the theory)
  - In principle one could improve the quality of the results by a suitable perturbative treatment of the Lioville field

Various observables with an effective string description

Three typical observables with a geometrically simple effective string picture



- Wilson loop: disk topology
- Correlator of Polyakov loops: cylinder topology
- Interfaces: torus topology

Polyakov loop correlators

The peculiar geometry of the Polyakov loop correlators implies that they are perfect tools to explore the range of scales where deviations with respect to the free bosonic effective string appear.

Important observation:

In the Wilson loop geometry $(T = 0)$ RL is simply the area of the loop. One can always choose large enough Wilson loops so as to reach the free string limit.

In the finite temperature geometry $L = 1/T$. The free string limit is reached only for very low temperatures. In particular at intermediate temperatures (say, $T \geq T_c/3$) higher order effects (which encode the self-interaction of the bosonic fields) become important and cannot be neglected.

First order approximation: transverse d.o.f. as free bosons

Let us define the free energy as

$$G(R) = \langle P(0)P^{\dagger}(R)\rangle = \exp\left[-F(R,L)\right]$$

$F(R,L)$ depends on the inverse temperature $L \equiv 1/T$ ($i.e.$ the lattice size in the compactified time direction) and the distance R, and is given by a classical and a quantum contribution:

$$F(R,L) = F_{\rm cl}(R,L) + F_{\rm q}(R,L)$$

The classical term corresponds to the area law:

$$F_{\rm cl}(R,L) = \sigma_0 LR + k(L)$$

while the quantum term turns out to be:

$$F_{\rm q}(R,L) = (d-2)\log\eta(\tau) \qquad \tau \equiv \frac{iL}{2R}$$

where η is again the Dedekind function

Important observation:

Due to the modular transformation

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\;\eta(\tau)$$

the asymptotic expansion is different in the two regimes:

$2R < L$

$$F_q(R,L) = (d-2)\left[-\frac{\pi L}{24R} + \sum_{n=1}^{\infty}\log(1 - e^{-\pi n L/R})\right]$$

$2R > L$

$$F_q(R,L) = (d-2)\left[-\frac{\pi R}{6L} + \frac{1}{2}\log\frac{2R}{L}\right]$$

$$+(d-2)\left[\sum_{n=1}^{\infty}\log(1 - e^{-4\pi n R/L})\right]$$

Hence for $R > L/2$ the string correction is linear in R and acts as a finite temperature renormalization of the string tension:

$$\sigma(T) = \sigma_0 - (d-2)\frac{\pi T^2}{6}$$

As T increases this string effect will induce a deconfinement transition.

N. G. effective action: perturbative approach in the physical gauge.

The next to leading order in $1/\sigma$ of the free energy can be evaluated in the framework of the zeta function regularization (*K. Dietz and T. Filk, Phys. Rev. D* **27** *(1983) 2944.*) the result in $d = 3$ is:

$$F_q^{(NLO)}(R, L) = -\frac{\pi^2 L}{1152\, \sigma R^3} \left[2E_4(\tau) - E_2^2(\tau) \right]$$

where E_2 and E_4 are the second and fourth order Eisenstein functions.

$$E_2(\tau) = 1 - 24 \sum_{n=1}^{\infty} \sigma(n) q^n \quad (5)$$

$$E_4(\tau) = 1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) q^n \quad (6)$$

$$q \equiv e^{2\pi i \tau} \ , \quad (7)$$

where $\sigma(n)$ and $\sigma_3(n)$ are, respectively, the sum of all divisors of n (including 1 and n), and the sum of their cubes.

N.G.: exact partition function in the covariant gauge.

- Action (in conformal gauge)

$$S = \frac{1}{4\pi\alpha'} \int d\xi^0 \int_0^\pi d\xi^1 \left[(\partial_0 X^M)^2 + (\partial_1 X^M)^2\right] + S_{\mathrm{gh.}}$$

- World-sheet parametrized by
  - $\xi^1 \in [0, \pi]$ (open string)
  - ξ^0 (proper time)

- The fields X^M $(M = 0, \ldots, d-1)$ describe the embedding of the world-sheet in the target space

- Boundary conditions:
  - Neumann in "time" direction:

$$\partial_0 X^0(\xi^0,\xi^1)\big|_{\xi^1=0,\pi} = 0$$

  - Dirichlet in spatial directions:

$$\vec{X}(\xi^0,0) = 0 \;, \quad \vec{X}(\xi^0,\pi) = \vec{R} \;.$$

"open string between D0-branes"

- The string fields have thus the expansion

$$X^0 = \hat{x}^0 + \frac{\hat{p}^0}{\pi\sigma} + \frac{\mathrm{i}}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\alpha^0}{n} \mathrm{e}^{-\mathrm{i}n\xi^0} \cos n\xi^1$$

$$\vec{X} = \frac{\vec{R}}{\pi}\xi^1 - \frac{1}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\vec{\alpha}}{n} \mathrm{e}^{-\mathrm{i}n\xi^0} \sin n\xi^1$$

$$\left[\alpha^M_m , \alpha^N_n\right] = m\,\delta_{m+n,0}\,\delta^{MN}$$

The free energy

Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string. The result is

$$\mathcal{F} = \mathcal{F}^{(0)} + 2\sum_{m=1}^{\infty} \mathcal{F}^{(m)}$$

with

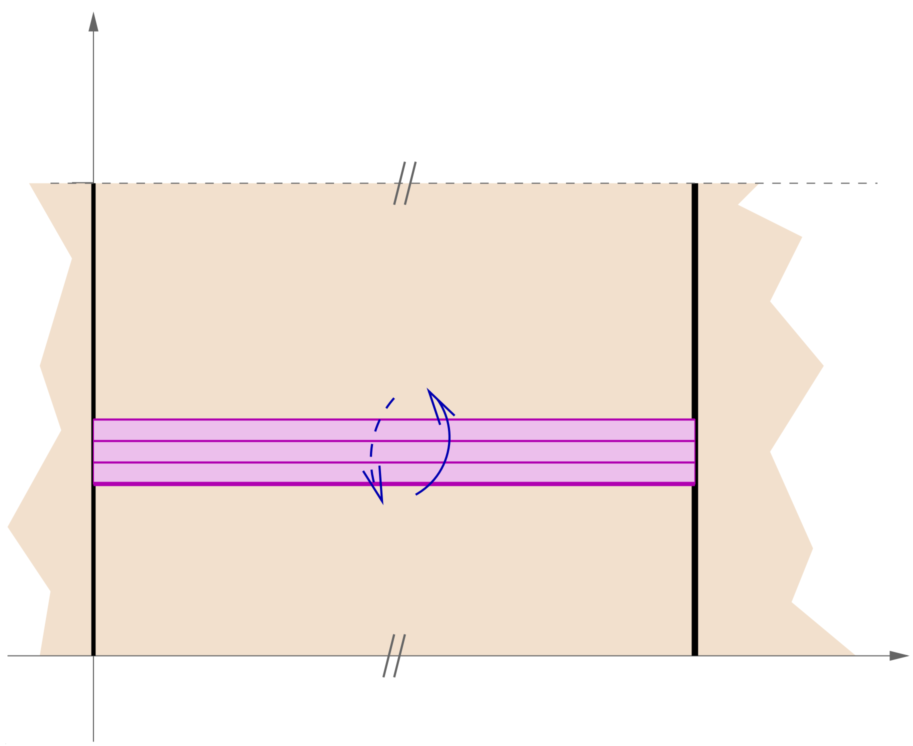
$$\mathcal{F}^{(m)} = \sqrt{\frac{\sigma L^2}{4\pi}} \int_0^\infty \frac{dt}{2t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\sigma L^2 m^2}{4t} - \sigma R^2 t} \left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2}$$

where the integer m is the # of times the open string wraps the compact time in its one loop evolution.

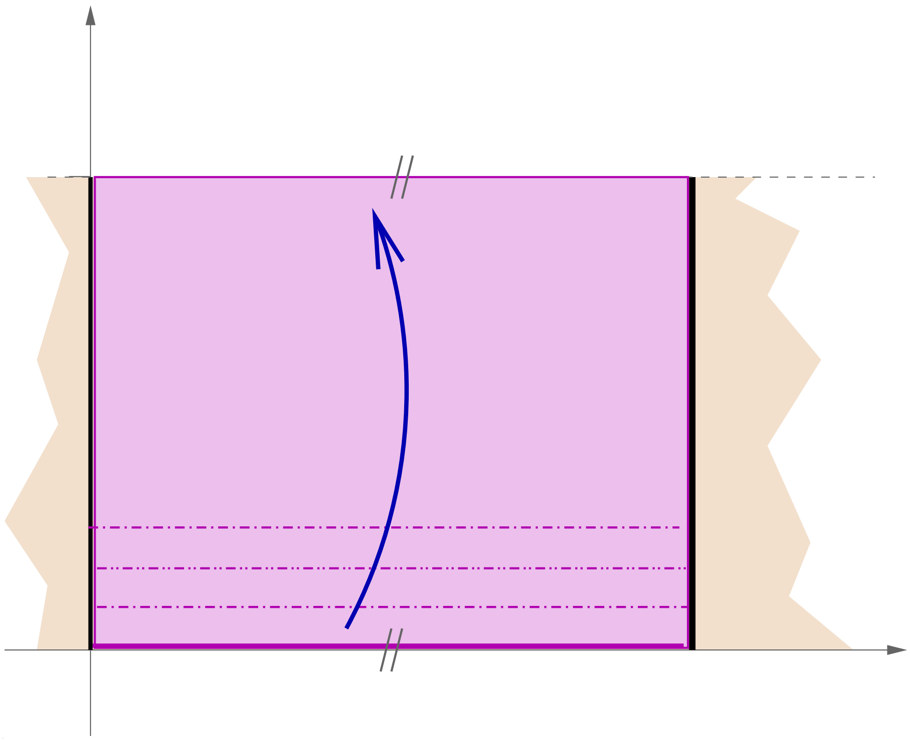
Each topological sector $\mathcal{F}^{(m)}$ describes the fluctuations around an "open world-sheet instanton"

$$X^0(\xi^0 + t, \xi^1) = X^0(\xi^0, \xi^1) + mL$$

An example with $m = 0$ (N.B. The classical solution degenerates to a line)



- The case $m = 1$. The world-sheet exactly maps to the cylinder connecting the two Polyakov loops.



- The sector with $m = 1$ of our free energy corresponds to the effective NG partition function we are looking for.

The NG result as a sum over excited states.

$$\mathcal{F}^{(m)} = \sqrt{\frac{\sigma L^2}{4\pi}} \int_0^\infty \frac{dt}{2t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\sigma L^2 m^2}{4t} - \sigma R^2 t} \left(\frac{1}{\eta(\mathrm{i}t)} \right)^{d-2}$$

- Expand in series the Dedekind functions:

$$\left(\prod_{r=1}^{\infty} \frac{1}{1-q^r} \right)^{d-2} = \sum_{k=0}^{\infty} c_k q^k$$

- Plug this into $\mathcal{F}^{(m)}$ and integrate over t using

$$\int_0^\infty \frac{dt}{t^{\frac{3}{2}}} \, \mathrm{e}^{-\frac{\alpha^2}{t} - \beta^2 t} = \frac{\sqrt{\pi}}{|\alpha|} \mathrm{e}^{-2|\alpha|\,|\beta|}$$

- The result is

$$\mathcal{F}^{(m)} = \frac{1}{2|m|} \sum_k c_k \mathrm{e}^{-|m| L E_k(R)} \,, \qquad (m \neq 0)$$

where the coefficient c_k are the partitions of integers (they come form the expansion of the Dedekind function) and

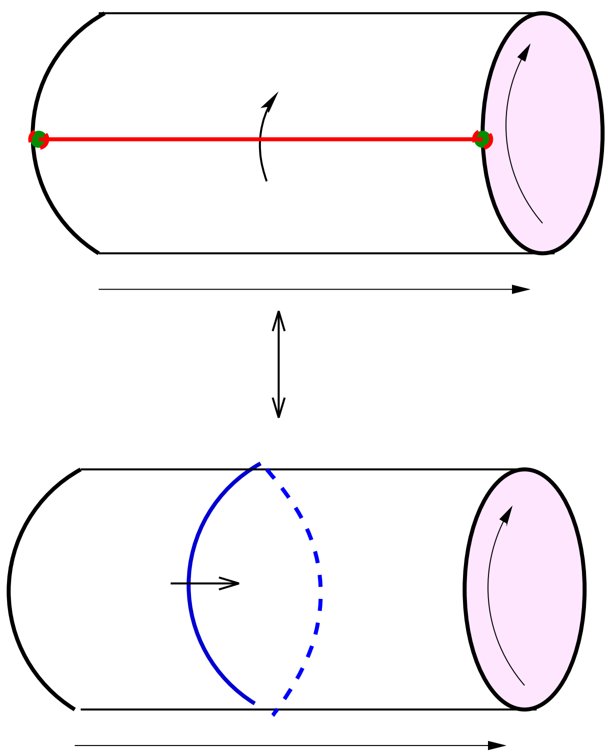
$$E_k(R) = \sigma R \sqrt{1 + \frac{2\pi}{\sigma R^2}\left(k - \frac{d-2}{24}\right)}$$

- first test: This spectrum coincides with that conjectured by Arvis long ago (1982) by (formal) quantization of the NG action in the physical gauge.

- second test: Expanding the above expression in powers of $\frac{1}{\sigma R^2}$ one exactly recovers at the second order the Dietz and Filk's result.

Duality and the closed string interpretation

Our first-order formulation is well-suited to give the direct closed string channel description of the correlator: The closed string channel tree level exchange between boundary states corresponds to the modular transformation $t \to 1/t$ of the open string channel 1-loop free energy



The result of the transformation is

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k c_k\, G\left(R; M(m,k)\right)$$

where $G(R;M)$ = propagator of a scalar field of mass M over the spatial distance $\vec{R}$ between the two D0-branes:

$$G(R;M) = \frac{1}{2\pi}\left(\frac{M}{2\pi R}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(MR) \quad ,$$

the mass $M(m,k)$ is that of a closed string state with k representing the total oscillator number, and m the wrapping number of the string around the compact time direction

$$M^2(m,k) = (m\sigma L)^2\left[1 + \frac{8\pi}{\sigma L^2 m^2}\left(k - \frac{d-2}{24}\right)\right]$$

and T_0 is the usual D0-brane tension in bosonic string theory:

$$T_0^2 = 8\pi\left(\frac{\pi}{\sigma}\right)^{\frac{d}{2}-2}$$

Third test: This expression agrees with that obtained by Lüscher and Weisz (2005) with a different approach.

Two important remarks

- From the lowest mass ($k = 0$, $m = 1$) in the closed string channel:

$$M^2(1,0) = (\sigma L)^2 \left[1 - \frac{\pi(d-2)}{3\sigma L^2}\right]$$

one can obtain an estimate for the deconfinement temperature (recall that $T = 1/L$) which in this framework appears as a consequence of the tachionic state present in the NG string (Olesen, 1985):

$$\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{3}{\pi(d-2)}}$$

This estimate turns out to be in very good agreement with MC simulations for several LGT's.

- In the large R and small L limit the NG partition function reduces to a single Bessel function. This means that the the NG effective string predicts the following behaviour for the Polyakov loop correlator:

$$\langle P(0)P^{\dagger}(R)\rangle \sim K_{\frac{d-3}{2}}(MR) \quad ,$$

with $M \sim \sigma L$. This is exactly the limit in which dimensional reduction occurs. In LGT this is known as the "Svetitsky-Yaffe" conjecture which states that the Polyakov loop correlator of a d dimensional LGT (in the confining phase) can be mapped into the spin-spin correlator of a suitably chosen $(d-1)$ spin model (in the high temperature phase). From the QFT approach to spin models we know that at high temperature and large distance whatever model we study the spin spin correlator will be dominated by the state of lowest mass whose propagator in d' dimensions is given by

$$G(R) \sim K_{\frac{d'-2}{2}}(mR)$$

Since $d' = d - 1$ this result exactly coincides with what we obtain with the NG string.

Comparison with MC simulations

Duality and a new algorithm: the snake algorithm allow high precision simulations for very large values of R and L in the gauge Ising model.

All the above predictions can be tested with precision $\frac{\delta G}{G}$ which in some cases reaches 10^{-4}.

In the comparison there is no free parameter. The figures are not the result of a fitting procedure.

The agreement at large distance with NG is impressive. At shorter distances deviations appear. Liouville mode?

The snake algorithm.

Goal: compute the ratio $G(R)/G(R+1)$.

Proposal: Use duality and factorize the ratio of partition functions in such a way that for each factor the partition functions differ just by the value of $J_{\langle ij \rangle}$ at a single link

$$\frac{Z_{L\times R}}{Z_{L\times(R+1)}} = \frac{Z_{L\times R,0}}{Z_{L\times R,1}} \cdots \frac{Z_{L\times R,M}}{Z_{L\times R,M+1}} \cdots \frac{Z_{L\times R,L-1}}{Z_{L\times R,L}} \, ,$$

where $L \times R, M$ denotes a surface that consists of a $L \times R$ rectangle with a $M \times 1$ column attached.

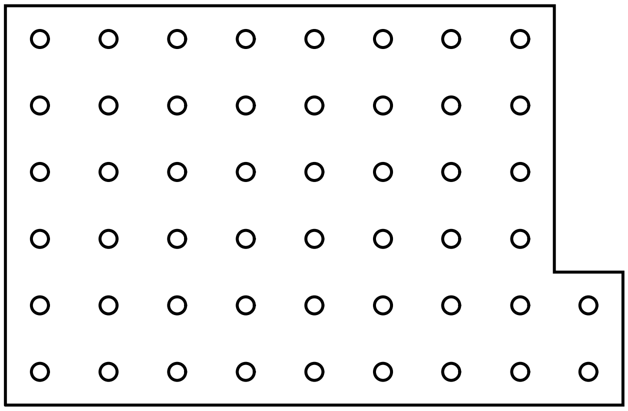


Figure 1: Sketch of the surface denoted by $L \times R, M$. In the example, $L = 6$, $R = 8$ and $M = 2$. The circles indicate the links that intersect the surface.

Each of the factors can be written as expectation value in one of the two ensembles:

$$\frac{Z_{L \times R, M+1}}{Z_{L \times R, M}} = \frac{\sum_{s_i = \pm 1} \exp(-\tilde{\beta} H_{L \times R, M}(s)) \; \exp(-2\tilde{\beta} s_k s_l)}{Z_{L \times R, M}} \; ,$$

where $< k, l >$ is the link that is added going from $L \times R, M$ to $L \times R, M + 1$.

Further improvement: hierarchical updates.

Result: the error show no dependence on R !!

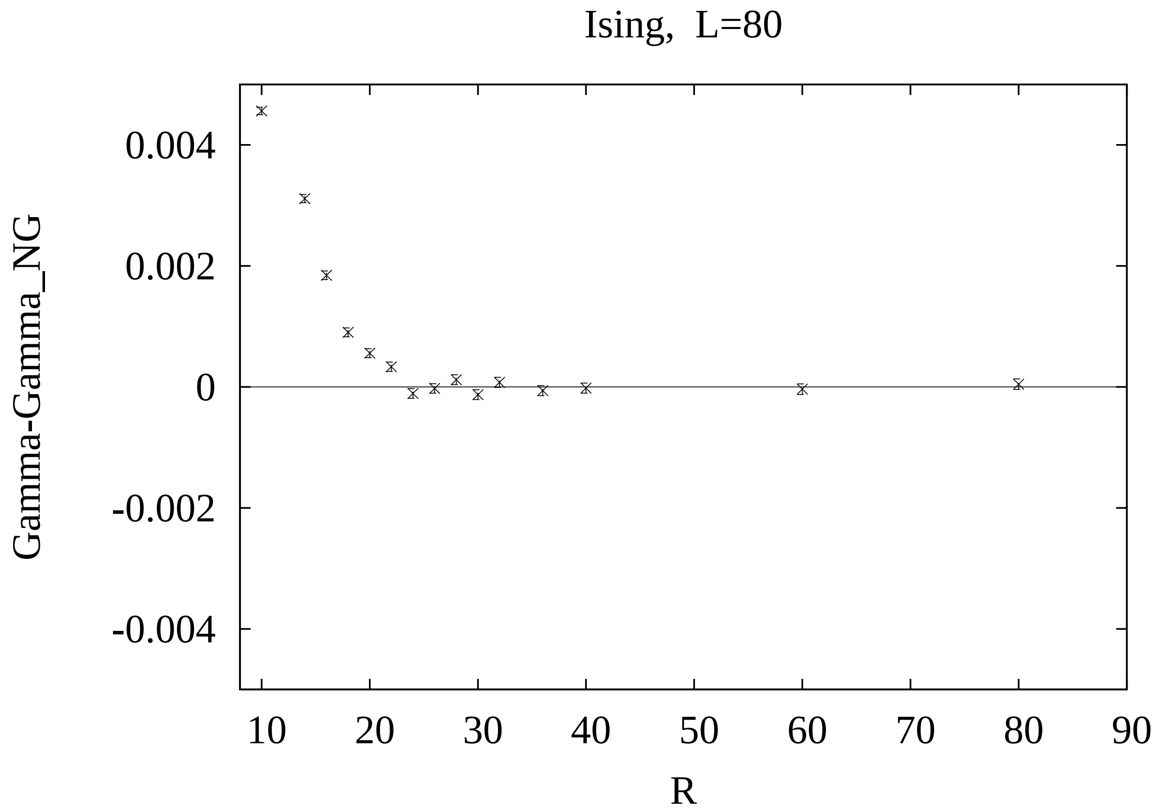


Figure 2: Montecarlo results for the Polyakov loop correlators in the (2+1) dimensional gauge Ising model. The data are taken at a fixed value of the lattice in the time direction: $L = 80$ (which corresponds to a very low temperature $T = T_c/10$) and a varying size of the interquark distance $(10 < R < 80)$. In the figure is plotted the deviation of Γ (the ratio $G(R+1)/G(R)$ of two correlators shifted by one lattice spacing) with respect to the Nambu-Goto string expectation Γ_{NG} (which with this definition of observables corresponds to the straight line at zero). Notice the remarkable agreement in the range $24 < R < 80$, which is not the result of a fitting procedure: in the comparison reported in the figure there is no free parameter.

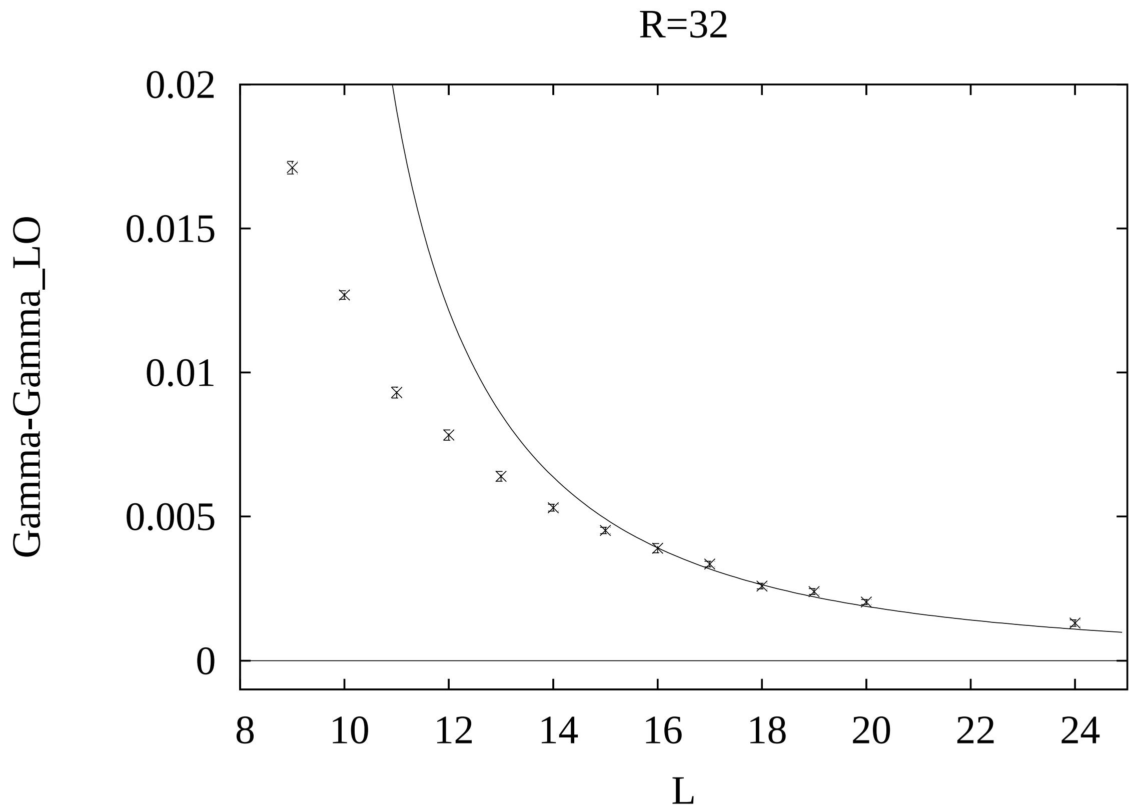
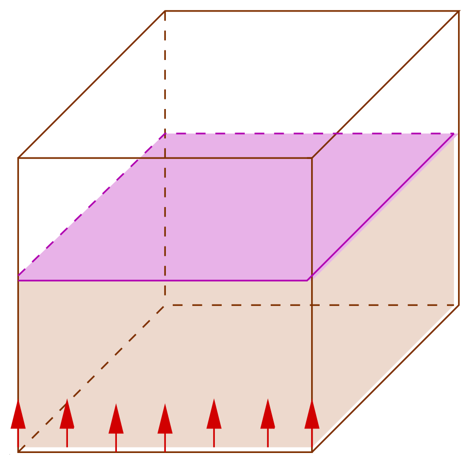


Figure 3: Montecarlo results for the Polyakov loop correlators in the (2+1) dimensional gauge Ising model. The data are taken at a fixed value $R = 32$ of the interquark distance and a varying size ($8 < L < 24$) of the lattice in the time directions. In the figure is plotted the deviation of Γ (defined as in the previous figure) with respect to the asymptotic free string expectation Γ_{LO} (which with this definition of observables corresponds to the straight line at zero). The curve is the Nambu-Goto prediction for this observable. Notice the remarkable agreement in the range $16 < L < 24$, which as for the previous figure is not the result of a fitting procedure: in the comparison reported in the figure there is no free parameter.

Interfaces in Ising-like models

- An interface separating regions with different magnetization can be forced in discrete spin models (Ising, etc.) by suitably fixing the boundary conditions in the direction orthogonal to the interface.



- in three dimensions such a spin model is related by duality to a gauge model. Example:

  3d Ising spin model $\leftrightarrow$ 3d Ising gauge model

- It is rather natural to try to describe the fluctuating interface by means of some effective string theory as for the Wilson loop and the P.L. correlators.

- This approach is known in condesed matter as the "capillary wave model" various results for Ising (and related) interfaces were obtained in past years using the first order approximation (free bosonic fields) of this model. No attempt was done to go beyond this level.

Nambu-Goto result

Also in this case it is possible to evaluate exactly the NG partition function, following the same lines of the Polyakov loops calculation. The final result depends only on the geometry of the target space, in particular on the area $A = L_1 L_2$ and the modulus $u = L_2/L_1$ of the interface plane:

$$\mathcal{I}^{(d)} = 2\left(\frac{\sigma}{2\pi}\right)^{\frac{d-2}{2}} V_T \sum_{m=0}^{\infty}\sum_{k=0}^{m} c_k c_{m-k}\left(\frac{\mathcal{E}}{u}\right)^{\frac{d-1}{2}} K_{\frac{d-1}{2}}(\sigma A \mathcal{E})$$

with V_T the transverse volume and

$$\mathcal{E} = \sqrt{1 + \frac{4\pi u}{\sigma A}(m - \frac{d-2}{12}) + \frac{4\pi u^2 (2k-m)^2}{\xi^2 A^2}}$$

We checked that the expansion at the second order in $\frac{1}{\sigma A}$ of this expression agrees with the Dietz and Filk calculation.

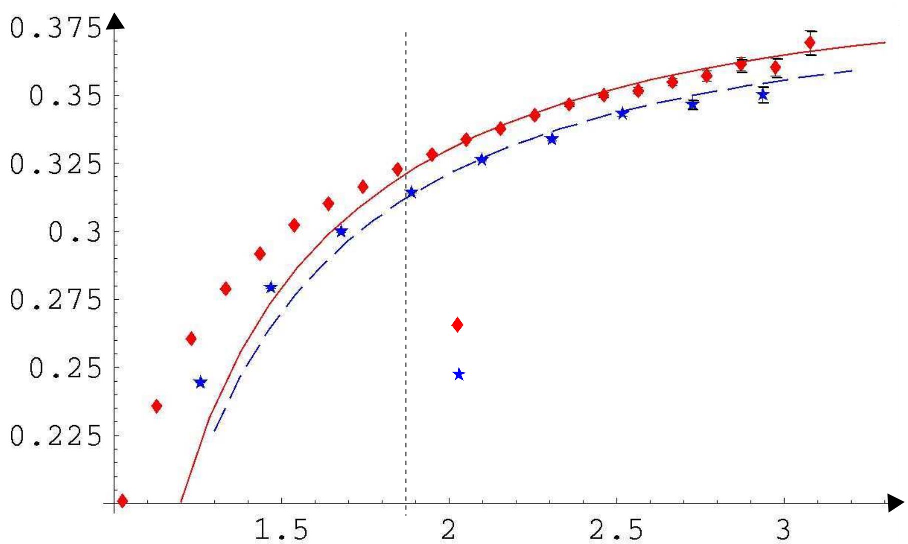
Comparison with MC simulations

Very accurate MC estimates for the free energy F_s of interfaces in the 3d Ising model can be obtained with snake-like algorithms.

We can compare the F_s MC data with the free energy F obtained from our partition function in $d = 3$:

$$F = -\log\left(\frac{\mathcal{I}^{(3)}}{V_T}\right) + \mathcal{N} \ .$$

The constant $\mathcal{N}$ is the only free parameter to be fitted.



Square lattices ($u = 1$):

$L_{\rm min}$	$(\sqrt{\sigma A})_{\rm min}$	$\mathcal{N}$	$\chi^2/(\rm d.o.f)$
Data set 1			
19	1.949	0.91957(18)	4.22
20	2.051	0.91891(22)	1.84
21	2.154	0.91836(27)	0.63
22	2.257	0.91829(33)	0.70
23	2.359	0.91797(45)	0.63

Rectangular lattices ($u \neq 1$)

L_1	L_2	$\sqrt{\sigma A}$	u	F_s	diff
10	12	2.29843	6/5	7.1670(6)	0.0016
10	15	2.56972	3/2	8.4449(12)	−0.0004
10	18	2.81498	9/5	9.6976(17)	−0.0009
10	20	2.96725	2	10.5235(25)	−0.0012
10	22	3.11208	11/5	11.3466(36)	0.0017

- No fitted parameters! (the normaliz. $\mathcal{N}$ was already fixed by previous fit).

Conclusions

- Covariant quantization of the free bosonic string attached to two parallel D0 branes leads to the effective Nambu-Goto string if one neglects the Liouville mode

- MC simulations strongly support the conjecture that Polyakov loops correlators and interfaces in the 3d gauge Ising model are well described at large enough distances and low enough temperatures by this Nambu-Goto effective string theory

- At smaller distances and/or higher temperatures significative deviations from the N-G predictions appear.

Open questions and future plans

- The large distance agreement with N-G is a peculiarity of the 3d Ising model or a general feature of LGT's?.

- The short distance deviations are a signature of the breaking of the effective string picture or may be interpreted in a string framework (maybe in terms of the Liouville mode)?

- Which is the meaning (in the context of LGT) of the other topological sectors that we obtain in covariant quantization? Is there a simple way to test these predictions on the lattice?

- Can this effective NG string shed some light on the real (i.e. consistent at the quantum level) string theory behind the 3d Ising model and QCD?