The rough phase in random percolation

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SM&FT 2006

Foreword

- This talk is based on
  - FG,S.Lottini, M.Panero, A.Rago, *Random percolation as a gauge theory,* Nucl.Phys.B 179 (2005) 255 [arXiv:cond-mat/0502339].
  - S.Lottini,FG, *The glue-ball spectrum of random percolation* PoS LAT2005:292 [arXiv:hep-lat/0510034]
  - FG, *Where is the confining string in random percolation* [arXiv:hep-lat/0601011]
- plus work in progress with Stefano Lottini and Pietro Giudice

Plan of the talk

1. Motivation
2. Percolation as a gauge theory
   - Confinement and scaling
   - Deconfinement at finite T
   - “Magnetic” monopole condensation
3. Roughening
   - The two confining phases
   - Quantum string fluctuations
   - A strong coupling expansion
4. Conclusions

Motivation

- ⋆ The main features of the confining phase of whatever gauge theory do not depend very much on the nature of the gauge group G
  - Area decay of large Wilson loops $\langle W(R,T)\rangle \propto e^{-\sigma RT}$
  - Universal, G independent, $1/R$ and $1/R^3$ corrections of the confining potential
    $V(R) = -\lim_{T\to\infty} \frac{\log W(R,T)}{T} \simeq \sigma R - \frac{\pi(D-2)}{24R} - \frac{\pi^2(D-2)^2}{1152\,\sigma\,R^3}$
  - glue-ball spectrum:
    $$\frac{m_{0^+}}{\sqrt{\sigma}} = 3.87(3)\ SU(2)\ ,\ D = 3+1$$
    $$\frac{m_{0^+}}{\sqrt{\sigma}} = 3.65(3)\ SU(3)\ ,\ D = 3+1$$
    $$\frac{m_{0^+}}{\sqrt{\sigma}} = 3.08(3)\ \ Z_2\quad ,D = 2+1$$
  - Deconfining temperature
    $T_c/\sqrt{\sigma} = 0.596(4) + 0.453(30)/N^2\ , SU(N),\ \ D = 3+1$.

- ⋆ Also the proposed confinement mechanisms (*magnetic monopole condensation , center vortices, confining string,...*) do not depend crucially on the gauge group
- ⋆ There is a good numerical support of these mechanisms but there is no proof
- ⋆ The various mechanisms are mutually inter-related, but the logical implications are not completely clear

- Search for a drastically simplified version of a confining gauge theory in which
  - ➪ The area decay of the Wilson loops is true by construction
  - ➪ The gauge group is trivial
  - ➪ Check whether the other general properties of the confining phase follow

Percolation as a gauge theory

Random Percolation

- ⋆ Percolation studies random graphs in a lattice. generated by switching on each link (or site) with probability p
- ⋆ Near the percolation threshold $p = p_c$ the connected components (clusters) of these random graphs show universal features
- ⋆ The theoretical description of the percolation processes is conventionally given in terms of the cluster sizes and most of the universal scalings deal with size distribution of clusters
- ⋆ The point of view which is taken here is different: We focus on topological entanglement of random clusters and use it to describe how *percolation theory* can be considered *as a paradigm of confining gauge theory*.

Observables

- ⋆ The most basic observables of any gauge theory are the *Wilson loops* $W_\gamma(C)$
- ⋆ Two inputs:
  - a closed path γ
  - A gauge configuration C
- ⋆ one output :
  - A real or complex number $W_\gamma(C)$

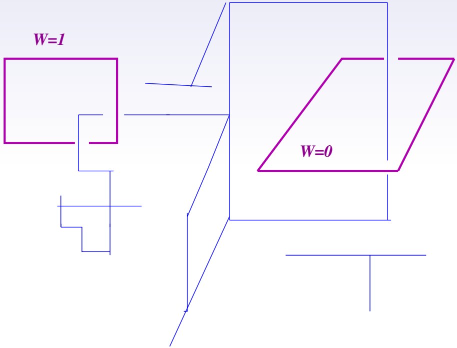
$$C \;\longrightarrow\; W_\gamma(C)$$

The setup

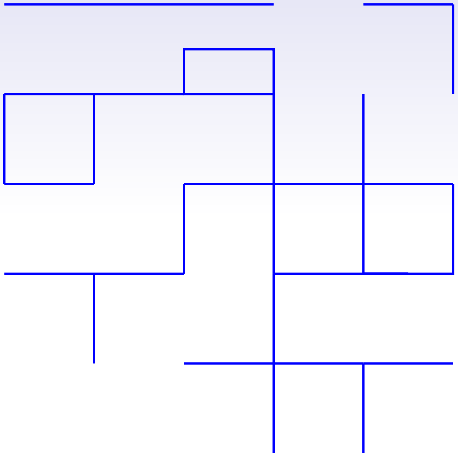
Define the following purely geometric setting

1. Generate a sample of random graphs $\{G_1, G_2, \dots\}$ simply by populating each of the links (or sites) of a (cubic) lattice *independently* with occupation probability p.
2. The physical observables of this system, that we call still Wilson operators W_γ, are associated to arbitrary loops γ of the dual lattice and obey the following rule
   - $W_\gamma(G_i) = 1$ if no cluster of the configuration G_i is topologically linked to γ;
   - $W_\gamma(G_i) = 0$ otherwise;
   - $\langle W_\gamma \rangle = \lim_{n\to\infty} \sum_{i=1}^{n} W_\gamma(G_i)/n$.

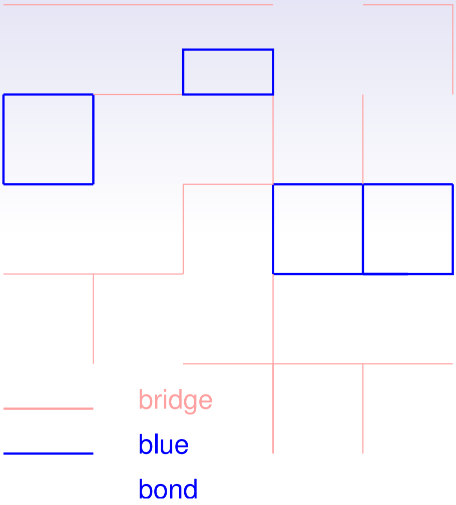
Topological linking of W



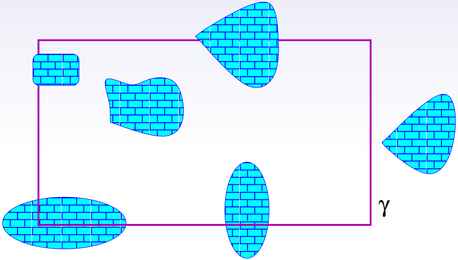
linking of W depends only on closed paths



linking of W depends only on closed paths



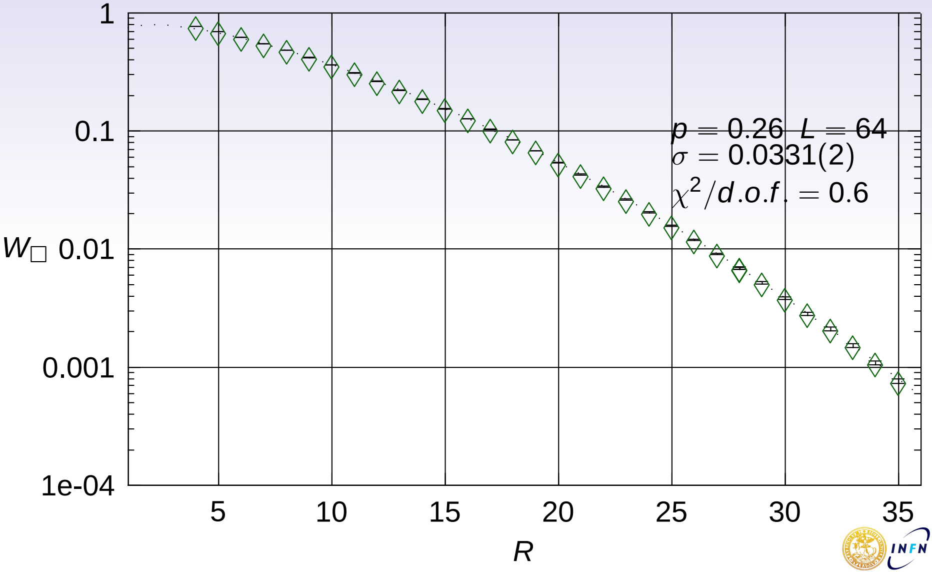
- Adding or removing bridges does not change the value of W
- The invariance of W under this local transformation is similar to a gauge transformation



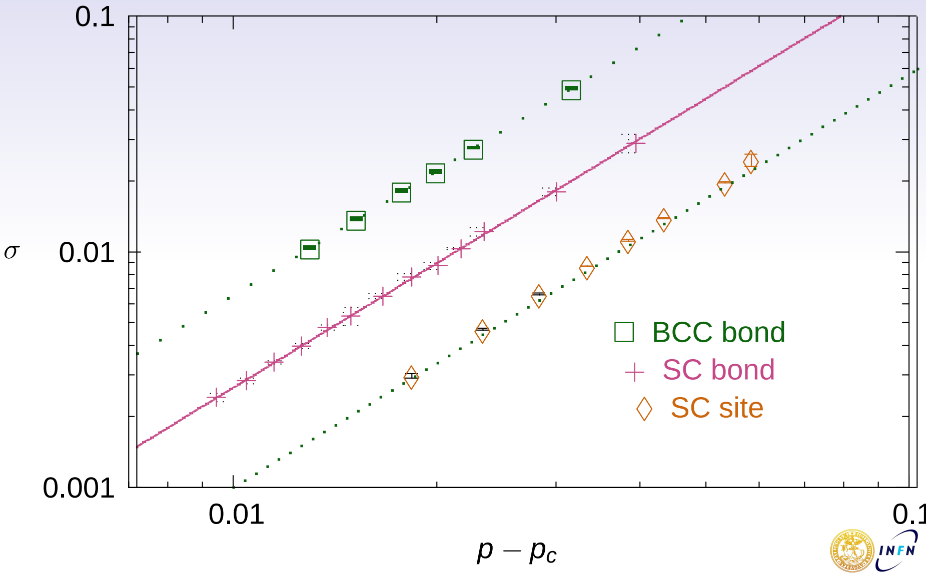
- ⋆ When $p < p_c$ only finite clusters form
- ➪ If the linear size of γ is much larger than the typical size of the clusters W_γ obeys a *perimeter law*
- ➪ when $p \geq p_c$ an infinite, percolating cluster form $\Rightarrow$ large Wilson loops obey an *area law*

Confinement and scaling

Area law for square Wilson loops



Scaling and Universality of σ



The expected behaviour of σ in the scaling region is

$$\sigma(p) = S\,(p - p_c)^{2\nu}$$

with

$$\nu = 0.8765(16)(2)$$

.

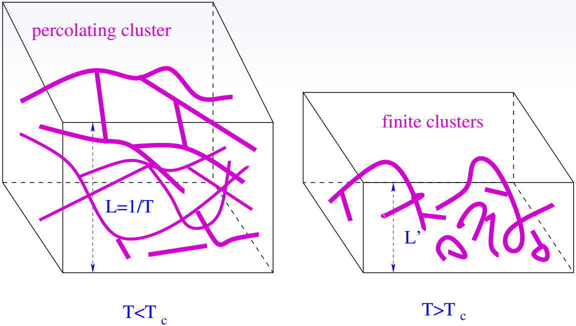
Lattice	p_c	S	$\chi^2/d.o.f$
SC site	0.3116081(7)(2)	3.370(8)	1.15
SC bond	0.2488126(5)	8.90(3)	0.30
BCC bond	0.1802875(10)	22.07(2)	0.98

Deconfinement at finite T

Deconfinement at finite T

- The percolation process explains also deconfinement at finite temperature as a *dimensional reduction*.
- Notice: percolation threshold p_c is a *decreasing* function of space-time dimension D

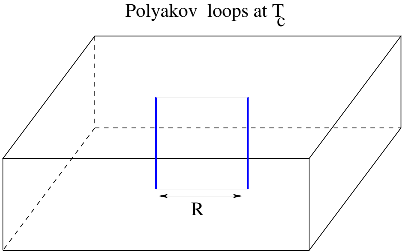
D	1	2	3
p_c	1	$\frac{1}{2}$	0.2488..



- When T reaches a threshold value T_c the percolating cluster crumbles away

Lattice	$1/T$	p_ℓ	$T_c/\sqrt{\sigma}$
SC site	7	0.3459514(12)	1.494(11)
BCC bond	3	0.21113018(38)	1.497(10)
BCC bond	4	0.20235168(59)	1.506(11)
SC bond	5	0.278102(5)	1.480(12)
SC bond	6	0.272380(2)	1.492(13)
SC bond	7	0.268459(1)	1.500(13)
SC bond	8	0.265615(5)	1.504(14)

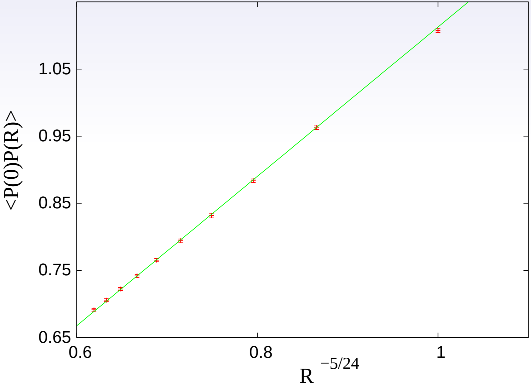
⋆ At T_c the system is *critical* $\Rightarrow$ deconfining transition expected to belong to the same universality class of 2 D percolation



Critical indices:

$\nu = \frac{4}{3}$ $\eta = \frac{5}{24}$

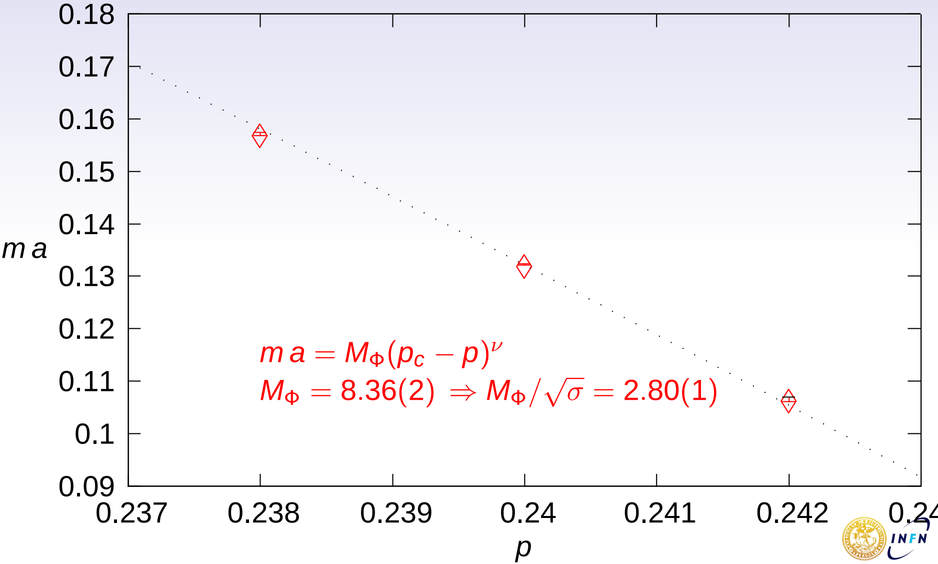
$\Rightarrow \langle P(0)\,P(R)\rangle \propto R^{-\frac{5}{24}}$ at $T = T_c$



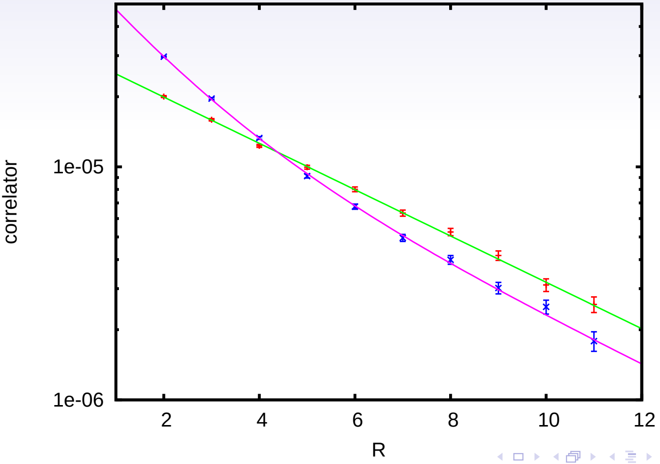
Monopole condensation

- t' Hooft and Mandelstam conjecture: The confining vacuum is a dual superconductor
- There should be a "disorder" operator $\Phi(x)$ such that
  - it is not local in the gauge variables
  - $\langle \Phi \rangle = 0$ in deconfined vacuum
  - $\langle \Phi \rangle \neq 0$ in confined vacuum *monopole condensation*
- In standard percolation: two-point function $G(x, y) \equiv$ probability that the sites x and y are in the same cluster
  - $G(x, y)$ cannot be rewritten in terms of Wilson loops: Connectivity (path between x and y) does not depend on linking properties (closed paths)
  - $\lim_{x\to\infty} G(0, x) = 0$ iff $p < p_c$
  - $\lim_{x\to\infty} G(0, x) \neq 0$ iff $p > p_c$
- This strongly suggests $G(x, y) = \langle \Phi(x) \Phi(y) \rangle$

$p < p_c$ Monopole mass

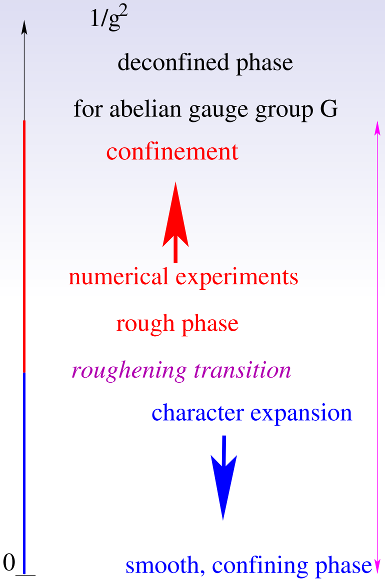


$p > p_c$ Glue-ball spectrum : $M_G/\sqrt{\sigma} = 4.17(3)$



Roughening and string vibrations

The two confining phases of gauge theories



perimeter law

$$\langle W_\gamma \rangle \propto e^{-b\,|\gamma|}$$

Area law

$$\langle W_\gamma \rangle \propto R_\gamma^{\frac{D-2}{4}}\, c_\gamma\, e^{-b\,|\gamma| - \sigma\, A_\gamma}$$

A_γ = minimal area of $\Sigma : \partial\Sigma = \gamma$
R_γ= linear size of γ
c_γ = shape function
$(c_{rectangle} = [\eta(it/r)]^{-\frac{D-2}{2}})$

$$\langle W_\gamma \rangle \propto e^{-b\,|\gamma| - \sigma\, A_\gamma}$$

$1/g^2 \sim 1-p$

deconfined phase

for abelian gauge group $G = 1$

confinement

numerical experiments

rough phase

roughening transition

character expansion

0

smooth, confining phase

Percolation

deconfined phase

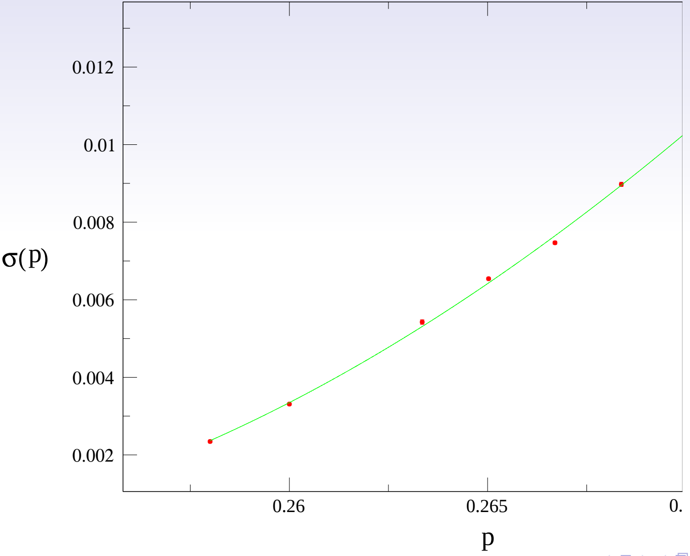
percolating phase

confinement

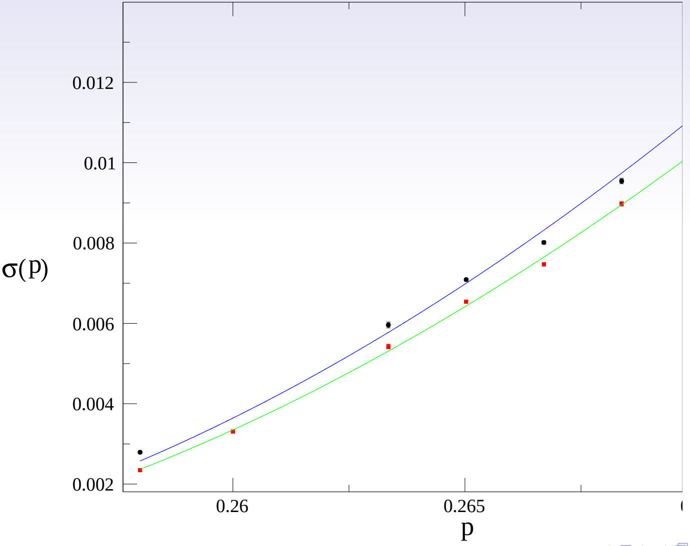
no charcter expansion

G=1

Fit to $c\,R^{\frac{1}{4}} \exp(-b\,R - \sigma\,R^2)$



Fit to $c\,\exp(-b\,R - \sigma\,R^2)$

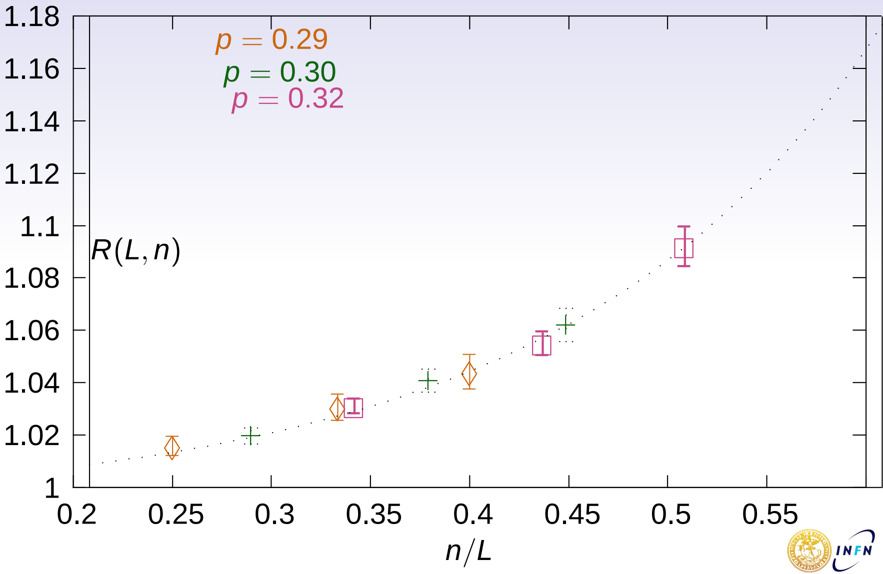


- A suitable quantity which is sensible to the universal shape effects is the function $\mathcal{R}(n,L) = \exp(-n^2\sigma)\frac{\langle W(L-n,L+n)\rangle}{\langle W(L,L)\rangle}$
- asymptotically (large L and $L-n$) (Gaussian limit) $\mathcal{R}$ becomes only a function $f(t)$ of the ratio $t = \frac{n}{L}$

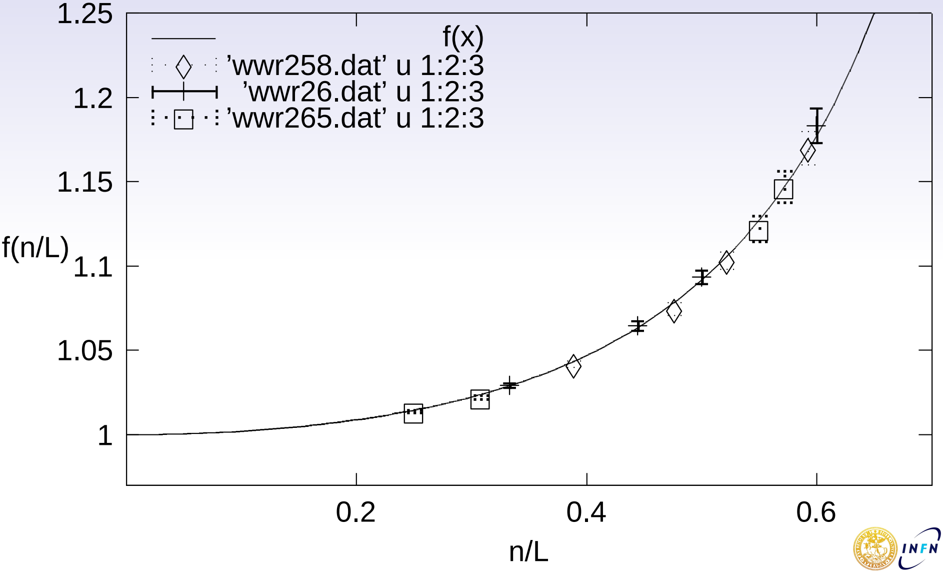
$$\mathcal{R}(n,L) \rightarrow f(t) = \left[\frac{\eta(i)\sqrt{1-t}}{\eta\left(i\frac{1+t}{1-t}\right)}\right]^{\frac{1}{2}}$$

$\eta \equiv$ Dedekind eta function

Numerical test

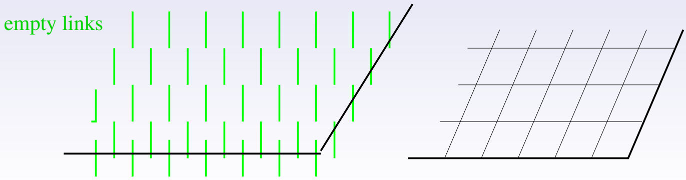


Numerical test



A strong coupling expansion

A strong coupling expansion?

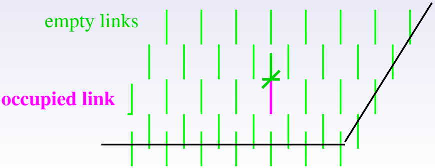


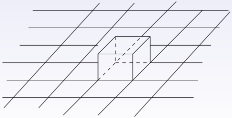
Wilson loop

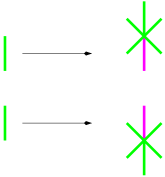
weight of configuration: $(1-p)^N$

$N=$ minimal area encircled by W

strong coupling expansion

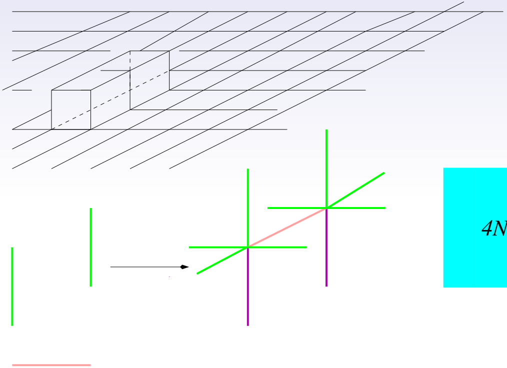






$$2N\; p\; (1-p)^{N+4}$$

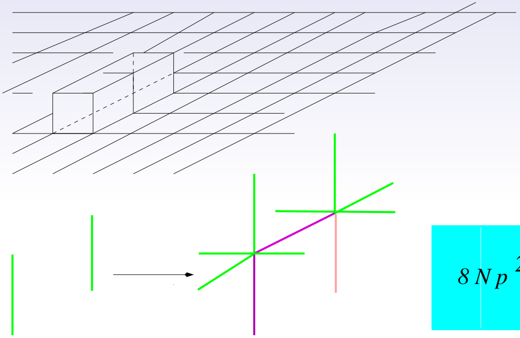
strong coupling expansion



arbitrary link

$$4N\,p^{2}\,(1-p)^{N+6}$$

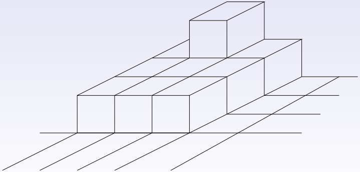
strong coupling expansion



$$8 N p^2 (1-p)^{N+6}$$

arbitrary link

generic term



$$N\, m_{\Sigma}\, n_G\, p^V (1-p)^{N+A}$$

- A =area of the deformation
- V = volume of the deformation
- m_{Σ} = multiplicity of the surface diagram
- m_G = number of graphs with V bonds drawn inside the deformed volume in such a way that switching on any link orthogonal to a plaquette yields percolation through the deformed volume

$m_G \sim V!$

strong coupling expansion in percolation

$$\sigma = -\log(1-p) - 2\,p(1-p)^4 - 12\,p^2(1-p)^6 + 4\,p^2(1-p)^8 - 96\,p^3(1-p)^8 -$$

$$-90\,p^4(1-p)^8 - 908\,p^4(1-p)^{10} + 96\,p^3(1-p)^{10} - 2304\,p^6(1-p)^{10} -$$

$$-1776\,p^5(1-p)^{10} + 72\,p^3(1-p)^{10} + O[(1-p)^{12}]$$

diverges!

➪ The percolating phase is rough in the whole range $p_c < p < 1$

Conclusion

- ⋆ Random percolating clusters of any percolating process captures the major features of the confining vacuum of gauge theories
  - Deconfinement at finite T
  - Magnetic monopole condensation
  - Non-trivial glue-ball spectrum
  - universal shape effects due to confining string fluctuations
  - The percolating phase is rough in the whole range $p_c < p < 1$