Evidence for diquarks from lattice QCD

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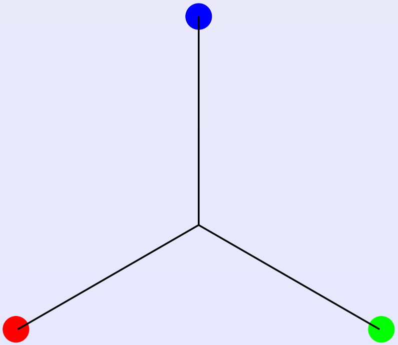
Diquark advocacy

Diquarks

- can explain the $\Delta I = 1/2$ rule in weak non-leptonic decays
- can explain some phenomena observed in deep inelastic scattering experiments
- are Cooper pairs of colour superconductivity
- can explain stability of some exotica (e.g. X and Y) and their general absense from the spectrum
- can explain some features of excited baryon spectrum

Diquarks in hadrons

Flux tube structure in baryons



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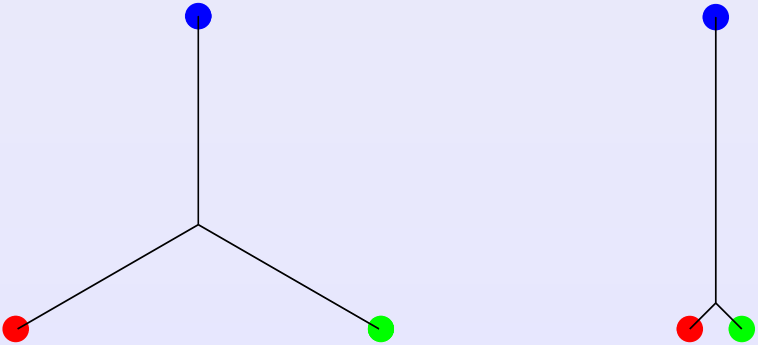

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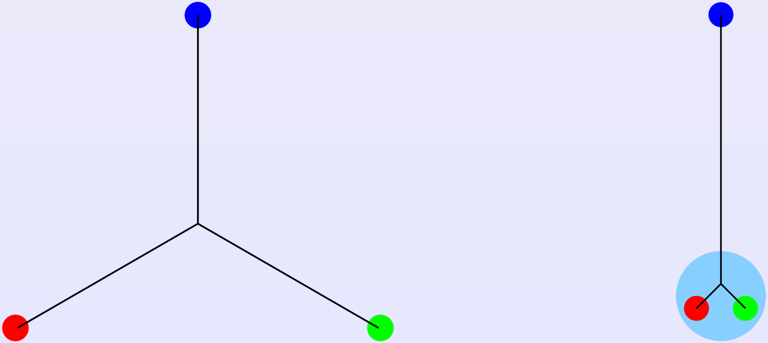

- qq (antisymmetrized in colour) behaves like $\bar{q}$

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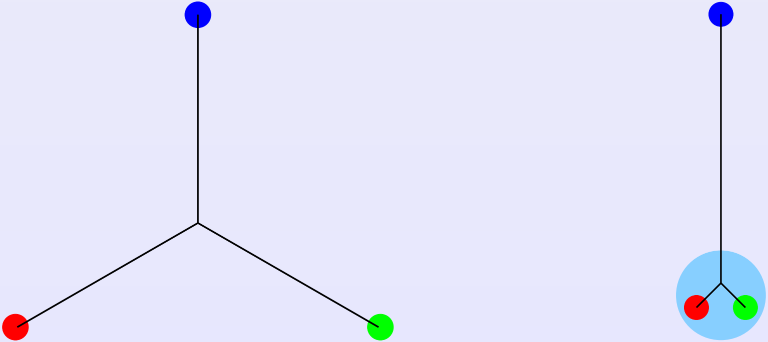

- qq (antisymmetrized in colour) behaves like $\bar{q}$
- one-gluon exchange: $V_{qq} = \frac{1}{2} V_{q\bar{q}}$ attractive

Complete Classification (R. Jaffe, hep-ph/0409065)

Diquarks are a combination of quarks in the colour antitriplet

$$3 \otimes 3 = \bar{3} \oplus 6$$

Quantum numbers and operators			
J^P	Colour	Flavor	Operator
0^+	$\bar{3}$	$\bar{3}$	$\bar{q}_C \gamma_5 q\ ,\bar{q}_C \gamma_0 \gamma_5 q$
1^+	$\bar{3}$	6	$\bar{q}_C \vec{\gamma} q\ ,\bar{q}_C \sigma_{0i} q$
0^-	$\bar{3}$	6	$\bar{q}_C q, \bar{q}_C \gamma_0 q$
1^-	$\bar{3}$	$\bar{3}$	$\bar{q}_C \vec{\gamma} \gamma_5 q, \bar{q}_C \sigma_{ij} q$

The parity even flavor antisymmetric spinless combination is the most attractive channel in this sector

Phenomenology

Spin-colour effective interaction

$$\mathcal{H} = \alpha_s \sum_{i \neq j} M_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i \cdot \vec{\lambda}_j$$

Predictions

- parity-odd states heavier (suppressed in non-relat. limit)
- $M(0^+) < M(1^+)$: 0^+ is "good" diquark, while 1^+ is "bad"
- ΔM from spin-spin interaction $\propto \frac{1}{m_1 m_2}$ for heavy quarks

Diquark masses from phenomenology

Using the effective spin-colour Hamiltonian one obtains

$$M_{\mathbb{Q}} \simeq 320 \text{ MeV} \qquad \text{and} \qquad \Delta M_{\mathbb{Q}\mathbb{Q}^*} \simeq 200\text{MeV}$$

As M_q increases, $M_{\mathbb{Q}}$ increases and $\Delta M_{\mathbb{Q}\mathbb{Q}^*}$ decreases

$M_{\mathbb{Q}} \simeq M_s + 500$ MeV and $\Delta M_{\mathbb{Q}\mathbb{Q}^*} \simeq 150$ MeV if one of the quarks is a strange quark

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Lattice setup

Problem: diquarks are coloured

$\hookrightarrow$ combine (diquark+static quark) into colour singlet:
static-light-light baryon

i.e. diquark in the background colour field of static quark

Baryon propagator

$$C(\vec{x},0;\vec{x},t) = \left\langle \mathbb{Q}(\vec{x},t) P \exp\left(-ig\int_0^t A_0(\vec{x},\tau)\,\mathrm{d}\tau\right)\mathbb{Q}^\dagger(\vec{x},0)\right\rangle$$

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Lattice action

Path integral

$$Z = \int (\mathcal{D}U_\mu(i))\,(\det M(U_\mu))^{N_f} e^{-S_g(U_{\mu\nu}(i))}$$

with

$$U_\mu(i) = Pexp\left(ig\int_i^{i+a\hat{\mu}} A_\mu(x)\mathrm{d}x\right)$$

and

$$U_{\mu\nu}(i) = U_\mu(i)U_\nu(i+\hat{\mu})U_\mu^\dagger(i+\hat{\nu})U_\nu^\dagger(i)$$

Gauge part

$$S_g = \beta\sum_{i,\mu}\left(1 - \frac{1}{N}\mathcal{R}e(U_{\mu\nu}(i))\right) \quad , \quad with\ \beta = 2N/g^2$$

Wilson fermions

Take the naive Dirac fermions and add an irrelevant term that goes like the Laplacian

$$M_{\alpha\beta}(ij) = (m + 4r)\delta_{ij}\delta_{\alpha\beta} - \frac{1}{2}\left[(r - \gamma_\mu)_{\alpha\beta} U_\mu(i)\delta_{i,j+\mu} + (r + \gamma_\mu)_{\alpha\beta} U^\dagger_\mu(j)\delta_{i,i-\mu}\right]$$

This formulations breaks explicitely the chiral symmetry

Define the hopping parameter

$$\kappa = \frac{1}{2(m + 4r)}$$

Chiral symmetry recovered in the limit $\kappa \to \kappa_c$ (κ_c to be determined numerically)

Quenched approximation

For an observable $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \frac{\int (\mathcal{D}U_\mu(i)) \, (\det M(U_\mu))^{N_f} f(M) e^{-S_g(U_{\mu\nu}(i))}}{\int (\mathcal{D}U_\mu(i)) \, (\det M(U_\mu))^{N_f} e^{-S_g(U_{\mu\nu}(i))}}$$

Assume $\det M(U_\mu) \simeq 1$ i.e. fermions loops are removed from the action

The approximation is exact in the $m \to \infty$ limit

$\hookrightarrow$ expected to describe reasonably well heavy quarks, but must fail at some point

Results are to be taken only as indications

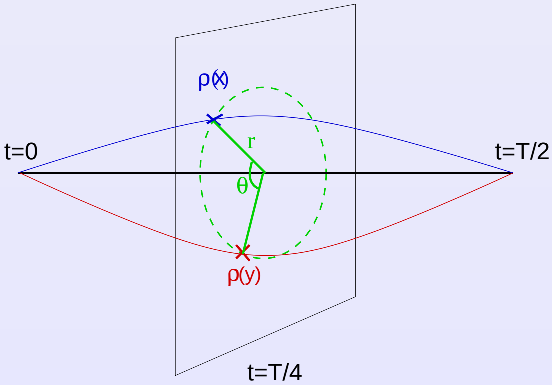
Summary of the calculations

Wilson fermions, quenched: $3 \times \beta$, $3 \times \kappa$ (heavy,medium,light)
unquenched (courtesy of the SESAM collaboration)

β	N_f	size	#conf	$a(r_0)$(fm)	κ	m_π (MeV)
5.8	0	$16^3 32$	2-500	0.136	0.156-0.159	690-910
6.0	0	$16^3 32$	2-500	0.093	0.153-0.155	620-900
6.2	0	$20^3 40$	200	0.068	0.151-0.1523	570-870
5.6	2	$24^3 40$	100	$\sim 0.1 (m_N)$	0.1575	530

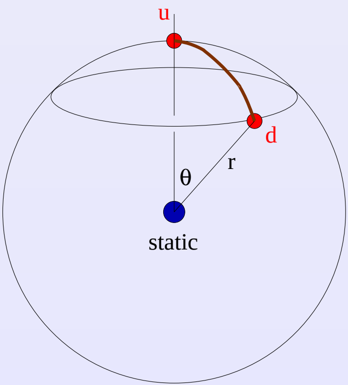
- I. Wavefunctions
- II. Masses

I. Wavefunction: density-density correlator



Fix distance from static quark → fixed background field
Look at angular distribution

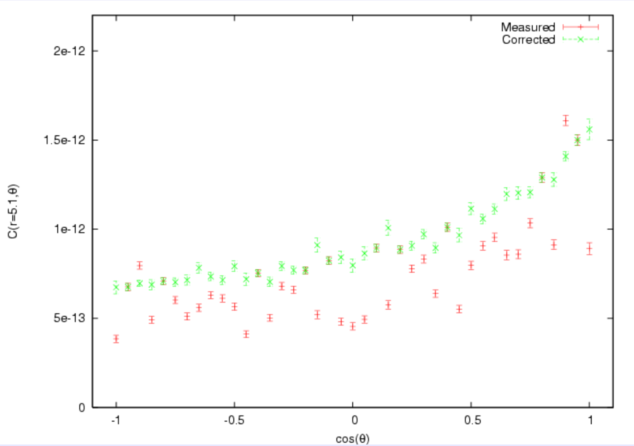
I. Wavefunction: density-density correlator



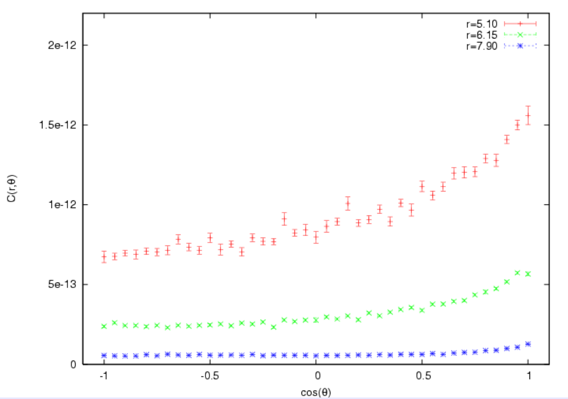
$$C_{\rm tot}(r) = \int \langle N|\rho^u \rho^d|N\rangle \; \sin\theta \mathrm{d}\theta \mathrm{d}\phi \; \equiv \int \mathrm{d}(\cos\theta) C(\theta, r), \quad \rho^q = : \bar{q}\gamma_0 q :$$

No correlation $\Leftrightarrow$ flat in $\cos\theta$

Angular distribution: Lattice vs. Continuum

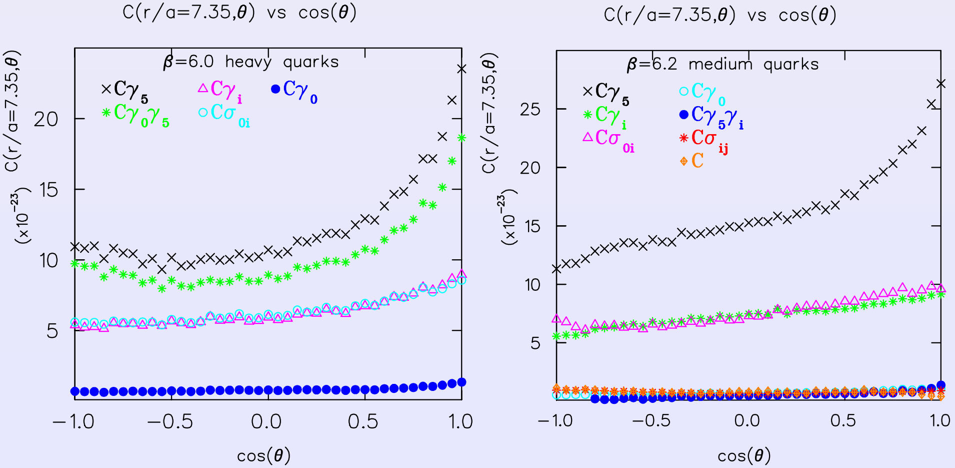


Scalar at various *r*



Robust w.r.t. the distance from the static source?

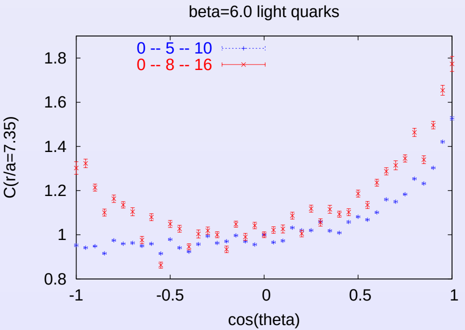
Spatial correlations



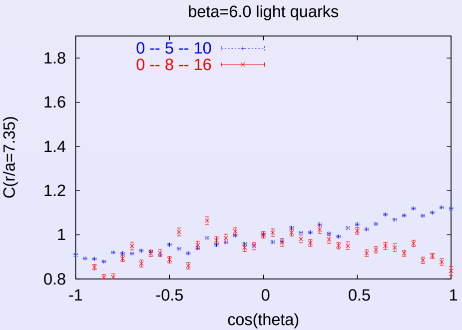
Attraction in scalar channel > vector channel as predicted

Excited states contamination?

Vary separations between source – measurement – sink:



Scalar

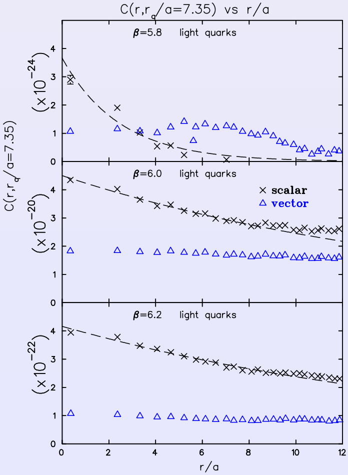


Vector

Better groundstate

$\longrightarrow$ more correlation in scalar diquark, less in vector diquark

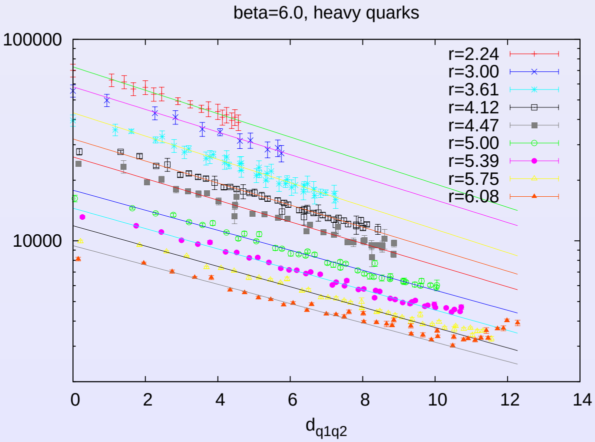
Diquark size



Very large size for vector; $\mathcal{O}(1)$fm for scalar

box size $\sim$ 1.5 fm $\rightarrow$ wrap-around effects

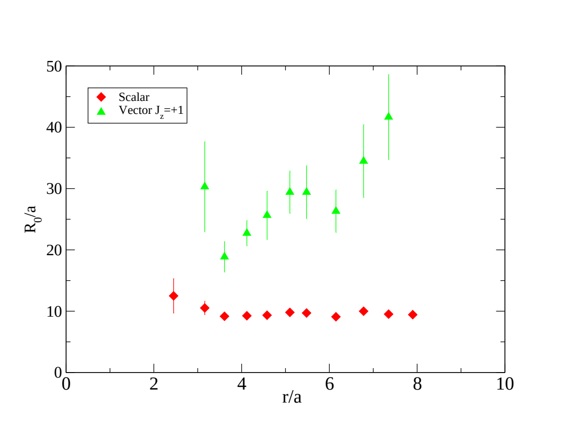
Size vs distance from static quark



Very stable (slowly increasing?)

Lighter quarks seem to give larger size – systematics when size $\gtrsim L_s/2$?

Size summary



From hep-lat/0509113

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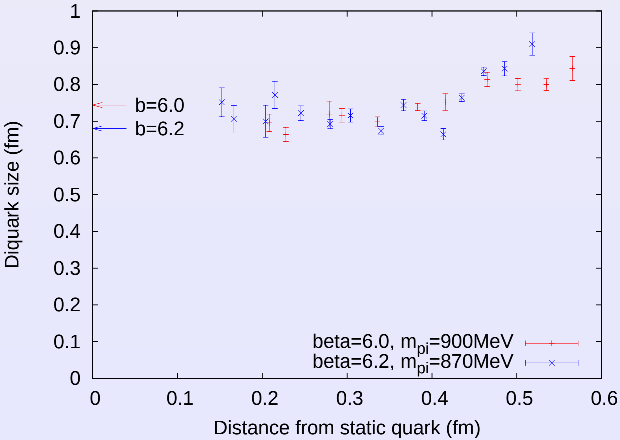
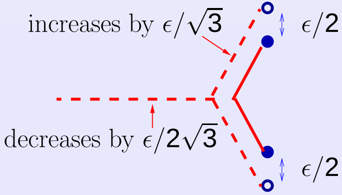


- Scalar: size $\lesssim$ 1 fm robust vs background field
- Vector: size $\gtrsim$ 2 fm

Why a large size?

- at small distances: $V_{qq} = \frac{1}{2} V_{q\bar{q}}$
- at large distances:



$$\delta L = (2/\sqrt{3} - 1/2\sqrt{3})\epsilon = \sqrt{3}\epsilon/2$$

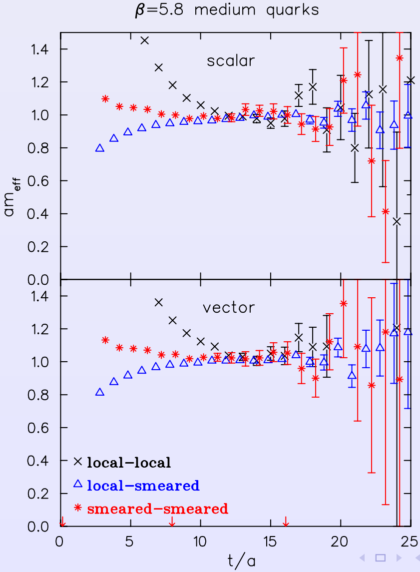
Diquarks are more loosely bound than mesons

II. Masses

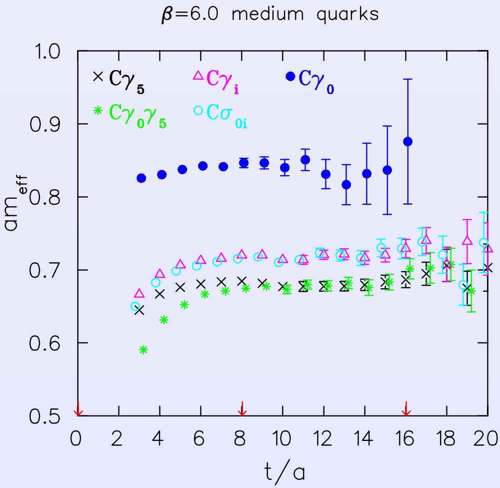
Correlation functions involving a static quark are very noisy
$\hookrightarrow$ need sophisticated techniques to isolate the ground state as quickly as possible

- HYP smearing for the temporal links entering in the construction of the static propagator
- Wuppertal smearing on the sink and the source using HYP smeared spatial links for the Wuppertal smearing function

Smearing


β=5.8 medium quarks
scalar
vector
am_eff
t/a
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25
local−local
local−smeared
smeared−smeared

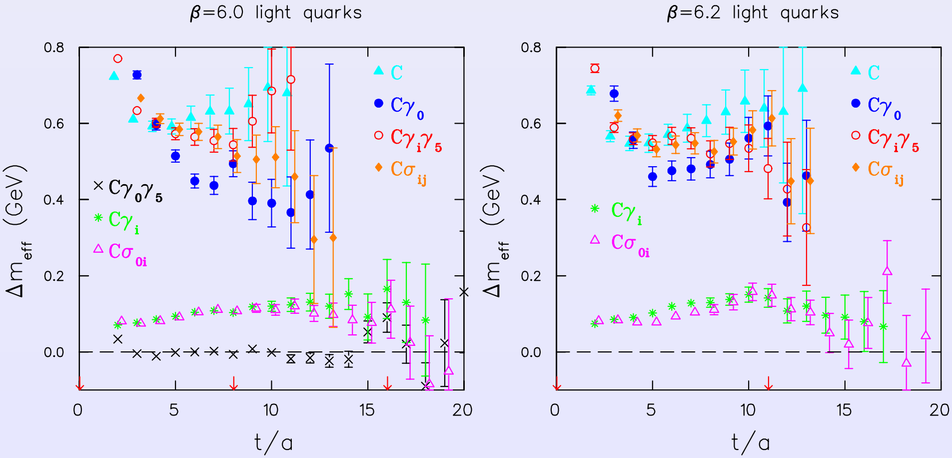
Effective masses: 3 groups



Static quark → mass UV divergent

Look at mass differences

Good, bad and worse



$m(\text{scalar}) < m(\text{vector}) \ll m(\text{others})$ as predicted

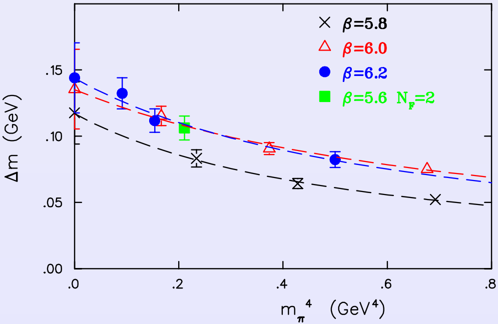
Controlling systematic effects

M(vector) - M(scalar) vs. *a* and *m*

β	κ	$\Delta M_{\mathbb{Q}\mathbb{Q}^*}$ (MeV)
5.8	0.1530	67(7)
5.8	0.1575	100(15)
6.0	0.1530	115(20)

- At our masses diquarks are quite heavy
- Our results for the mass difference are not incompatible with theoretical estimates
- Lattice artifacts for the mass difference are under control

m(vector) - m(scalar) versus pion mass



Ansatz $\Delta m = \frac{c_1}{c_2 + m_\pi^4}$ $\rightarrow$ extrapolation $\sim$ 150 MeV (200 MeV expected)

Larger in QCD? (K. Orginos, hep-lat/0510082)

Conclusions

> ”Good” diquark more than phenomenological construct
> diquarks are for real!

- All measurements consistent with predictions.
- Scalar diquark $\sim$ robust versus background field
  $\rightarrow$ unchanged in colour superconductivity?
- Tighter binding, smaller size for lighter quarks?
  $\rightarrow$ fit diquark inside nucleon?
- Outlook:

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  static $\bar{q}$ + diquark + diquark $\longrightarrow$ diquark-diquark interactions
- Density-density correlators: powerful gauge-invariant tool
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