Three flavor crystalline color superconductivity

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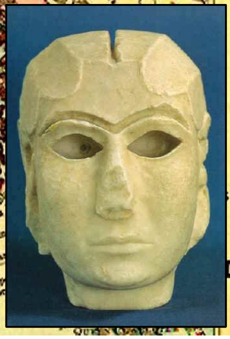


Superconductivity in QCD: Asymptotia

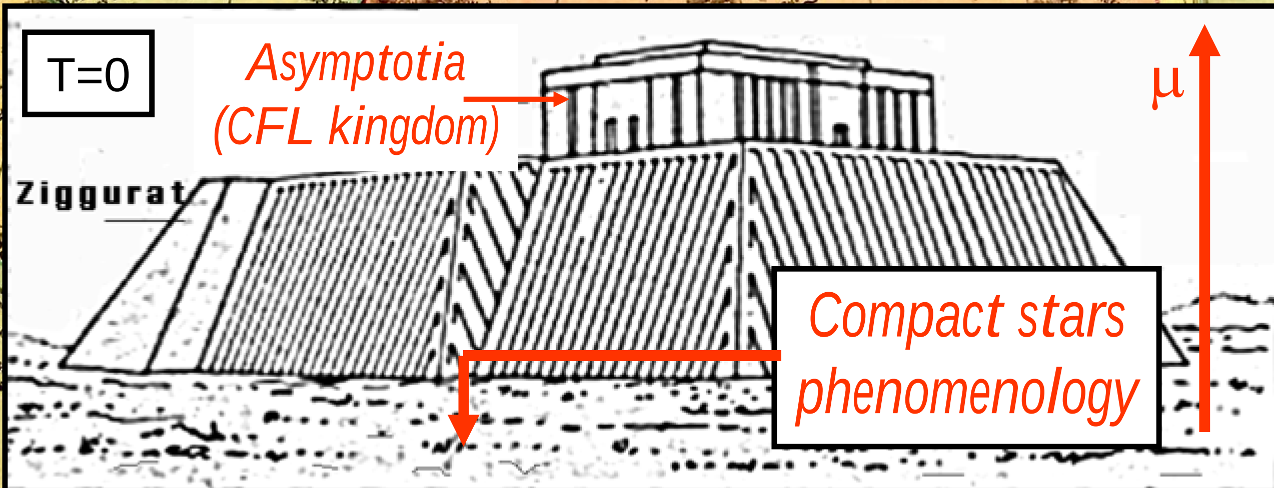
- *Asymptotically high density QCD: Deconfinement (Collins&Perry, 1975): quarks live in large Fermi spheres*
- *Attractive interaction among quarks (O.G.E. in the antisymmetric channel): Cooper pairing and Color Superconductivity*
- *BCS pairing: zero total momentum, zero total spin for the paired quarks; pair wavefunction antisymmetric in color and flavor*
- *BCS state is likely to be realized with non-mismatched Fermi spheres*

This is just the case for quarks living in Asymptotia, i.e. in the case of infinite baryon chemical potential:

1) All the quarks can be considered massless

2) Superconductive (three flavor) quark matter is neutral without need of leptons

From Asymptotia to the real world

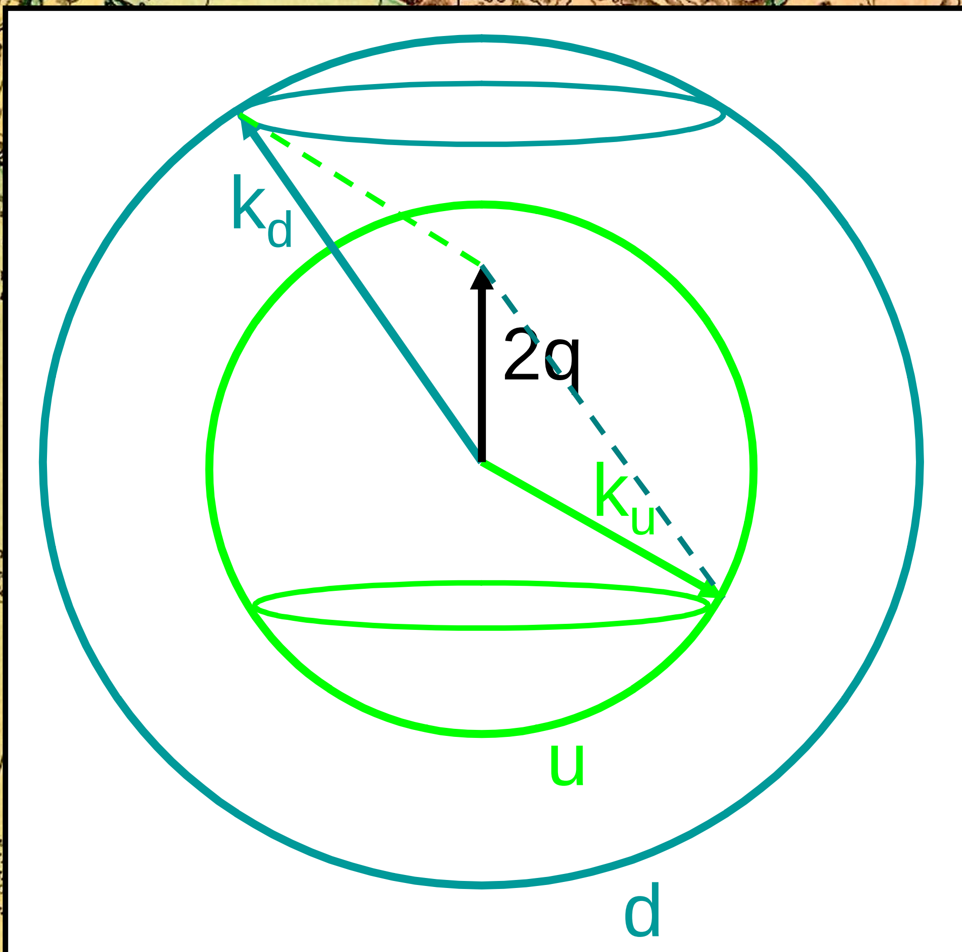


Electrical and color neutrality conditions PLUS finite strange quark mass cause the Fermi momenta of quarks with different color and flavor to be different:

$$\mu_{ij}^{\alpha\beta} = (\mu\delta_{ij} - \mu_Q Q_{ij})\,\delta^{\alpha\beta} + \delta_{ij}\left(\mu_3 T_3^{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8 T_8^{\alpha\beta}\right)$$

The Fermi spheres of the pairing quarks are likely to be mismatched, and the BCS ground state can be disfavored.

LOFF phase: the two component case



$$\langle u_g d_r \rangle \propto \Delta_3 \exp\{2\imath \vec{q}\cdot\vec{r}\}$$

Inhomogeneous superconductivity (LOFF)

(Fulde, Ferrel, 1964; Larkin, Ovchinnikov, 1964).

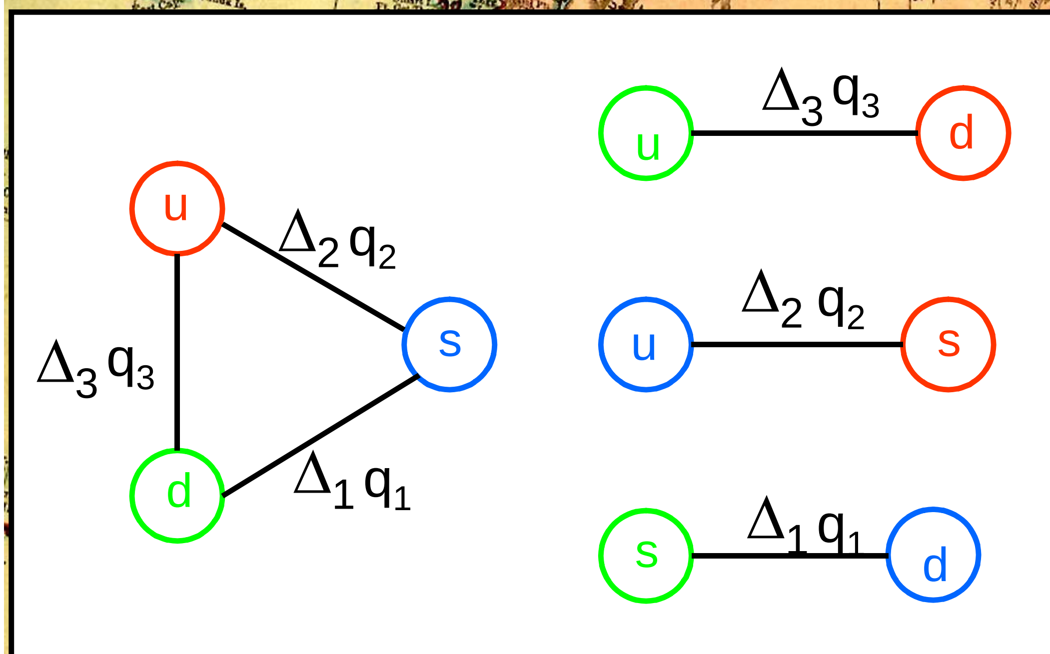
The two flavor LOFF state has been introduced in QCD by Alford et al., 2002

LOFF phase: the three flavor case

For the three flavor case (massless u and d, massive s) we consider the ansatz:

$$\langle \psi_{\alpha i}\psi_{\beta j}\rangle \propto \sum_{I=1}^{3} \Delta_I \exp\{2\imath \vec{q}_I \cdot \vec{r}\}\, \epsilon_{\alpha\beta I}\epsilon_{ijI}$$

M.R. et al., 2005
Mannarelli et al., 2006
M.R. et al., 2006

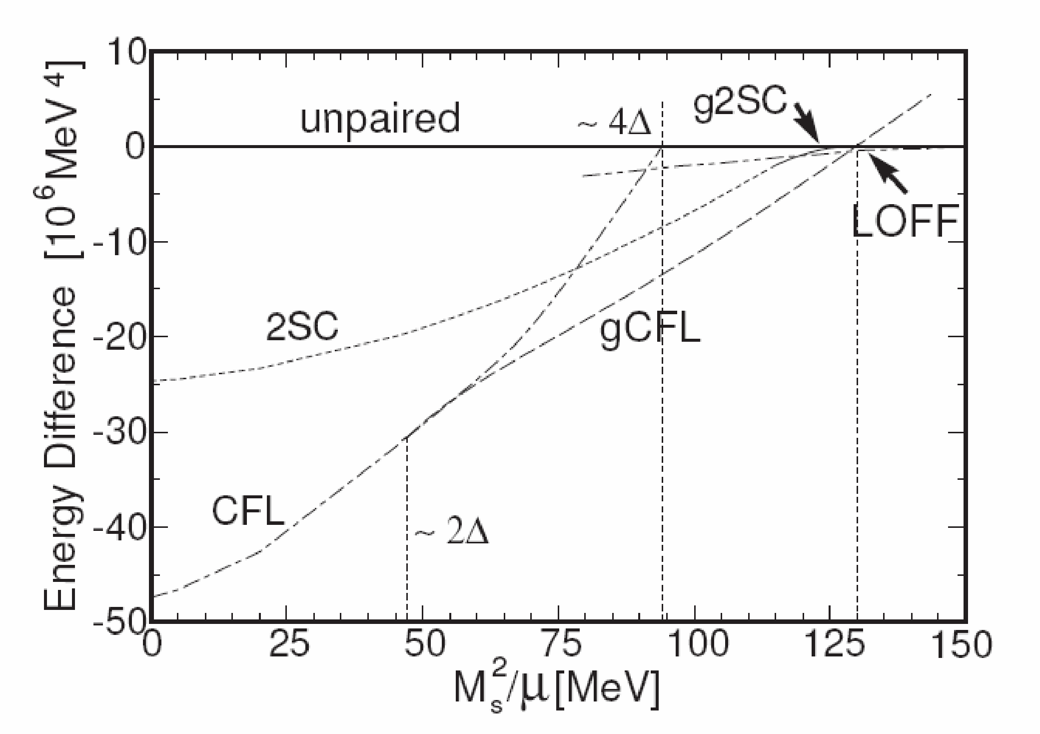


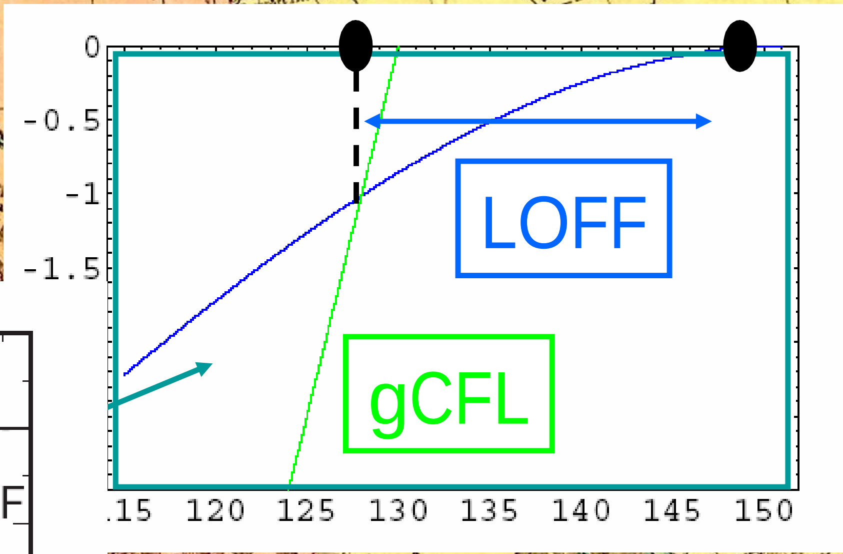
GOAL:
Free Energy computation

Ginzburg – Landau (GL) expansion, with coefficients evaluated assuming FOUR-FERMION interaction among quarks

Free energy: LOFF window

Strange quark mass leading order effect:
Shift of the strange Fermi momentum.
The mass enters in the model only
by the combination $M_s^2/$❍





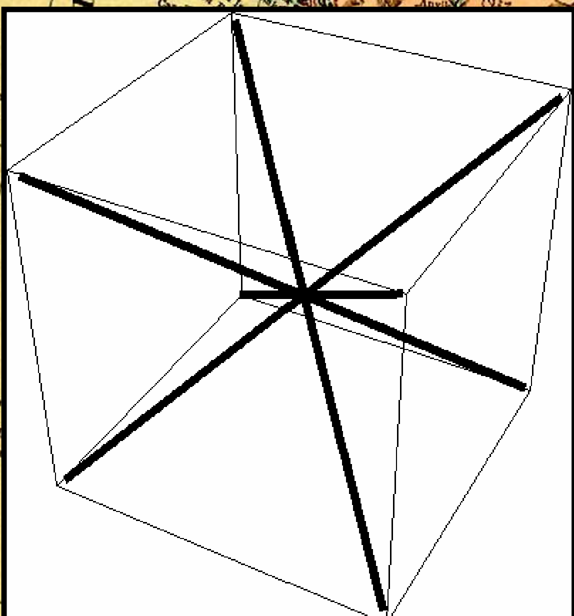
<u>LOFF window</u>:
M_s^2/μ 2 128 – 150 MeV

Crystallography, 1

The one plane wave ansatz examined till now can be improved to enlarge the LOFF window, by assuming a <u>linear combination of plane waves</u> for each quark pairing channel (Rajagopal and Sharma, 2006):

$$\langle ud \rangle \sim \Delta_3 \sum_a \exp\left(2i\mathbf{q}_3^a \cdot \mathbf{r}\right)$$

$$\langle us \rangle \sim \Delta_2 \sum_a \exp\left(2i\mathbf{q}_2^a \cdot \mathbf{r}\right)$$

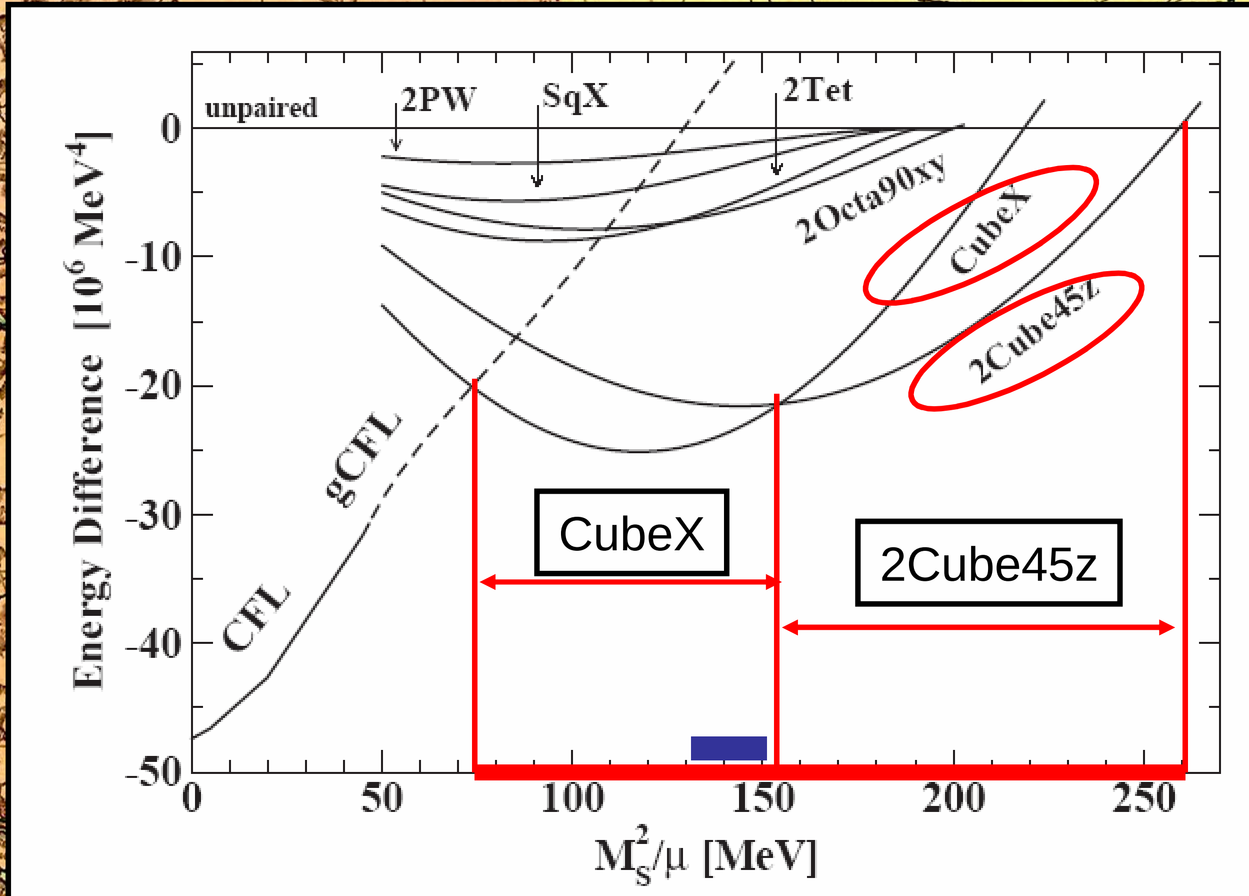


Structure	Description
2PW	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ coincide; each contains one vector. (So, 2 plane waves with $\mathbf{q}_2 \parallel \mathbf{q}_3$.)
SqX	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain two antiparallel vectors. The four vectors together form a square; those from $\{\mathbf{q}_2\}$ and those from $\{\mathbf{q}_3\}$ each form one stroke of an "X".
Tetrahedron	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain two vectors. The four together form a tetrahedron.
2Triangles	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ coincide; each contains three vectors forming a triangle.
Cube X See Eq. (90)	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain 4 vectors forming a rectangle. The 8 vectors together form a cube. The 2 rectangles intersect to look like an "X" if viewed end-on.
2Tet	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ coincide; each contains four vectors forming a tetrahedron.
Twisted Cube	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain four vectors forming a square which could be one face of a cube. Instead, the eight vectors together form the polyhedron obtained by twisting the top face of a cube by 45° relative to its bottom face.
2Octa90xy	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain 6 vectors forming an octahedron. The $\{\mathbf{q}_2\}$ vectors point along the positive and negative axes. The $\{\mathbf{q}_3\}$-octahedron is rotated relative to the $\{\mathbf{q}_2\}$-octahedron by 90° about the $(1,1,0)$-axis.
2Octa45xyz	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain 6 vectors forming an octahedron. The $\{\mathbf{q}_2\}$ vectors point along the positive and negative axes. The $\{\mathbf{q}_3\}$-octahedron is rotated relative to the $\{\mathbf{q}_2\}$-octahedron by 45° about the $(1,1,1)$-axis.
2Cube45z See Eq. (87)	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain 8 vectors forming a cube. The $\{\mathbf{q}_2\}$ vectors point along $(\pm1,\pm1,\pm1)$. The $\{\mathbf{q}_3\}$-cube is rotated relative to that by 45° about the z-axis.
2Cube45xy	$\{\mathbf{q}_2\}$ and $\{\mathbf{q}_3\}$ each contain 8 vectors forming a cube. The $\{\mathbf{q}_2\}$ vectors point along $(\pm1,\pm1,\pm1)$. The $\{\mathbf{q}_3\}$-cube is rotated relative to that by 45° about the $(1,1,0)$-axis.

Crystallography, 2

Again a GL expansion using a four fermion interaction among quarks

N.B.: this analysis has to be considered only qualitative, since the gap parameters in the cubic structures are large and G.L. calculations are not reliable !!!!!



LOFF window enlarged

(Rajagopal and Sharma, 2006)

Conclusions

- *In deconfined quark matter at high density and low temperature the Fermi surfaces of the quarks are likely to be mismatched and the BCS state can be disfavored*
- *Free energy computation shows that crystalline color superconductivity offers a good alternative to BCS pairing when the Fermi surfaces of the paired quarks are mismatched*
- *Crystalline color superconductivity can be improved by more complicated crystallographic ansatz, resulting in an enlargement of the LOFF window*
- *The LOFF phase has gapless fermions that affect the cooling of a compact star if a LOFF core is present inside the star itself : this offers a good chance to compare experimental observations with theoretical calculations (next talk)*

Outlook

- Chromo-magnetic stability of complex crystalline structures (1PW is stable, M.R et al., 2006)
- Merging with lattice simulations (effective theories seem do not suffer of the sign problem)
- Low energy effective lagrangian for the LOFF phase (fermions + goldstones and phonons)
- Transport properties of the crystalline superconductor (shear and bulk viscosity, heat conducibility)
- Effect of a gluon condensate on the color superconductive vacuum in three flavor and three color QCD

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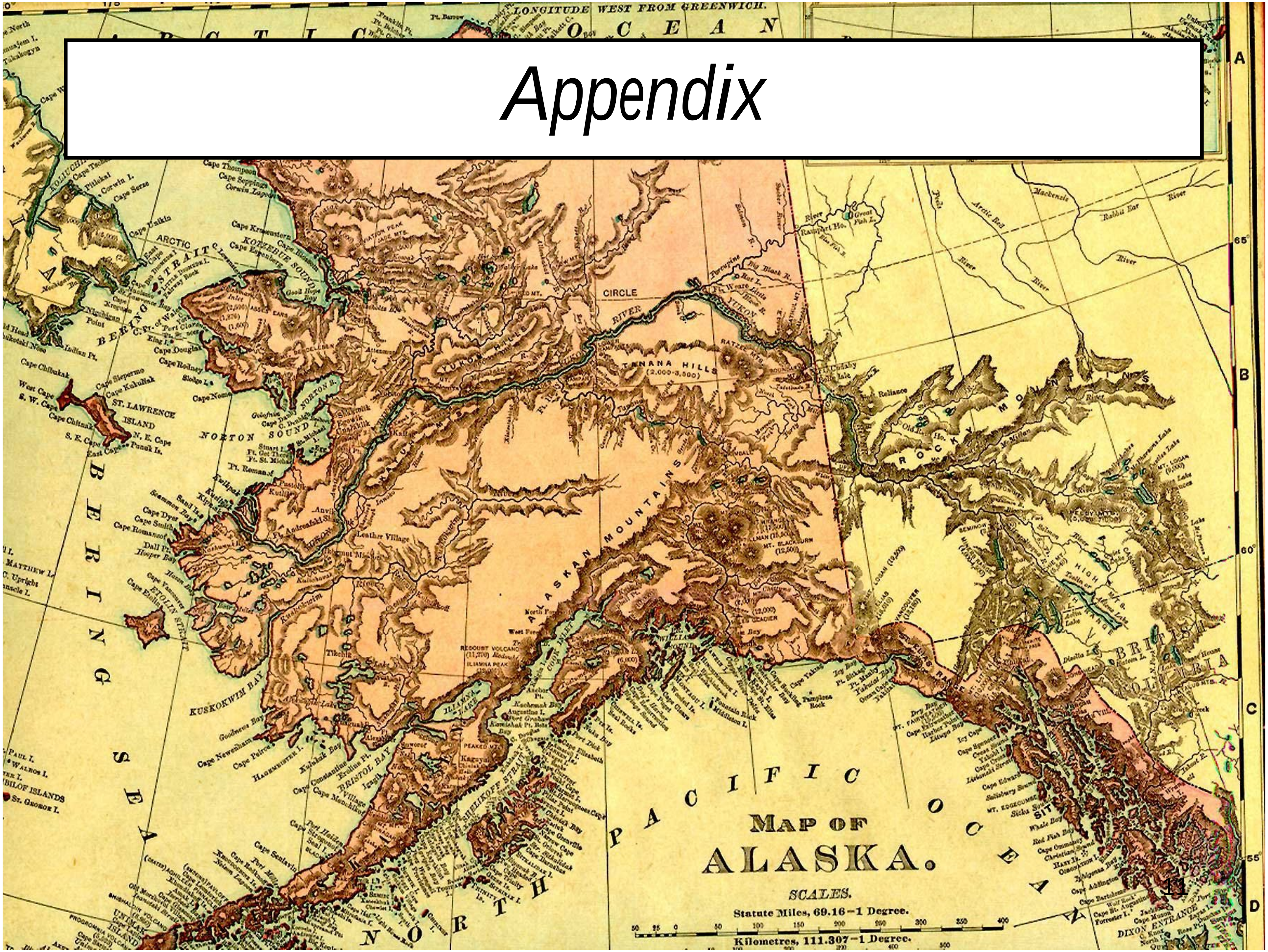
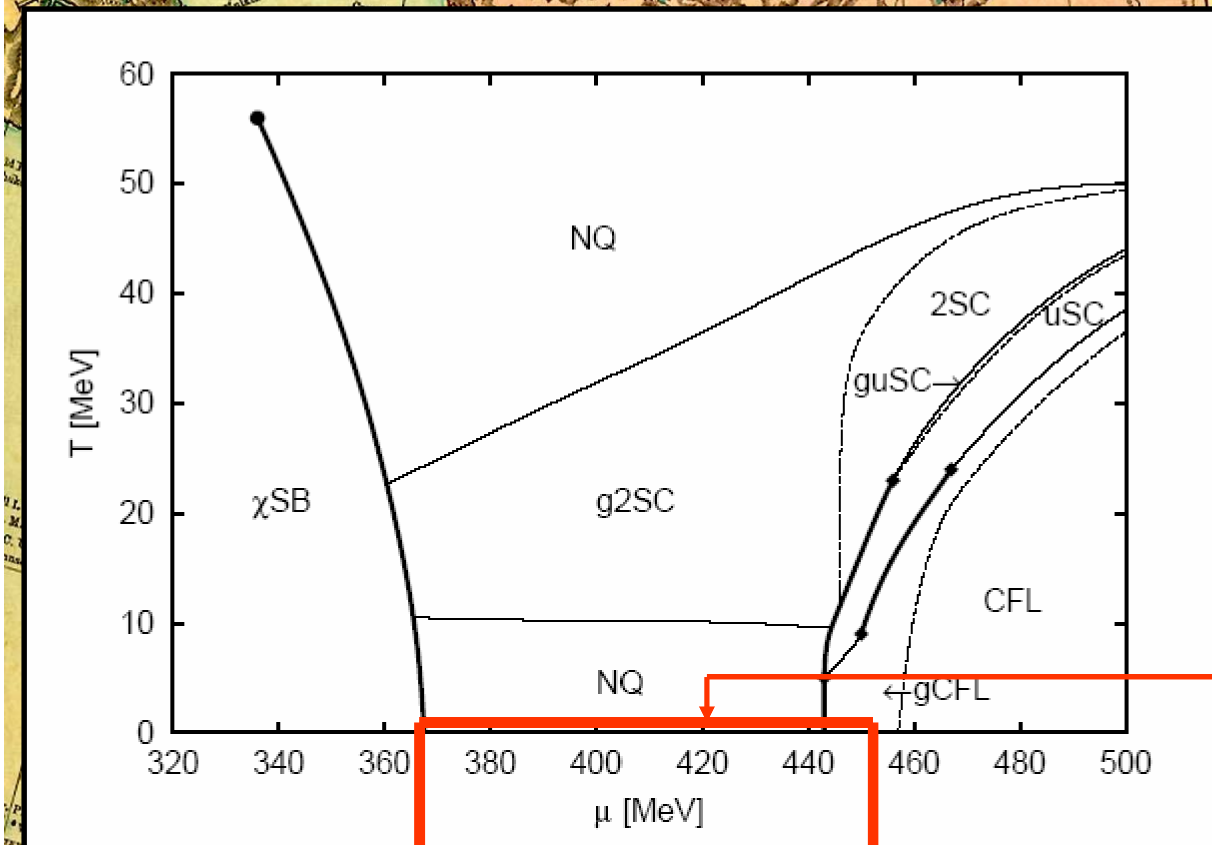
"The Lady Who ascends into the Heavens is radiant on the horizon."

Appendix


LONGITUDE WEST FROM GREENWICH.
MAP OF
ALASKA.
SCALES.
Statute Miles, 69.16 – 1 Degree.
Kilometres, 111.307 – 1 Degree.
PACIFIC OCEAN
NORTH
BERING SEA
ALASKAN MOUNTAINS
TANANA HILLS (2,000-3,500)
NORTON SOUND
KUSKOKWIM BAY
BRISTOL BAY
ST. LAWRENCE ISLAND
CIRCLE
ROCKY MOUNTAINS
BRITISH COLUMBIA
DIXON ENTRANCE

A sight to the phases of QCD



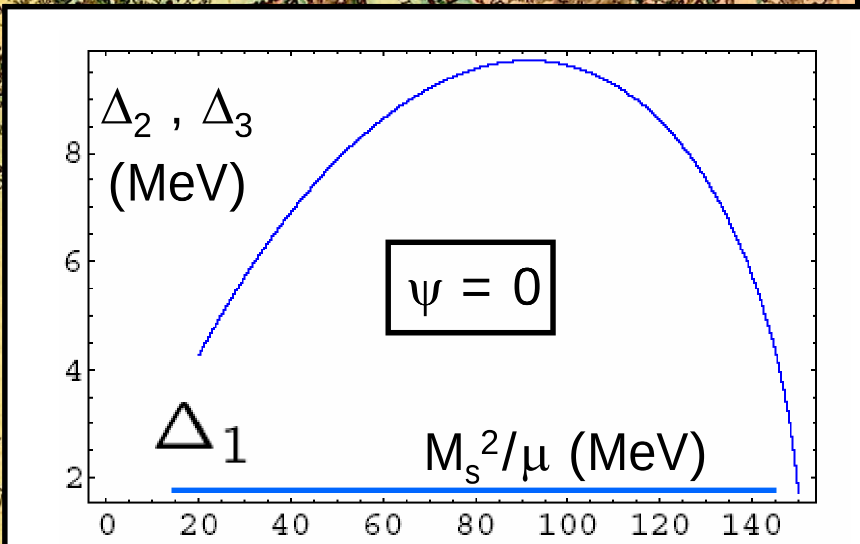
Buballa et al., 2005

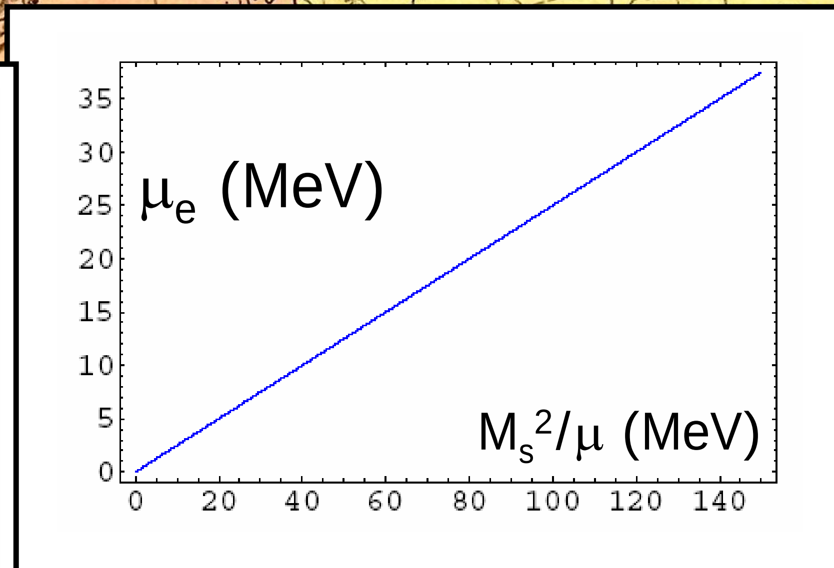
Interaction among quarks mimicked by a four-fermion NJL lagrangian

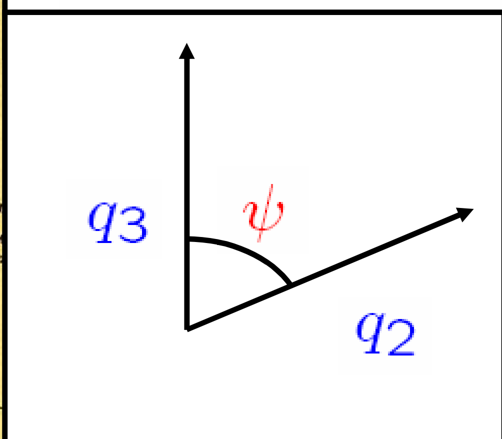
Higly mismatched Fermi surfaces

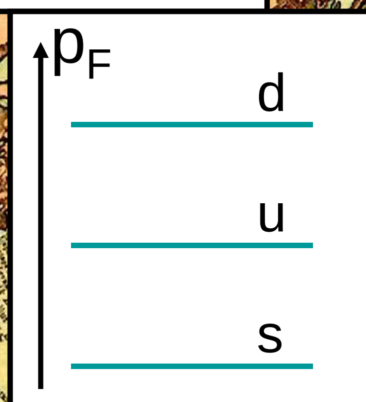
A good window to look for exotic pairing patterns !!!

Some results









$$\mu_e \approx \frac{M_s^2}{4\mu} \quad \rightarrow \quad \delta\mu_{us} \approx \delta\mu_{ud} \approx \frac{M_s^2}{8\mu}$$

$$\mu_3, \mu_8 = \mathcal{O}\left(\frac{\Delta^4}{q^3}\right)$$