Symmetry Breaking in Twisted Eguchi-Kawai Models

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Overview

- Eguchi-Kawai reduced models and the ${Z_N}^4$ symmetry: periodic vs. twisted boundary conditions;
- Phase structure of “small” N TEK models: good behaviour;
- Phase structure of “large” N TEK models: symmetry breaking;
- Implications for the study of large N gauge theories.

Eguchi-Kawai (EK) Models: construction

SU(N) YM in a L^4 periodic lattice	$\underset{N\to\infty}{=}$	SU(N) YM in a 1^4 periodic lattice

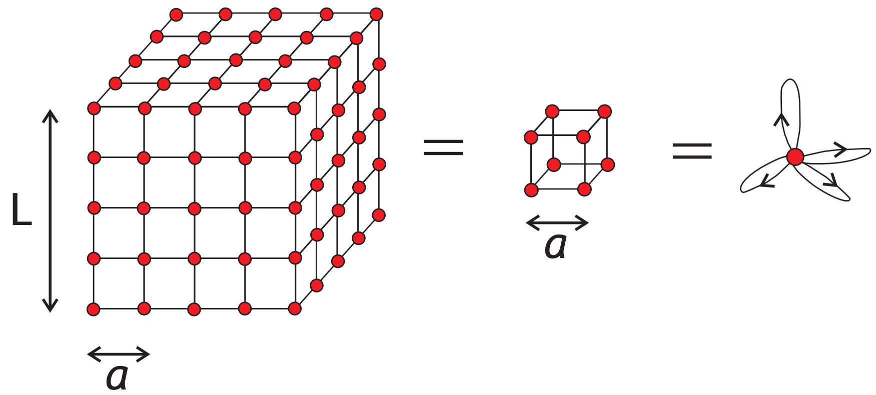
$$S_{\rm EK}[\widetilde{U}] = bN \sum_{\mu>\nu}^{4} {\rm ReTr}\left(I - \widetilde{U}_\mu \widetilde{U}_\nu \widetilde{U}_\mu^\dagger \widetilde{U}_\nu^\dagger\right) \geq 0$$

Prescription:

1) $U_\mu(x) \rightarrow \widetilde{U}_\mu$

2) $\mathcal{O}[U] \rightarrow \widetilde{\mathcal{O}}[\widetilde{U}] = \mathcal{O}[\widetilde{U}]$

3) $\langle\ \mathcal{O}[U]\ \rangle_{\rm W} = \langle\ \widetilde{\mathcal{O}}[\widetilde{U}]\ \rangle_{\rm TEK}$

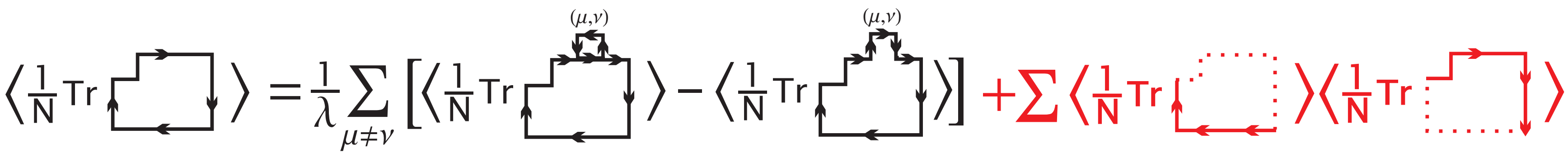


EK Models: justification

heuristic: Approaching the $N = \infty$ theory via translational invariant (constant) gauge fields: Witten's Master field;

perturbative: Reduced models have the same **planar diagrams** as the original Wilson's lattice gauge theory;

nonperturbative: Reduced models have the same **Schwinger--Dyson equations** as the original Wilson's lattice gauge theory, up to contact terms involving open lines:



EK Models: ${Z_N}^4$ symmetry

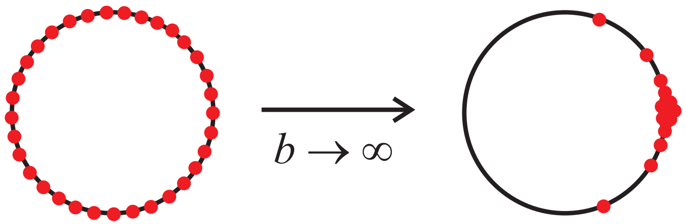
The action of the EK model:

$$S_{\rm EK}[U] = bN \sum_{\mu>\nu}^{4} {\rm Re Tr}\left(I - U_\mu U_\nu {U_\mu}^\dagger {U_\nu}^\dagger\right) \geq 0$$

has a global ${Z_N}^4$ symmetry:

$$U_\mu \to z_\mu U_\mu \ , \qquad z_\mu \in Z_N$$

which is spontaneously broken due to collapse of eigenvalues of U_μ in the weak-coupling regime $\Longrightarrow$ contact terms do not vanish!



Twisted Eguchi-Kawai (TEK) Models

SU(N) YM in a L^4 periodic lattice	$\underset{N\to\infty}{=}$	SU(N) YM in a 1^4 **twisted** lattice

$$S_{\rm TEK}[U;n] = bN \sum_{\mu>\nu}^{4} {\rm ReTr} \left(I - Z_{\mu\nu}(n) U_\mu U_\nu {U_\mu}^\dagger {U_\nu}^\dagger \right) \geq 0$$

$$Z_{\mu\nu}(n) = Z_{\nu\mu}(n)^* = e^{-i \frac{2\pi}{N} n_{\mu\nu}}$$

symmetric twist: $n_{\mu\nu} = L \ , \quad \forall \mu > \nu$

colour conversion: $N^2 = L^4$

Wilson loops: $W(I,J) = \frac{1}{N} {\rm Tr}\ {Z_{\mu\nu}}^{IJ} {U_\nu}^I {U_\mu}^J {U_\mu}^{\dagger I} {U_\nu}^{\dagger J}$

Polyakov loops: $P_\mu = \frac{1}{N} {\rm Tr}\ {U_\mu}^L$

TEK Models: ${Z_N}^4$ symmetry

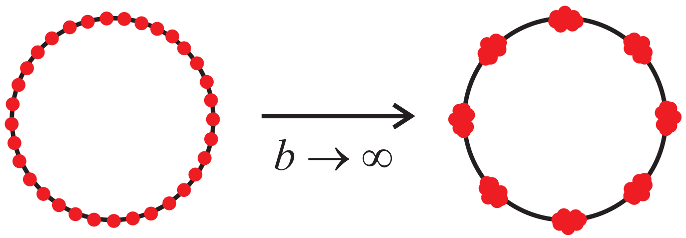
In weak-coupling, the link matrices fluctuate around the classical extrema (Γ_μ) of the TEK model, the so-called **twist-eaters**:

$$S_{\rm TEK}[\Gamma; n] = 0 \quad \Longrightarrow \quad \Gamma_\mu \Gamma_\nu {\Gamma_\mu}^\dagger {\Gamma_\nu}^\dagger = Z_{\mu\nu}(n)^*$$

which preserve the ${Z_N}^4$ symmetry:

$$P_\mu(\alpha) = \frac{1}{N} {\rm Tr}\ {\Gamma_\mu}^\alpha = \delta_{\alpha L}$$

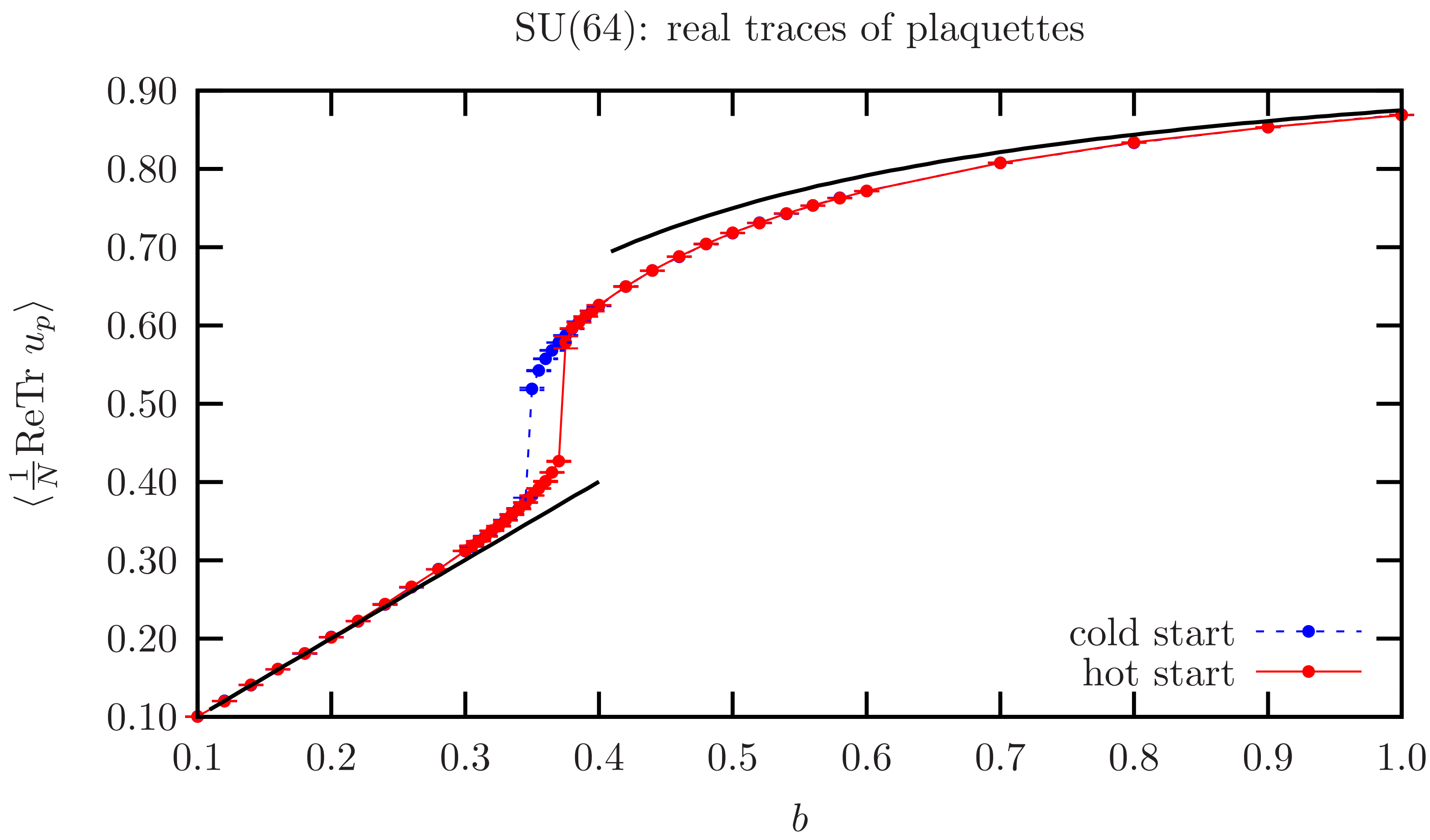
because the eigenvalues collapse symmetrically on the unit circle.



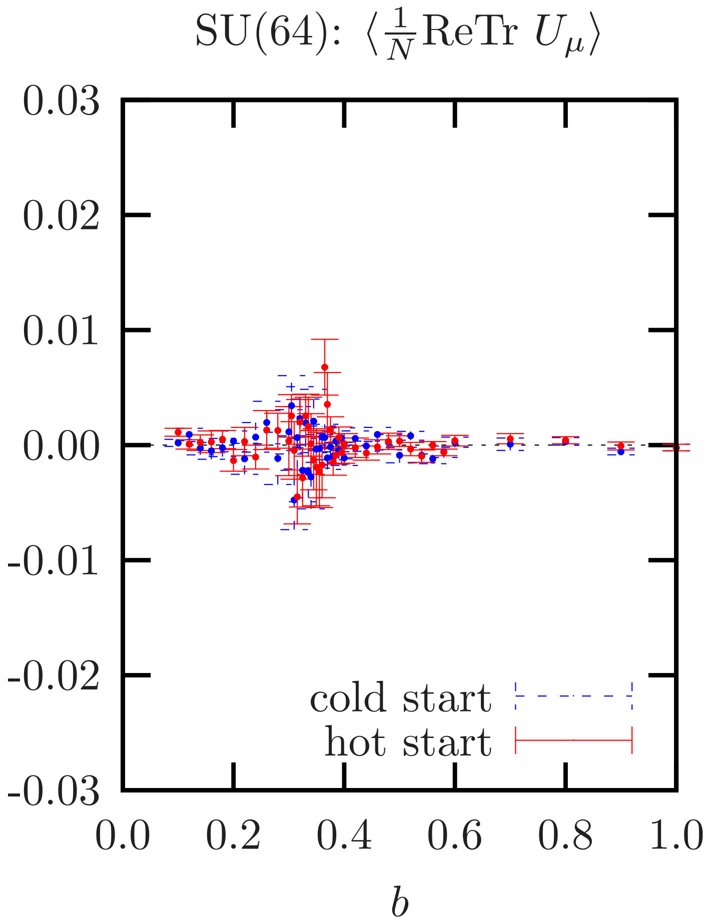
Phase structure of *small*-N TEK models

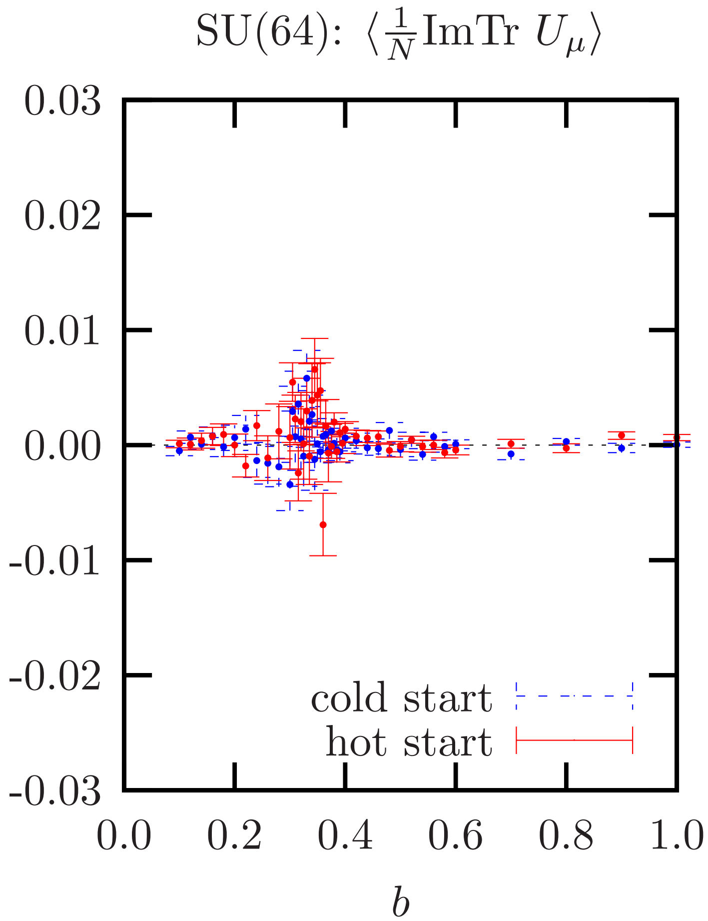
$(N < 81)$

For $N < 81$, the real part of the plaquette shows the typical behaviour expected for this model:

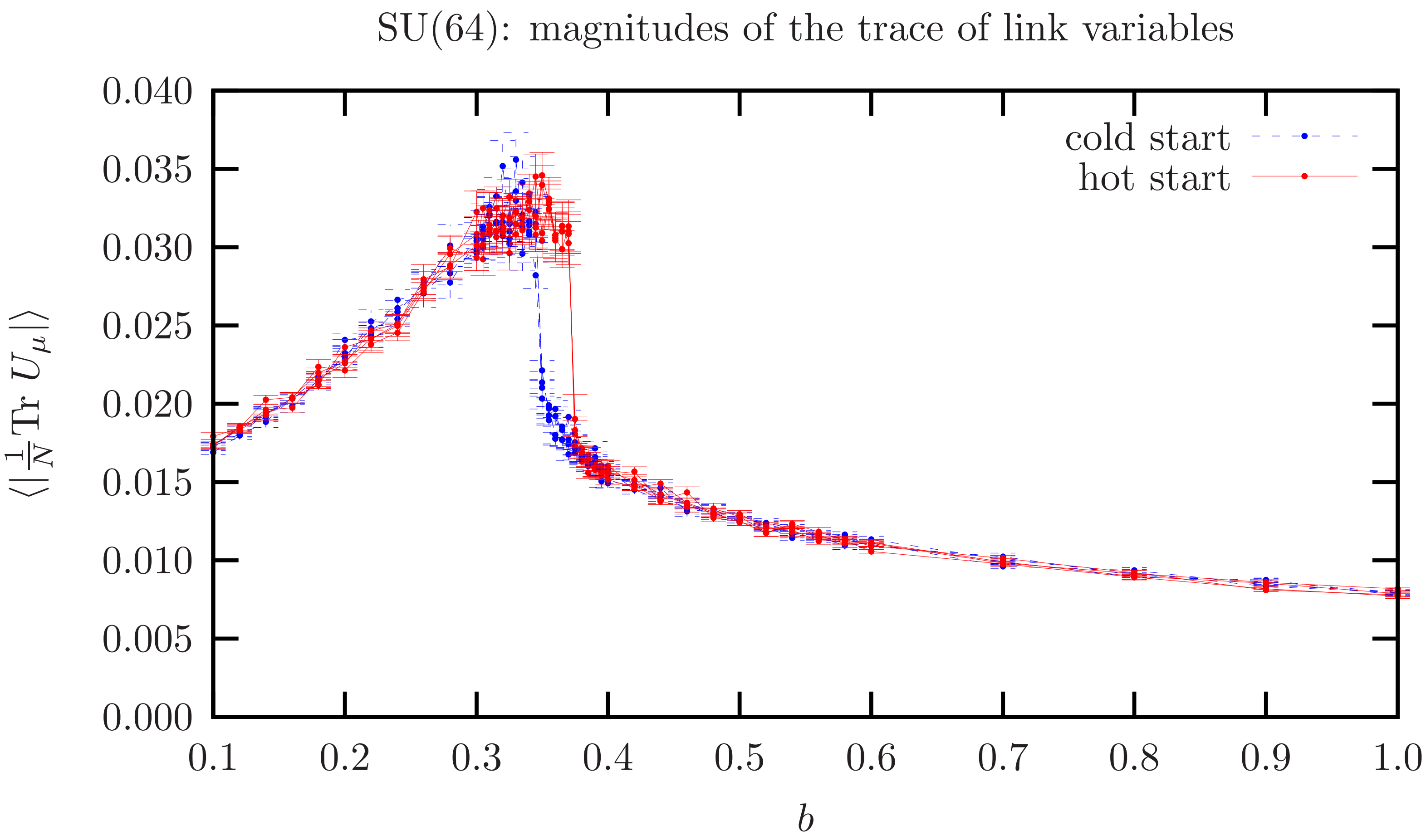


Also, the real and imaginary parts of the traces of link variables are zero in average for all couplings...

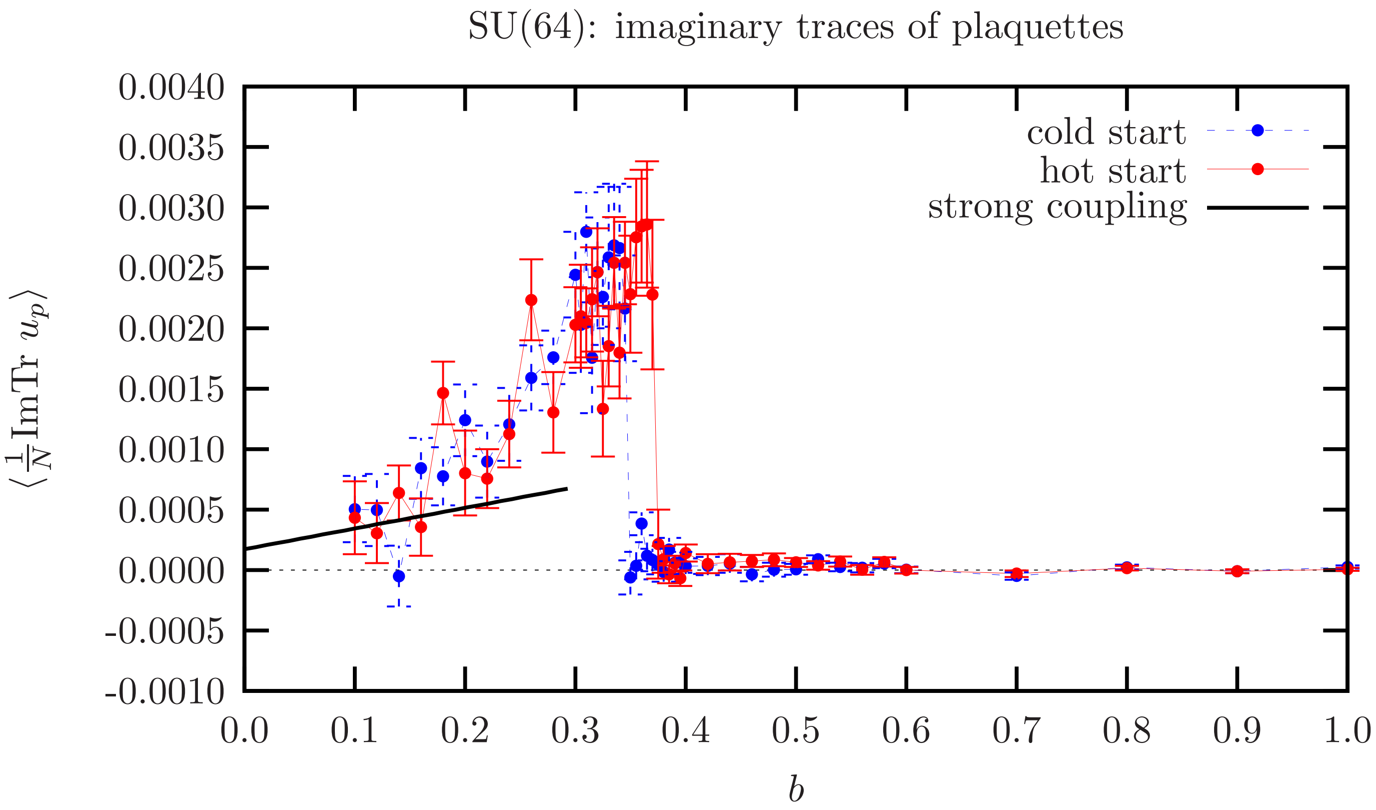




... even though at intermediate couplings the magnitude of the trace changes significativelly.

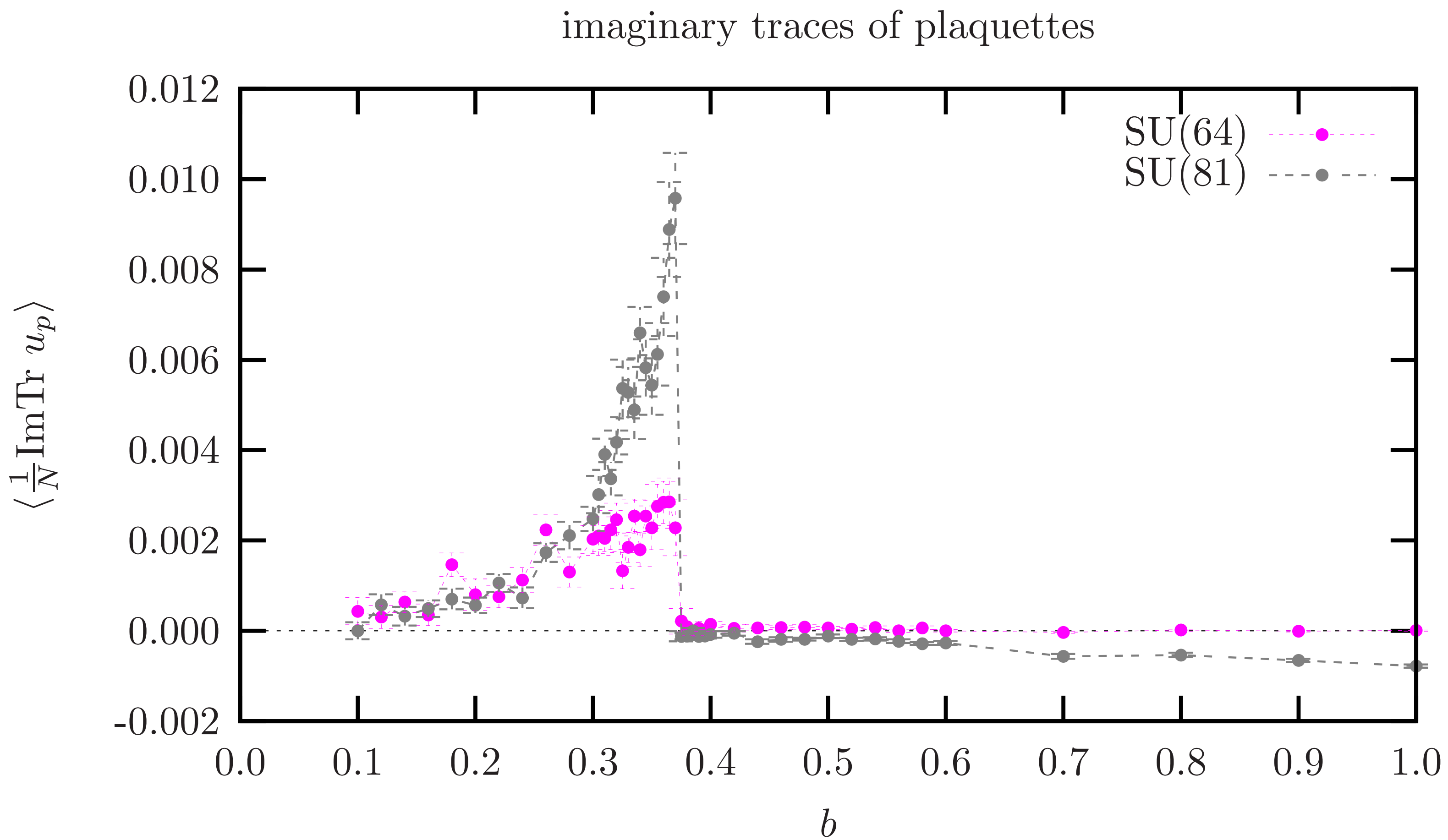


The imaginary part of the trace of elementary plaquettes follow the same behaviour:



which can be explained by the *absence* of CP-invariance in S_{TEK}.

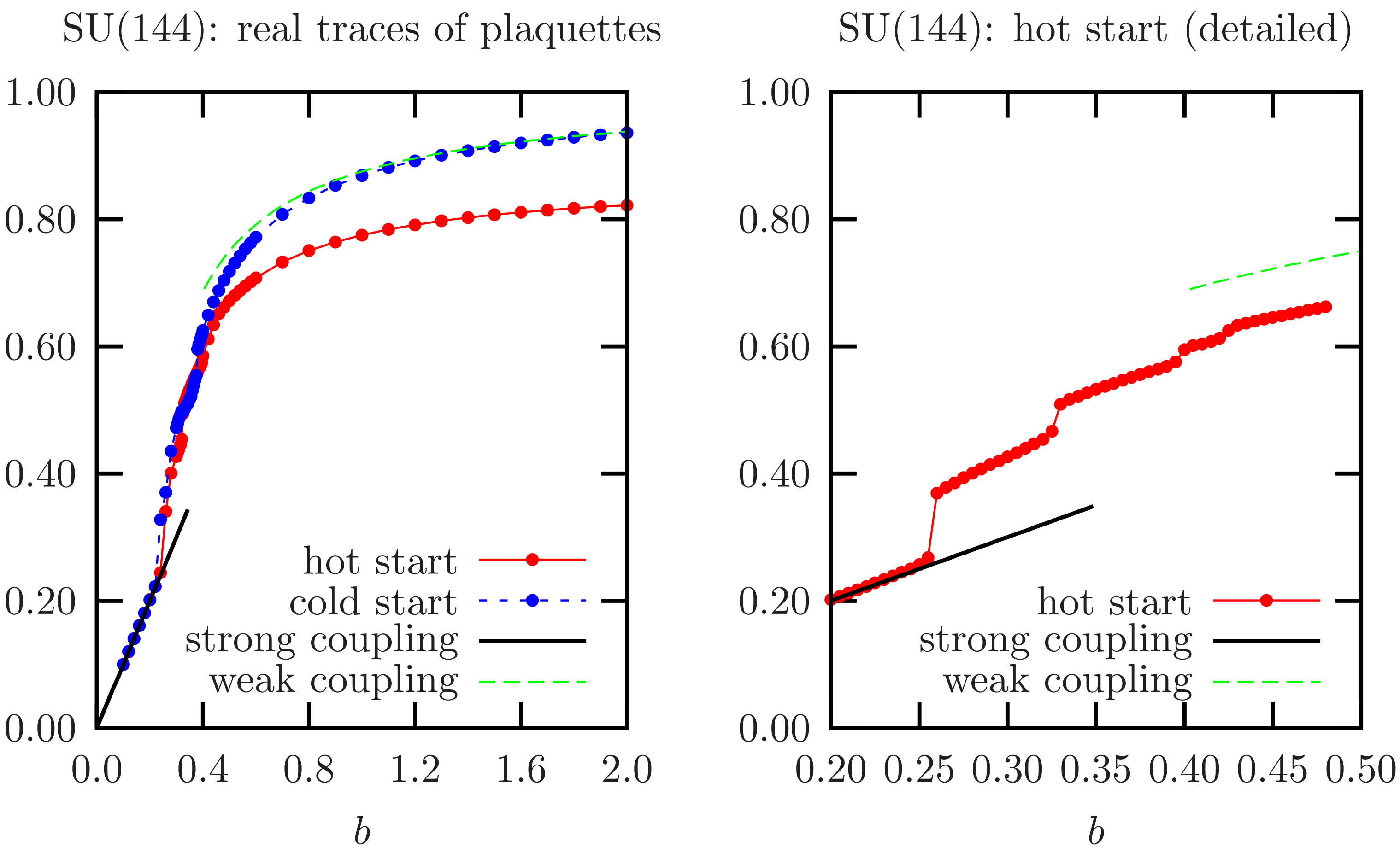
But for $N = 81$ the signal is stronger for intermediate couplings, even though it vanishes faster (as expected) for small couplings:



Phase structure of *large*-N TEK models

$(N \geq 100)$

For $N > 81$, we observed the existence of several transitions affecting the plaquettes in the TEK model:



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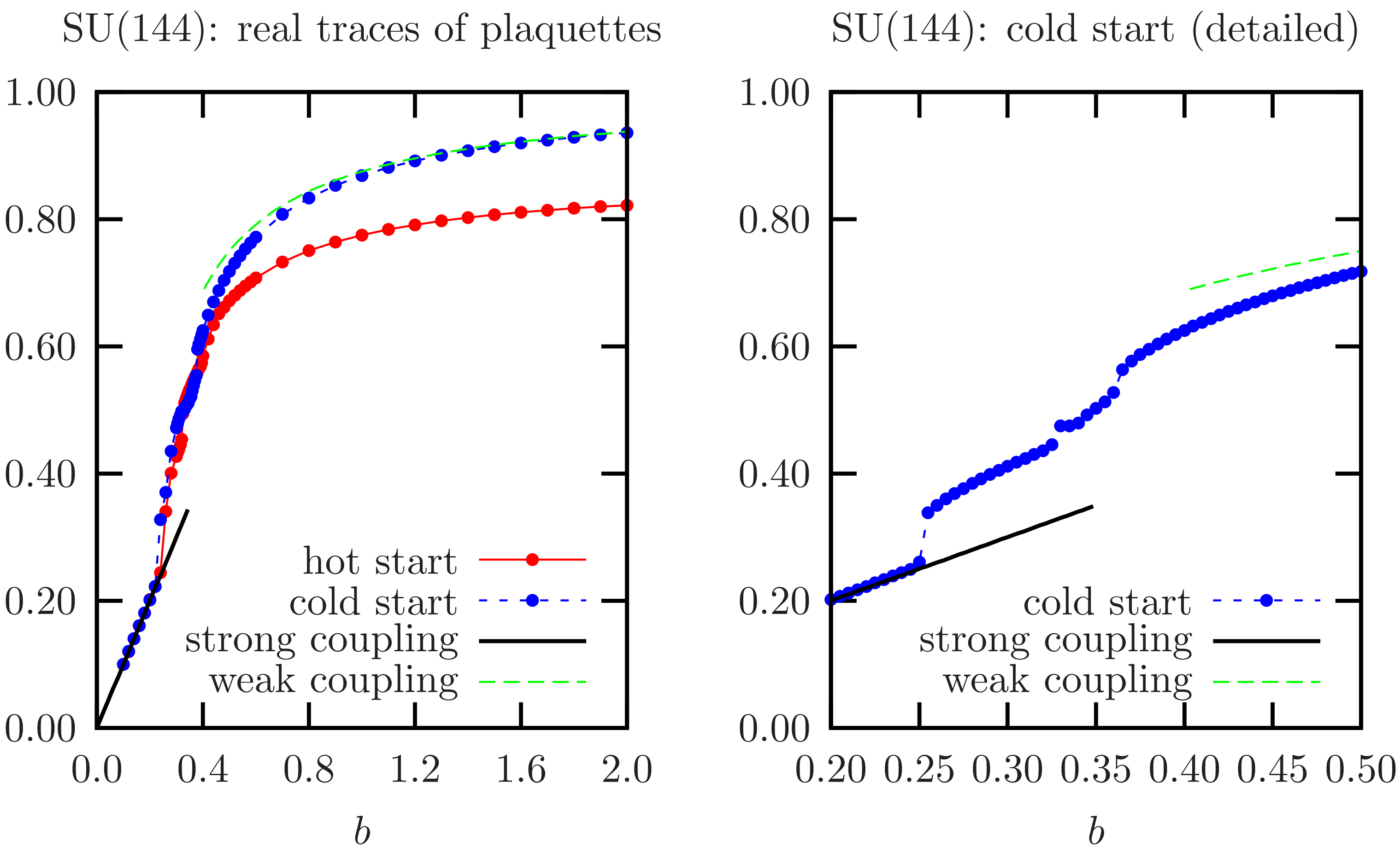
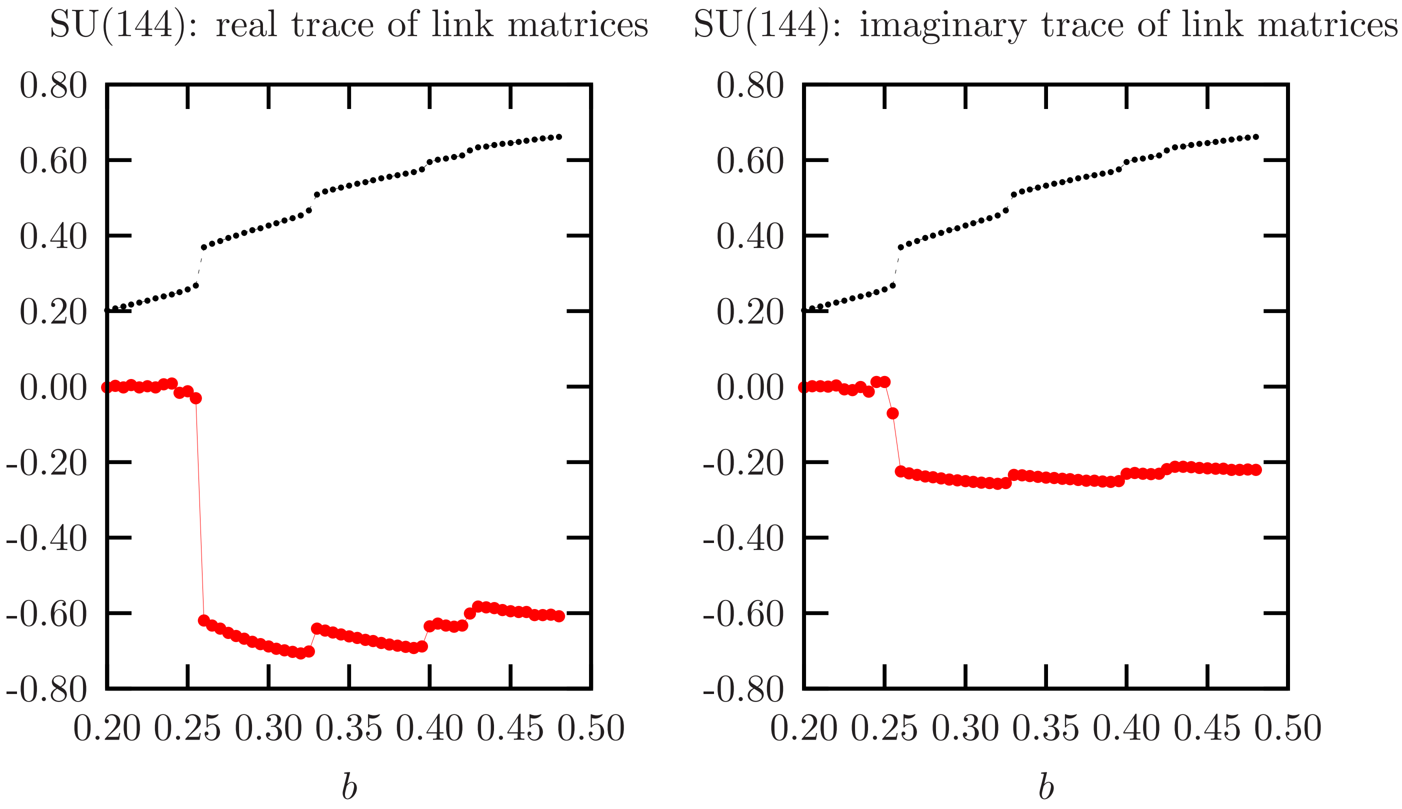


Each transition is usually associated with the breaking of one Z_N symmetry. This can be seen from the expectation values of the traces of each link matrix; other open lines also show the same behaviour.



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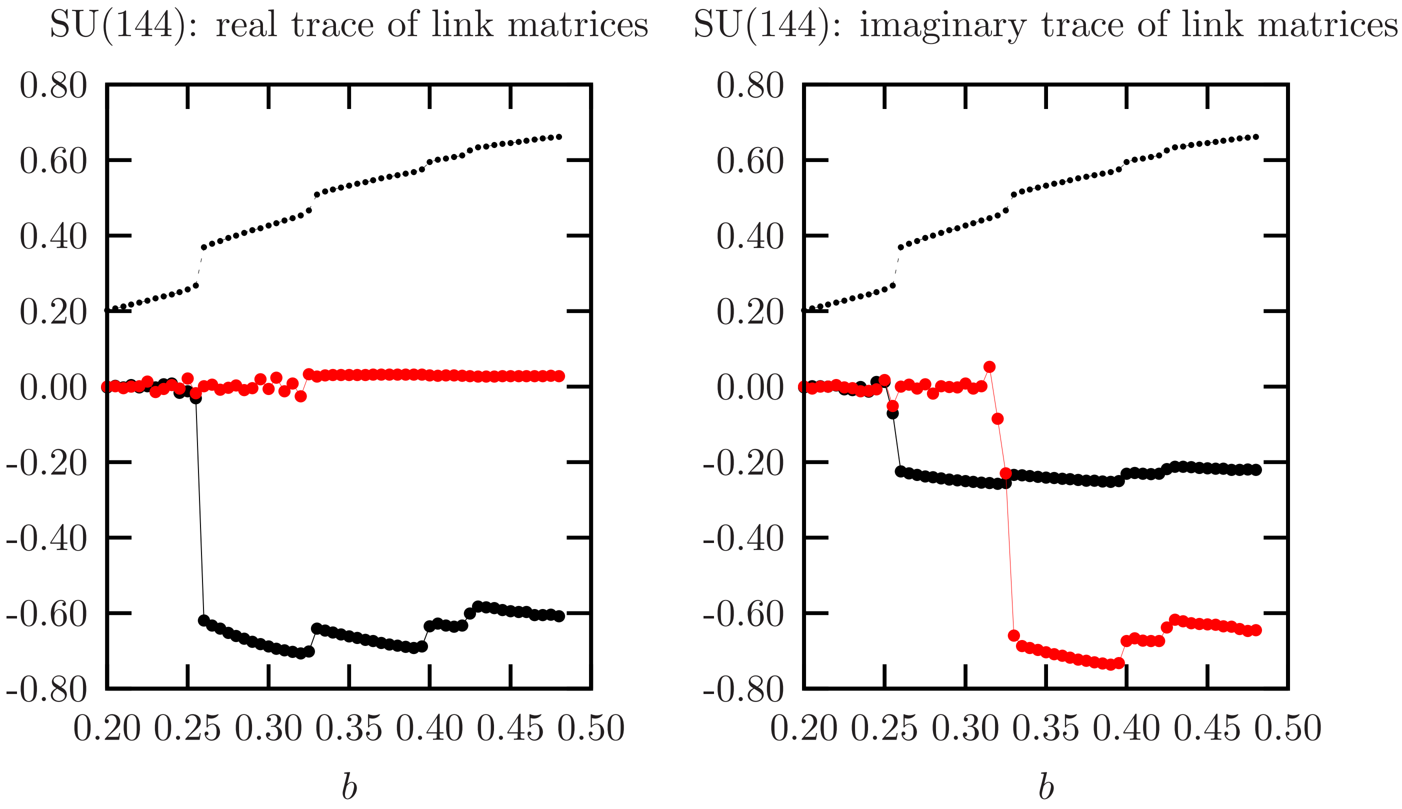

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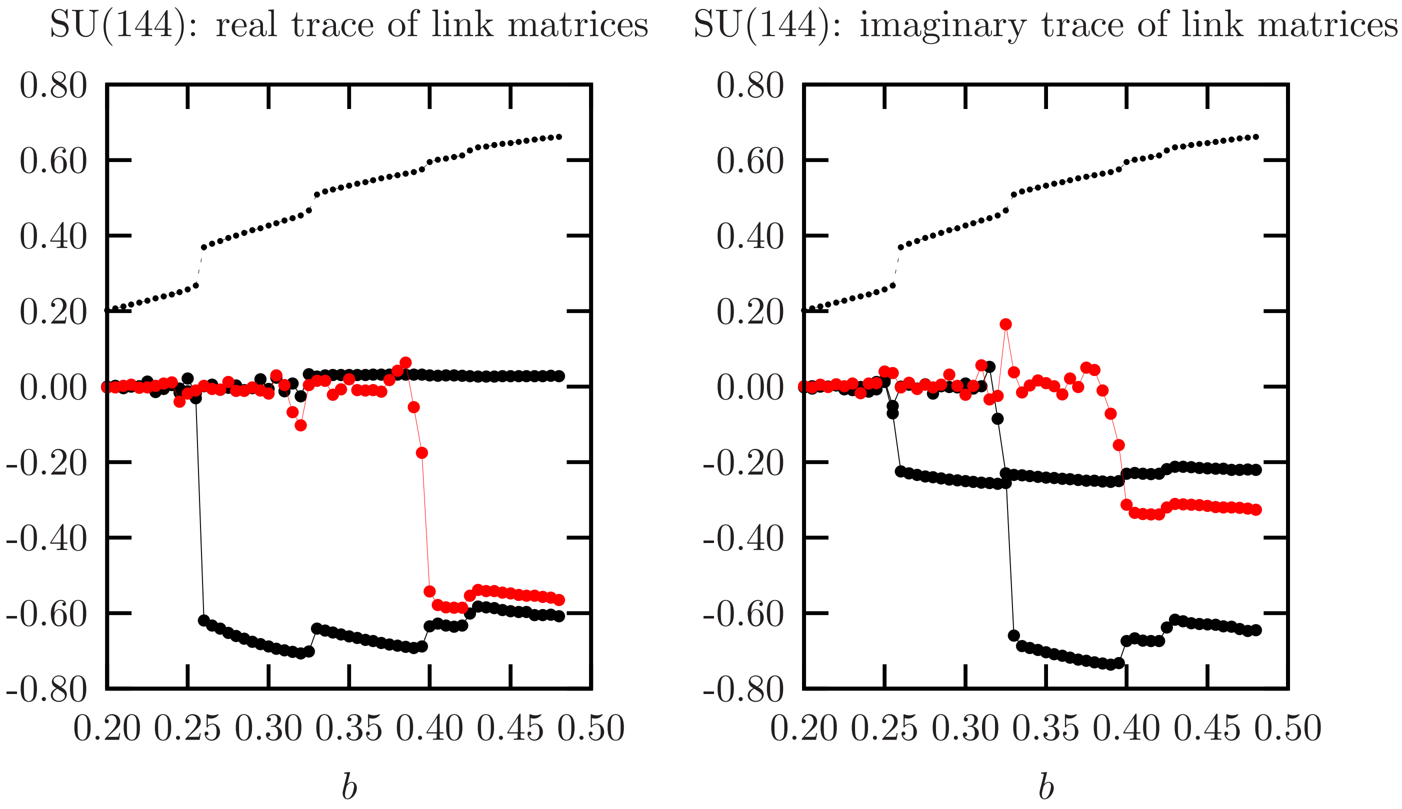

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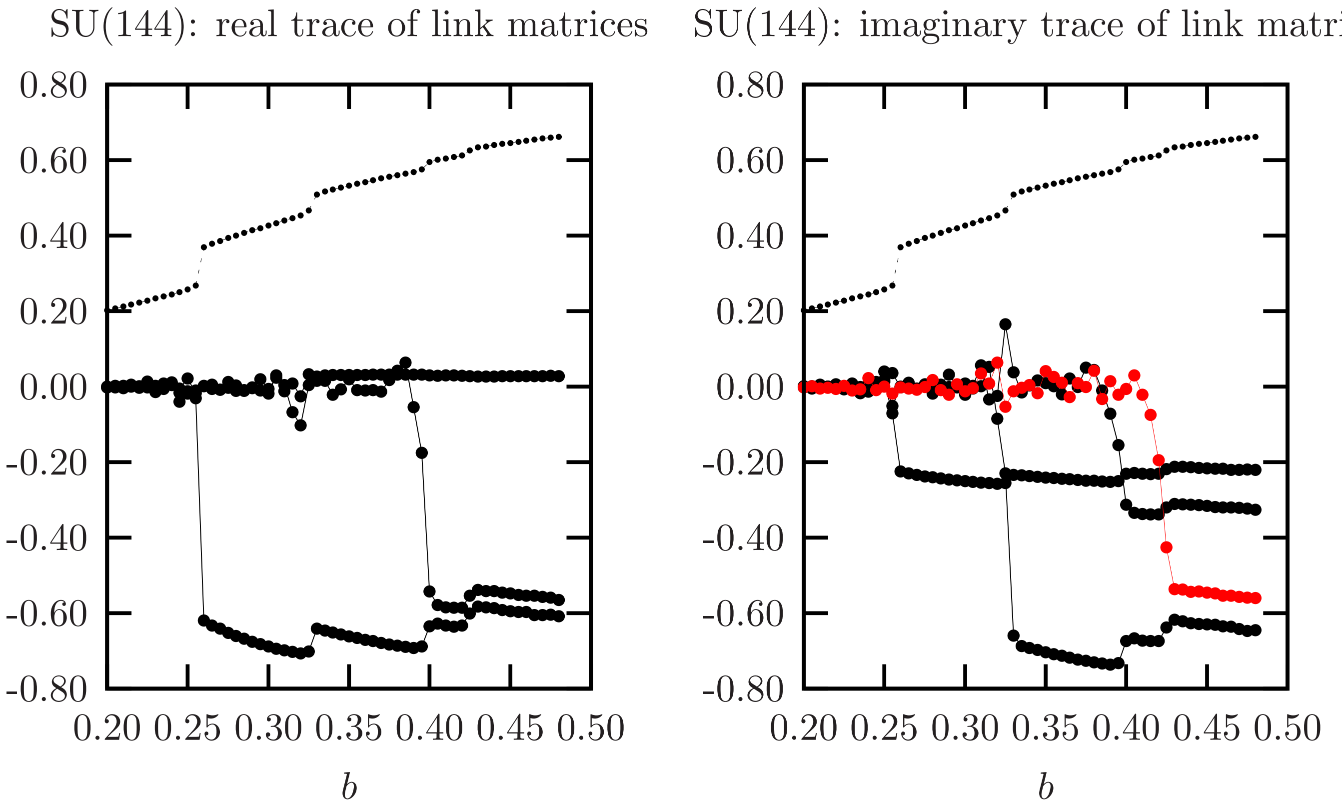
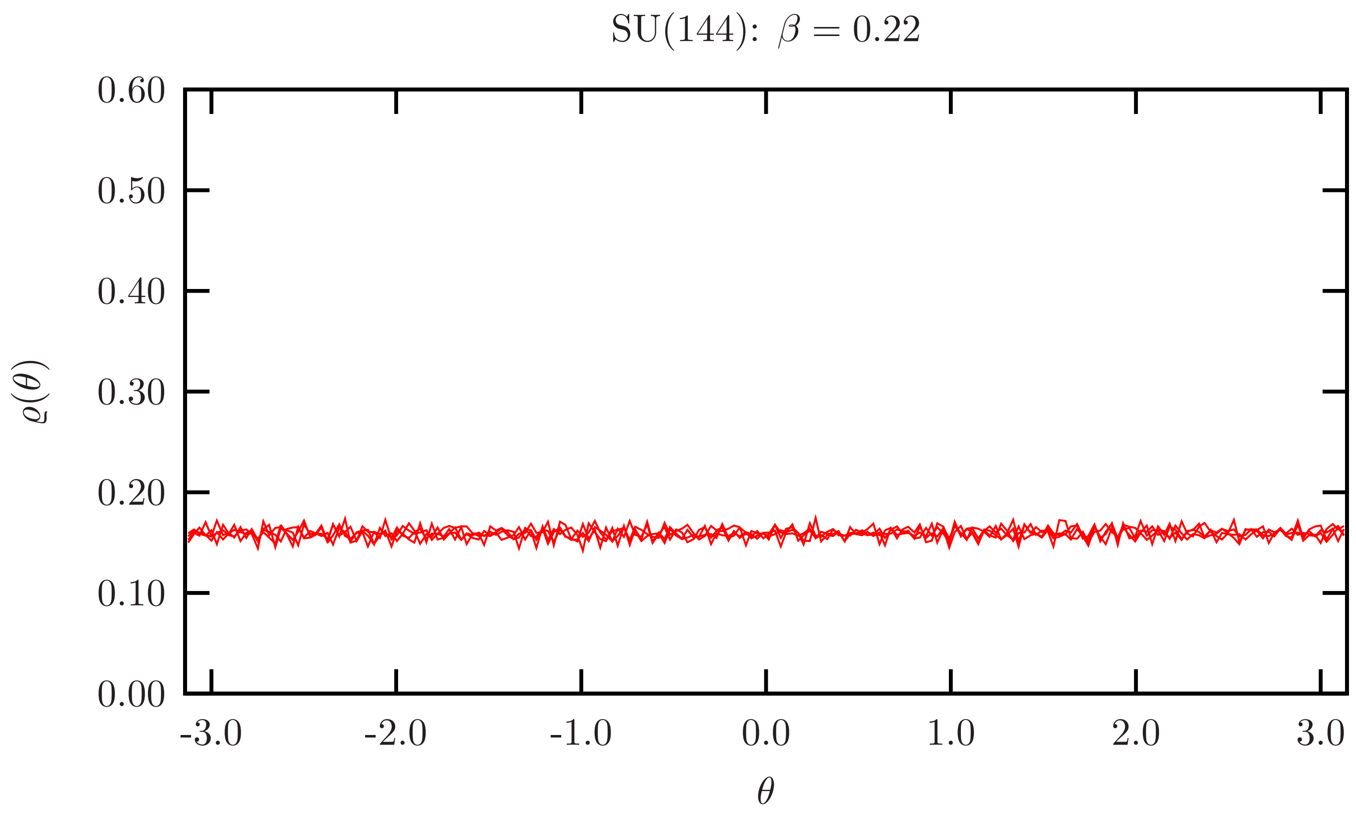


The eigenvalue spectrum of the link matrix associated with the Z_N symmetry breaking suffers a dramatic change at these transitions:



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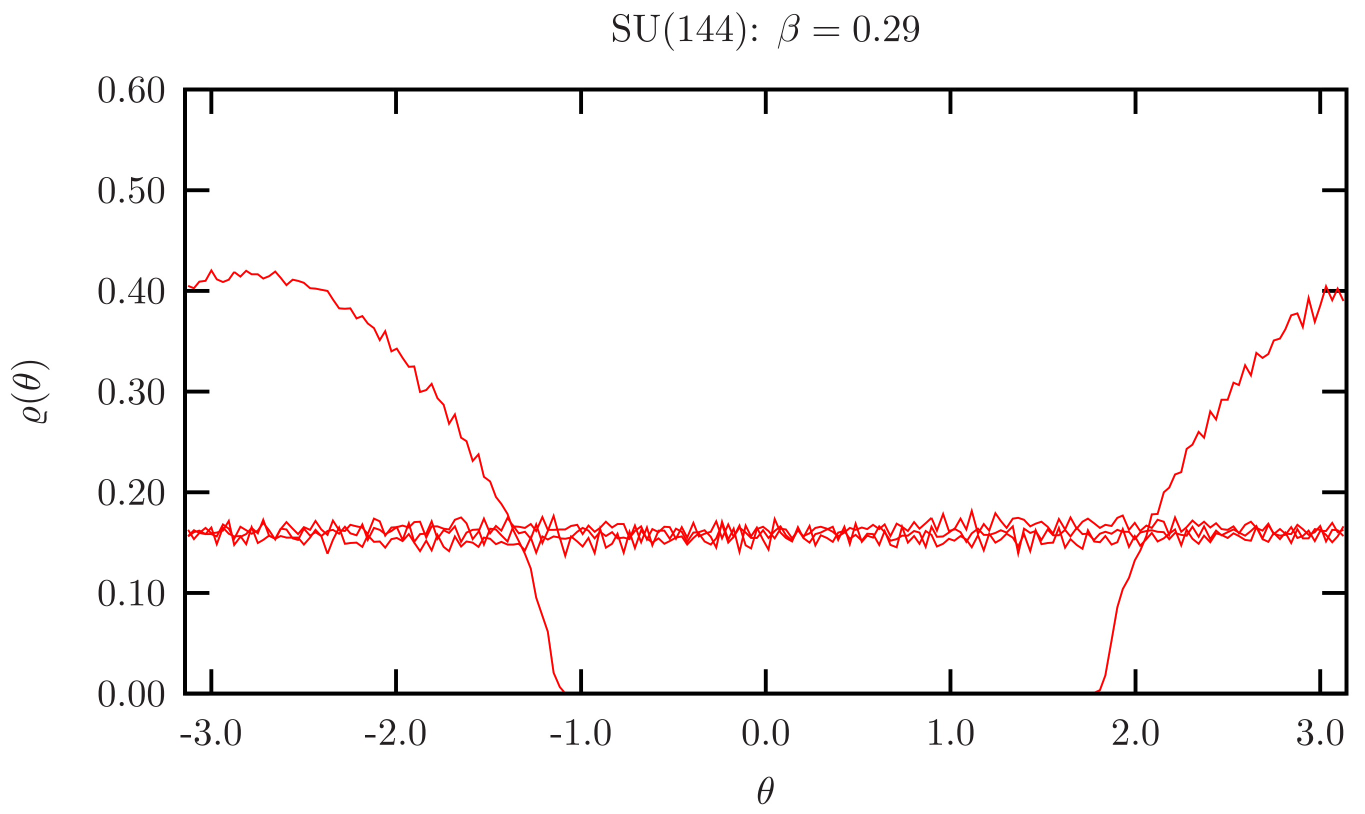

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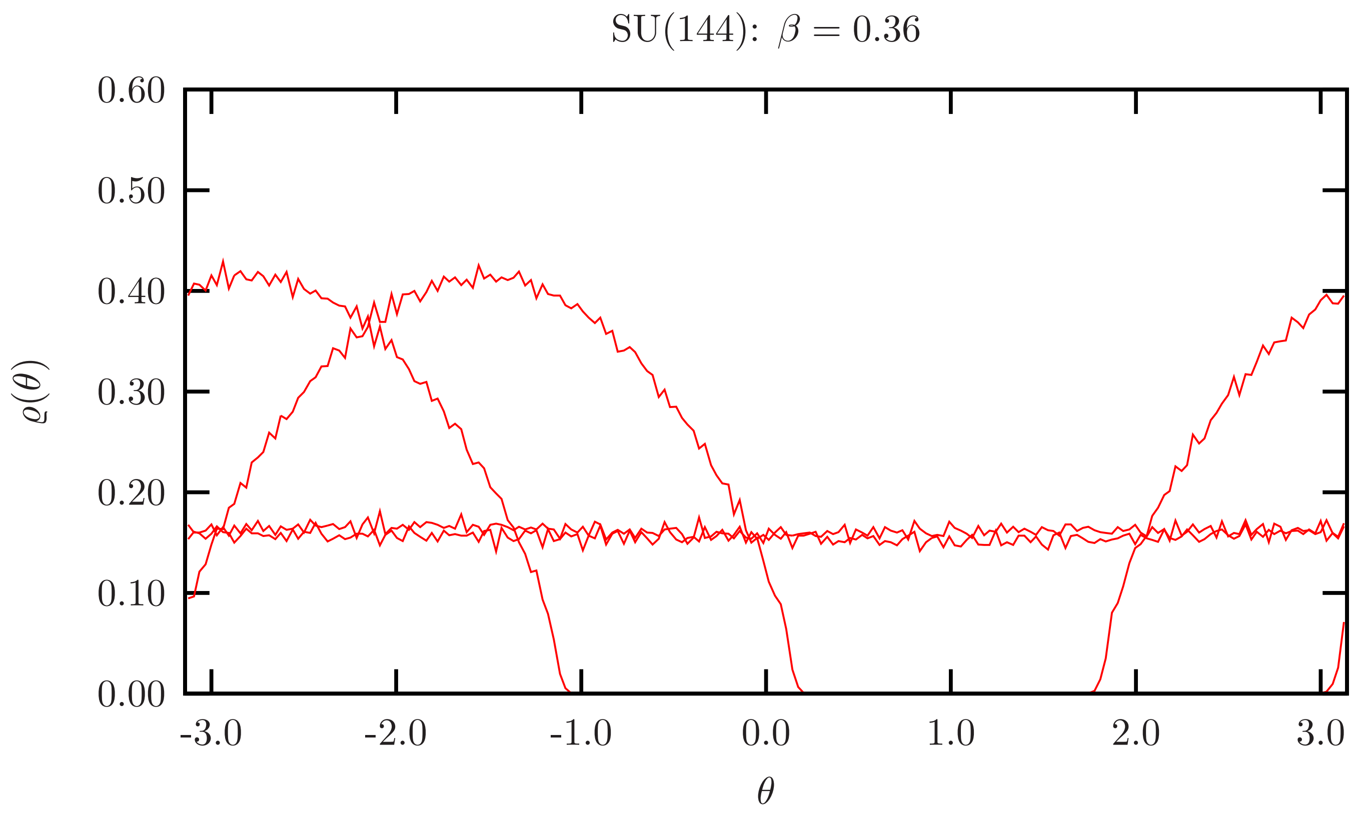

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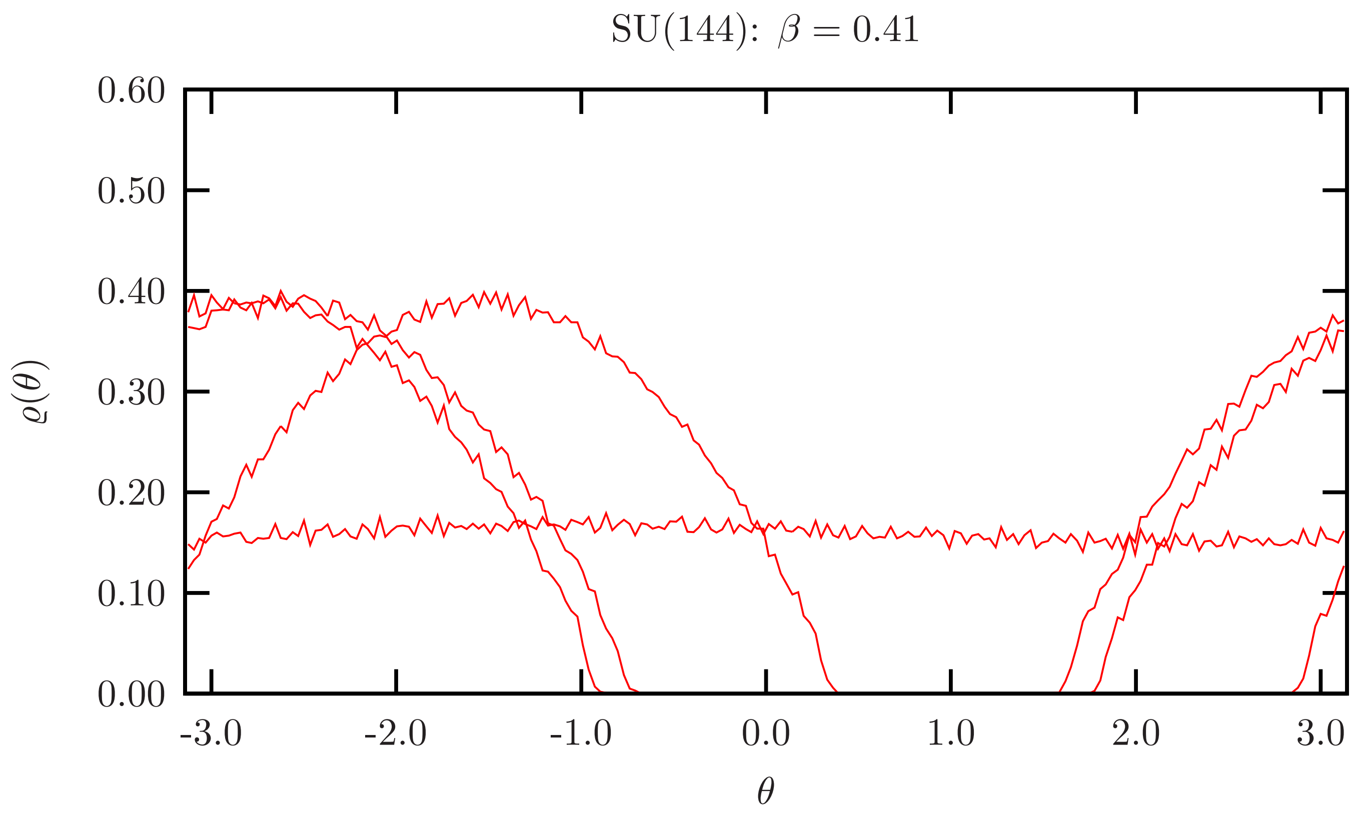

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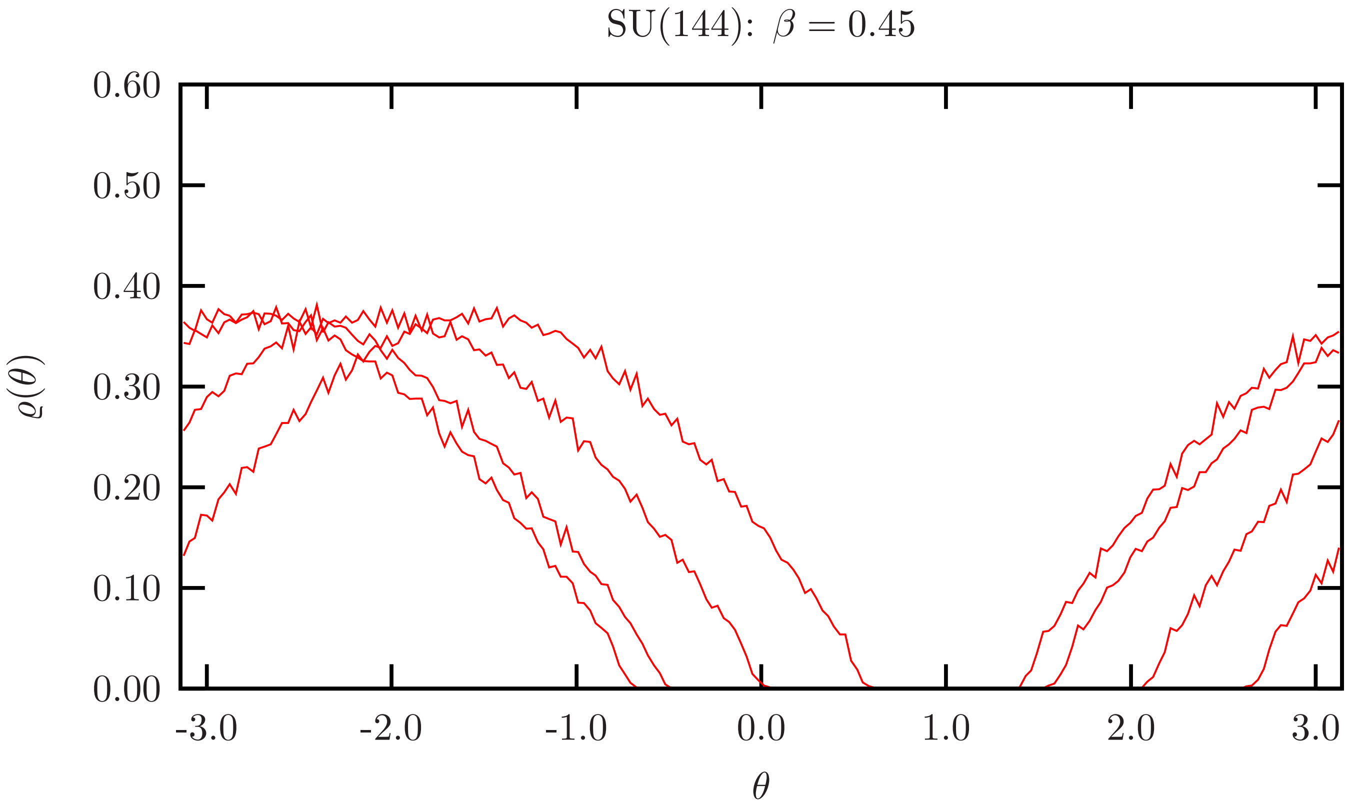
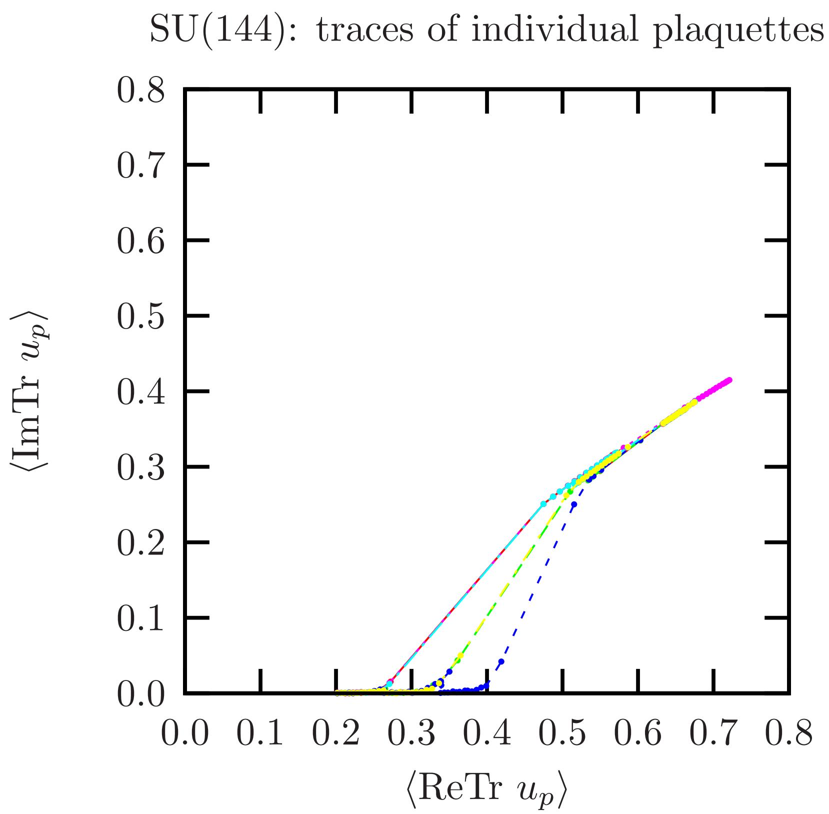


The real and imaginary parts of individual plaquettes tend to a common value, which is an element of Z_N:



The existence of extrema other than the twist-eaters is a consequence of the **stationarity condition**:

$$\delta S_{\mathrm{TEK}}[U;n] = 0 \implies \sum_{\mu\neq\nu} [(P_{\mu\nu} - P_{\nu\mu}) - U_\nu (P_{\mu\nu} - P_{\nu\mu}) {U_\nu}^\dagger] = 0$$

where $P_{\mu\nu} = Z_{\mu\nu}(n) {U_\mu}^\dagger {U_\nu}^\dagger U_\mu U_\nu$; and the **stability condition**:

$$\cos \frac{2\pi}{N} (n_{\mu\nu} - m_{\mu\nu}) \geq 0$$

These conditions allow several possible classes of extrema, some of which survive the large N limit:

twist-eaters: $P_{\mu\nu} = 1$

fluxons: $P_{\mu\nu} = Z_{\mu\nu}(n - m)$

diagonals(?): $P_{\mu\nu} = Z_{\mu\nu}(n)$

Conclusions

- TEK models for "small" N are well-behaved with respect to the LGT-TEK correspondence, but they lack information about the large N physics, e.g. deconfining phase transition.
- "Large" N TEK models are ill-behaved numerically: they tend to fall in vacua lacking the correct symmetries of the LGT-TEK correspondence. Ergodicity problem? Or new surviving extrema?
- Possibly, an artifact of the full reduction; other formulations are in principle freed of such problems, e.g. quenched EK or partial reduction.