Errata Corrige to the text book
Leonardo Angelini
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Chapter 2 One-Dimensional systems

2.4 Particle confined on a segment II

In formula (2.13) on page 19, the result is exact, but the previous step

$$= \frac{2L^2}{n^3\pi^3}\left(\frac{n^3\pi^3}{6} - \frac{1}{4}n\pi\cos(2n\pi) - \frac{1}{8}(1-6n^2\pi^2)\sin(2n\pi)\right) =$$

should be corrected by

$$= \frac{2L^2}{n^3\pi^3}\left(\frac{n^3\pi^3}{6} - \frac{1}{4}n\pi\cos(2n\pi) - \frac{1}{8}(1-2n^2\pi^2)\sin(2n\pi)\right) =$$

and the first formula at page 21

$$(\triangle x)^2 = \langle x^2\rangle - \langle x\rangle^2 = L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right) - \frac{a^2}{4} = L^2\left(\frac{1}{12} - \frac{1}{2n^2\pi^2}\right)$$

by

$$(\triangle x)^2 = \langle x^2\rangle - \langle x\rangle^2 = L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right) - \frac{L^2}{4} = L^2\left(\frac{1}{12} - \frac{1}{2n^2\pi^2}\right)$$

2.5 Particle confined on a segment III

In the intermediate steps of the last formula on page 22 symbol a ***should be replaced by*** L***.***

2.6 Scattering by a square-well potential

Insert an imaginary i ***in the numerator of the expression (2.17) for*** R***.***

Chapter 3 Two and three-dimensional systems

3.2 Spherical harmonic oscillator

The formula on page 60:

$$\begin{cases} -\frac{1}{i}\, a = m\, b \\ \\ \frac{1}{i}\, b = m\, a \end{cases} \Rightarrow m^2 = \hbar^2 \qquad \Rightarrow \qquad m = \pm 1\,,$$

should be corrected by

$$\begin{cases} -\frac{1}{i}\, a = m\, b \\ \\ \frac{1}{i}\, b = m\, a \end{cases} \Rightarrow m^2 = 1 \qquad \Rightarrow \qquad m = \pm 1\,,$$

3.8 Measurements of angular momentum (II)

The formula on page 68:

$$\begin{aligned} &\psi(r,\theta,\phi) = \\ &= C\, r^2 e^{-\alpha r^2} (\sin^2\theta\, \sin\phi\, \cos\phi + \sin\theta\, \cos\theta\, \sin\phi + \sin\theta\, \cos\theta\, \cos\phi) = \\ &= \frac{C}{2i}\, r^2 e^{-\alpha r^2} \left\{ \frac{1}{2}\, \sin^2\theta\, (e^{2i\phi} - e^{-2i\phi}) + \sin\theta\, \cos\theta\, \left[e^{i\phi}(1+i) - e^{-i\phi}(1-i) \right] \right\} = \\ &= \frac{C}{2i}\, r^2 e^{-\alpha r^2} \sqrt{\frac{8\pi}{15}}\, \left[Y_{2,-2} - Y_{2,2} - (1+i) Y_{2,1} + (1-i) Y_{2,-1} \right], \end{aligned}$$

should be corrected by

$$\begin{aligned} &\psi(r,\theta,\phi) = \\ &= C\, r^2 e^{-\alpha r^2} (\sin^2\theta\, \sin\phi\, \cos\phi + \sin\theta\, \cos\theta\, \sin\phi + \sin\theta\, \cos\theta\, \cos\phi) = \\ &= \frac{C}{2i}\, r^2 e^{-\alpha r^2} \left\{ \frac{1}{2}\, \sin^2\theta\, (e^{2i\phi} - e^{-2i\phi}) + \sin\theta\, \cos\theta\, \left[e^{i\phi}(1+i) - e^{-i\phi}(1-i) \right] \right\} = \\ &= \frac{C}{2i}\, r^2 e^{-\alpha r^2} \sqrt{\frac{8\pi}{15}}\, \left[Y_{2,2} - Y_{2,-2} - (1+i) Y_{2,1} + (1-i) Y_{2,-1} \right], \end{aligned}$$

3.13 Particle inside a sphere

Replace the last sentence with The zeros of spherical Bessel functions are tabulated (see, for example, Abramowitz e Stegun) or can be obtained from mathematical manipulation programs.

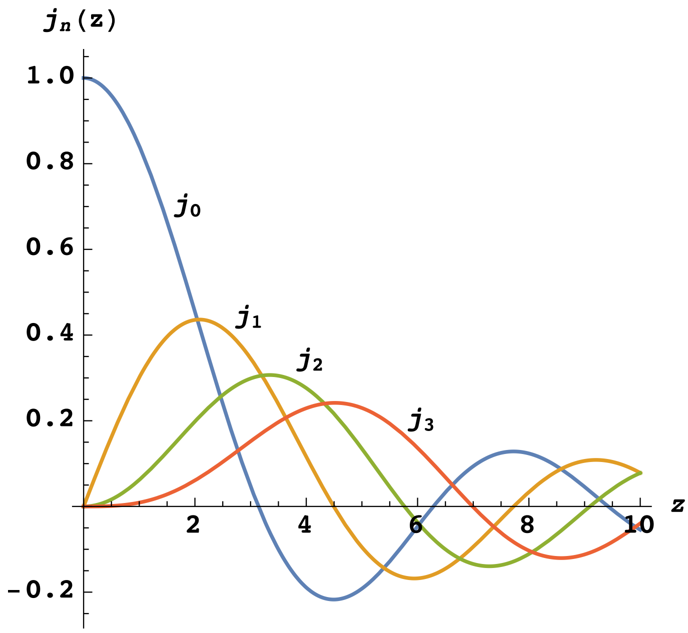

Figura 1: The first spherical Bessel functions

Figure 1 shows the first spherical Bessel functions ($\ell = 0, 1, 2, 3$). We can see that the ground state is obtained from the first zero of j_0, which is π. In ascending order, the following levels are obtained from the first zero of j_1 and j_2, the third excited level by the second zero of j_0, the fourth excited level by the first zero of j_3.

3.21 Determining the State of a Hydrogen atom

To be coherent with the definitions of Spherical Harmonics, as given in the Appendix, we have to modify the expressions regarding the probability. The correction does not change the result.

$$P(\phi)\,d\phi = \left|\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2}}e^{i\phi} + \frac{1}{\sqrt{2}}e^{-i\phi+i\delta}\right)\right|^2 d\phi\,.$$

$$\begin{aligned} P\left(0<\phi<\frac{\pi}{2}\right) &= \int_0^{\frac{\pi}{2}} P(\phi)d\phi = \\ &= \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} \left[1+\cos(2\phi-\delta)\right]d\phi = \\ &= \frac{1}{4} + \frac{1}{2\pi}\sin\delta = \frac{1}{4}\,. \end{aligned} \tag{3.50}$$

in

$$P(\phi)\,d\phi = \left| \frac{1}{\sqrt{2\pi}} \left(\frac{1}{-\sqrt{2}} e^{i\phi} + \frac{1}{\sqrt{2}} e^{-i\phi + i\delta} \right) \right|^2 d\phi\,.$$

$$\begin{aligned} P\left(0 < \phi < \frac{\pi}{2}\right) &= \int_0^{\frac{\pi}{2}} P(\phi) d\phi = \\ &= \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} \left[1 - \cos(2\phi - \delta)\right] d\phi = \\ &= \frac{1}{4} + \frac{1}{2\pi} \sin\delta = \frac{1}{4}\,. \end{aligned} \tag{3.50}$$

Chapter 5 Time Evolution

5.6 Particle confined on a segment (I)

In the calculation of $\langle p \rangle_t$ at page 119 the result is not

$$-\frac{8\hbar}{3L} \sin(\alpha + \omega t)\,,$$

but

$$-\frac{8\hbar}{3L} \sin(\alpha - \omega t)\,,$$

Chapter 6 Time-independent Perturbation Theory

6.23 β decay in a Hydrogenlike atom

The sentence on page 154 "Therefore, the first-order change in the energy levels is given by" becomes clearer if modified as follows:
The perturbation depends on r and, therefore, it commutes with the operators L^2 and L_z, whose quantum numbers are used to label the degeneracy; it follows that the matrix of $\mathcal{V}_1$ in the non-perturbed basis is diagonal. Thus, the first-order correction, as in the non-degenerate case, is given by

6.24 Stark Effect

Formulas (6.9) and (6.10) on page 181:

$$\begin{aligned} |3\rangle &\rightarrow \psi_{2,1,1}(r,\theta,\phi) = \frac{1}{8\sqrt{2\pi}}\, a_0^{-\frac{3}{2}}\, \frac{r}{a_0}\, \mathrm{e}^{-\frac{r}{2a_0}} \sin\theta \mathrm{e}^{\imath\phi} \\ |4\rangle &\rightarrow \psi_{2,1,-1}(r,\theta,\phi) = \frac{1}{8\sqrt{2\pi}}\, a_0^{-\frac{3}{2}}\, \frac{r}{a_0}\, \mathrm{e}^{-\frac{r}{2a_0}} \sin\theta \mathrm{e}^{-\imath\phi} \end{aligned}$$

should be corrected by (a factor $\sqrt{2}$ in the normalization constant)

$$|3\rangle \;\rightarrow\; \psi_{2,1,1}(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\, a_0^{-\frac{3}{2}}\, \frac{r}{a_0}\, \mathrm{e}^{-\frac{r}{2a_0}} \sin\theta \mathrm{e}^{\imath\phi}$$

$$|4\rangle \;\rightarrow\; \psi_{2,1,-1}(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\, a_0^{-\frac{3}{2}}\, \frac{r}{a_0}\, \mathrm{e}^{-\frac{r}{2a_0}} \sin\theta \mathrm{e}^{-\imath\phi}$$

6.25 Hydrogen: relativistic corrections

The formula on page 183 before eq. (6.14):

$$\begin{aligned}
E_n^1 &= -\frac{1}{8m^3c^2}\langle n,\ell,m|p^4|n,\ell,m\rangle = \\
&= \frac{1}{2mc^2}\langle n,\ell,m|\left(\mathcal{H}_0 + \frac{\mathrm{e}^2}{r}\right)^2|n,\ell,m\rangle = \\
&= -\frac{1}{2mc^2}\left((E_n^0)^2 + 2E_n^0\,\langle n,\ell,m|\frac{\mathrm{e}^2}{r}|n,\ell,m\rangle + \langle n,\ell,m|\frac{\mathrm{e}^4}{r^2}|n,\ell,m\rangle\right).
\end{aligned}$$

should be corrected by (a minus sign was forgotten in the second step)

$$\begin{aligned}
E_n^1 &= -\frac{1}{8m^3c^2}\langle n,\ell,m|p^4|n,\ell,m\rangle = \\
&= -\frac{1}{2mc^2}\langle n,\ell,m|\left(\mathcal{H}_0 + \frac{\mathrm{e}^2}{r}\right)^2|n,\ell,m\rangle = \\
&= -\frac{1}{2mc^2}\left((E_n^0)^2 + 2E_n^0\,\langle n,\ell,m|\frac{\mathrm{e}^2}{r}|n,\ell,m\rangle + \langle n,\ell,m|\frac{\mathrm{e}^4}{r^2}|n,\ell,m\rangle\right).
\end{aligned}$$

Chapter 8 Identical particles

8.7 Three interacting fermions on a segment

In the problem text formula:

$$V = \alpha\left(\vec{S}_1\cdot\vec{S}_2 + \vec{S}_1\cdot\vec{S}_2 + \vec{S}_2\cdot\vec{S}_3\right)$$

should be changed in

$$V = \alpha\left(\vec{S}_1\cdot\vec{S}_2 + \vec{S}_1\cdot\vec{S}_3 + \vec{S}_2\cdot\vec{S}_3\right)$$

8.8 Two Interacting Fermions in a Sphere

The sentence:
The first excited state is obtained for $\ell = 1$ and $n = 1$ and is three times degenerate ($m = 0, \pm 1$).
should be changed in
The first excited state is obtained for $\ell = 1$ and $n = 1$, see problem 3.13 (as modified by the present Errata Corrige), and is three times degenerate ($m = 0, \pm 1$).

Chapter 9

9.2 Gaussian potential

The calculation of the total cross section:

$$\begin{aligned}\sigma &= \int d\Omega \frac{d\sigma}{d\Omega} = 2\pi \frac{\pi m^2 V_0^2}{4\hbar^4\alpha^6} \int_{-1}^{+1} d\cos\theta e^{-\frac{k^2}{\alpha^2}(1-\cos\theta)} = \\ &= \frac{\pi^2 m^2 V_0^2}{2\hbar^4\alpha^6} \left(1 - e^{-2\frac{k^2}{\alpha^2}}\right) = \\ &= \frac{\pi^2 m V_0^2}{4\hbar^2\alpha^4 E} \left(1 - e^{-4m\frac{E}{\hbar^2\alpha^2}}\right) ,\end{aligned}$$

should be changed in

$$\begin{aligned}\sigma &= \int d\Omega \frac{d\sigma}{d\Omega} = 2\pi \frac{\pi m^2 V_0^2}{4\hbar^4\alpha^6} \int_{-1}^{+1} d\cos\theta e^{-\frac{k^2}{\alpha^2}(1-\cos\theta)} = \\ &= \frac{\pi^2 m^2 V_0^2}{2\hbar^4\alpha^4 k^2} \left(1 - e^{-2\frac{k^2}{\alpha^2}}\right) = \\ &= \frac{\pi^2 m V_0^2}{4\hbar^2\alpha^4 E} \left(1 - e^{-4m\frac{E}{\hbar^2\alpha^2}}\right) ,\end{aligned}$$

Chapter 11 Variational Method

11.3 First Energy Levels of a linear potential

The formula on page 233:

$$\left[-\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{\rho^2} - \rho\right]\psi(\rho) = \varepsilon\psi(\rho) ,$$

should be corrected by

$$\left[-\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{\rho^2} + \rho\right]\psi(\rho) = \varepsilon\psi(\rho) ,$$

and the first formula on page 234:

$$\begin{aligned}E(\alpha) &= \frac{\langle\psi|\mathcal{H}|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\int_0^\infty d\rho\,\rho^{\ell+1}\,e^{-\alpha\rho}\left[-\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{\rho^2} - \rho\right]\rho^{\ell+1}\,e^{-\alpha\rho}}{\int_0^\infty d\rho\,\rho^{2(\ell+1)}\,e^{-2\alpha\rho}} = \\ &= \frac{(2\ell+3)!\,(2\alpha)^{-2(\ell+2)}(2\alpha^3+2\ell+3)}{(2\ell+3)!\,(2\alpha)^{-(2\ell+3)}} = \frac{2\alpha^3+2\ell+3}{2\alpha} ,\end{aligned}$$

should be corrected by

$$\begin{aligned}E(\alpha) &= \frac{\langle\psi|\mathcal{H}|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\int_0^\infty d\rho\,\rho^{\ell+1}\,e^{-\alpha\rho}\left[-\frac{d^2}{d\rho^2} + \frac{\ell(\ell+1)}{\rho^2} + \rho\right]\rho^{\ell+1}\,e^{-\alpha\rho}}{\int_0^\infty d\rho\,\rho^{2(\ell+1)}\,e^{-2\alpha\rho}} = \\ &= \frac{(2\ell+3)!\,(2\alpha)^{-2(\ell+2)}(2\alpha^3+2\ell+3)}{(2\ell+3)!\,(2\alpha)^{-(2\ell+3)}} = \frac{2\alpha^3+2\ell+3}{2\alpha} ,\end{aligned}$$

Appendix A: Useful formulas

A.1.1 Gaussian integrals

***After formula (A.3)*, insert:** If we integrate the same function between 0 and $+\infty$, we obtain the integral

$$I_n(\alpha) = \int_0^{+\infty} dx\, x^n\, e^{-\alpha x^2}\,.$$

Note that $I_n(\alpha) = -\frac{\partial}{\partial\alpha} I_{n-2}(\alpha)$. So, to obtain $I_n(\alpha)$, it is sufficient to know $I_0(\alpha)$ and $I_1(\alpha)$.
Indeed, if n is even, we only need to halve the results obtained for the previous integral (A.2); so,

$$I_0(\alpha) = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\,.$$

It remains to calculate I_1.

$$I_1(\alpha) = \int_0^{+\infty} dx\, x e^{-\alpha x^2} = -\frac{1}{2\alpha}\int_0^{+\infty} dx\, \frac{\partial}{\partial x}\, e^{-\alpha x^2} = -\frac{1}{2\alpha}\, e^{-\alpha x^2}\Big|_0^{+\infty} = \frac{1}{2\alpha}$$

The results for the first values of n are

$$I_2 = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}; \qquad I_3 = \frac{1}{2\alpha^2}; \qquad I_4 = \frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}; \qquad I_5 = \frac{1}{\alpha^3}\,.$$

A.3.2 Position basis treatment

Formula (A.18)

$$\int_{-\infty}^{+\infty} d\xi\, H_n(\xi) H_m(\xi) = \sqrt{\pi}\, 2^n\, n!\, \delta_{n,m}$$

should be corrected as follows

$$\int_{-\infty}^{+\infty} d\xi\, \mathrm{e}^{-\xi^2}\, H_n(\xi) H_m(\xi) = \sqrt{\pi}\, 2^n\, n!\, \delta_{n,m}$$

A.5.2 Spherical Harmonics

Formula (A.28)

$$P_\ell^m(z) = (1-z^2)^{\frac{|m|}{2}}\, \frac{d^{|m|}}{dz^{|m|}}\, P_\ell(z)\,,$$

should be corrected as follows

$$P_\ell^m(z) = (-1)^m (1-z^2)^{\frac{|m|}{2}}\, \frac{d^{|m|}}{dz^{|m|}}\, P_\ell(z)\,,$$

and formula (A.29)

$$P_\ell(z) = \frac{1}{2^\ell \ell!}\,\frac{d^\ell}{dz^\ell}\,(1-z^2)^\ell\,.$$

should be corrected as follows

$$P_\ell(z) = (-1)^\ell \frac{1}{2^\ell \ell!}\,\frac{d^\ell}{dz^\ell}\,(1-z^2)^\ell\,.$$

1 A.8 Hydrogen atom first energy eigenfunctions

Formula (A.67)

$$\psi_{2,1,\pm 1} = \frac{1}{8\sqrt{2\pi}}\,a_0^{-\frac{3}{2}}\,\frac{r}{a_0}e^{-\frac{r}{2a_0}}\,\sin\theta\,e^{\pm\imath\varphi}\,.$$

should be corrected as follows

$$\psi_{2,1,\pm 1} = \frac{1}{8\sqrt{\pi}}\,a_0^{-\frac{3}{2}}\,\frac{r}{a_0}e^{-\frac{r}{2a_0}}\,\sin\theta\,e^{\pm\imath\varphi}\,.$$