On the Observable Differences between Proper and Improper Mixtures. - II

N. CUPARO PETRONI

Istituto di Fisica dell'Università - Bari
Istituto Nazionale di Fisica Nucleare - Sezione di Bari

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Summary. — The aim of this paper is to construct some observables which can distinguish, by means of measurements of correlation functions, whether a quantum system is described by a state vector of the I type (proper mixture) or by a state vector of the II type (improper mixture). We apply a theorem proved in a previous paper to the "singlet" state of two spin-$\frac{1}{2}$ particles. The widest class of sensitive observables is constructed and shown to depend on four real parameters. A particular subclass is studied in detail: its sensitivity is such that one has to distinguish experimentally between 1 and 3 in the most meaningful case. An inverse application of our theorem to the Friedberg-Jammer observable leads to the determination of the state for which this observable is sensitive. This state is a superposition of products of linear-polarization states of the two particles along orthogonal axes. Also in this case one has to distinguish experimentally between 1 and 3.

1. — Introduction.

In a recent paper (1), we have outlined the importance of a thorough experimental check of the quantum-mechanical description of a compound system. From this standpoint, we have proved a theorem which allows one to construct or to analyze suitable observables. Now we shall show the usefulness of this tool in order to point out some indications for experimental works; namely we shall develop some particular cases for two-particle systems which pick out a first set of sensitive observables, i.e. a first set of physical quantities on the ground of which it is possible to plan new experiments capable

to distinguish whether a system is described by a I-type or by a II-type state vector (*).

In this paper, we consider only examples based on dichotomic spin and polarization variables, the analysis for observables with continuous eigenvalues (or, however, for infinite-dimensional Hilbert spaces) being left to the future research.

Moreover, we remark here that, as one can see later from our examples, all sensitive observables hitherto analysed (***) and suggested for experimental research (****) can be built, starting from our theorem, which, furthermore, shows the way to find any suitable observable for new experiments.

2. – Photons and electrons.

We shall examine here the systems \( \Sigma_\gamma \) composed by two correlated photons and \( \Sigma_e \) composed by two correlated electrons. These systems, as is well known, are described by state vectors of two 4-dimensional Hilbert spaces \( H_2(\gamma) \) and \( H_2(e) \), respectively (**).

Now, \( H_2(\gamma) \) and \( H_2(e) \) being, respectively, the tensorial products \( H_1(\gamma) \otimes H_4(\gamma) \) and \( H_1(e) \otimes H_4(e) \) of the spaces which describe the subsystems (namely single photon or single electron), we can construct the base vectors for \( H_2(\gamma) \)

(* ) We remember here that we call « of I type » a state vector of a compound system \( \Sigma \) if it is factorable into individual state vectors of the subsystems which compose \( \Sigma \); on the contrary, a state vector is « of II type » if it is not factorable. Moreover, an observable is called « sensitive » if it is capable to distinguish between these two situations. For greater detail on these concepts see ref. (1).

(1) J. S. BELL: Physics, 1, 185 (1965).
(10) R. A. HOLT and F. M. PIPKIN: Harvard University, preprint.

(**) Therefore, we are here confronted with the most favourable situation for the application of our theorem: indeed, \( H_2(\gamma), H_2(e) \) being two finite-dimensional Hilbert spaces, the set of the eigenvalues \( \lambda_n \) of a linear Hermitian operator is always a bounded set, and hence it is always possible to keep the difference between the expectation values of our observables for I- and II-type state vectors finite. For details see ref. (1).
and $H_z(e)$ on the ground of the base vectors of $H_1(\gamma)$, $H_z(\gamma)$ and $H_1(e)$, $H_z(e)$. We have, therefore, that

\textit{a}) for correlated photons, if $|x\rangle$, $|y\rangle$ are orthonormal state vectors for the photon 1 (base in $H_1(\gamma)$) with linear polarization along the $x$, $y$ axes respectively, and $|x'\rangle$, $|y'\rangle$ similarly describe the photon 2 (base in $H_z(\gamma)$), a set of orthonormal base vectors for $H_z(\gamma)$ is

\begin{align*}
|\xi_1\rangle &= |x\rangle |x'\rangle, \\
|\xi_2\rangle &= |x\rangle |y'\rangle, \\
|\xi_3\rangle &= |y\rangle |x'\rangle, \\
|\xi_4\rangle &= |y\rangle |y'\rangle;
\end{align*}

\textit{b}) likewise, for correlated electrons, if $|u_+\rangle$, $|u_-\rangle$ are the orthonormal state vectors for the electron 1 (base in $H_1(e)$) with eigenvalues $\pm 1$ for the $z$-component of the spin respectively, and $|v_+\rangle$, $|v_-\rangle$ similarly describe the electron 2 (base in $H_z(e)$), a set of orthonormal base vectors for $H_z(e)$ is

\begin{align*}
|\varphi_1\rangle &= |u_+\rangle |v_+\rangle, \\
|\varphi_2\rangle &= |u_+\rangle |v_-\rangle, \\
|\varphi_3\rangle &= |u_-\rangle |v_+\rangle, \\
|\varphi_4\rangle &= |u_-\rangle |v_-\rangle.
\end{align*}

In this notation, for example, the singlet and the triplet states for $\Sigma_\ast$ are

\begin{align*}
\frac{1}{\sqrt{2}} \left[ |\varphi_2\rangle - |\varphi_3\rangle \right], \\
\frac{1}{\sqrt{2}} \left[ |\varphi_2\rangle + |\varphi_3\rangle \right],
\end{align*}

respectively.

Furthermore, if we put for $H_z(\gamma)$ (with $P'$, $Q'$, $R'$, $P''$, $Q''$, $R''$ being Hermitian operators)

\begin{align*}
|x\rangle \langle x'| &= \frac{I + R'}{2}, \\
|y\rangle \langle y'| &= \frac{I - R'}{2}, \\
|x'\rangle \langle x'| &= \frac{I + R''}{2}, \\
|y'\rangle \langle y'| &= \frac{I - R''}{2}, \\
|x''\rangle \langle x''| &= \frac{P' + iQ'}{2}, \\
|y''\rangle \langle y''| &= \frac{P'' + iQ''}{2},
\end{align*}

and for $H_z(e)$ (with $\sigma'_z$, $\sigma''_z$, $\sigma'_y$, $\sigma''_y$, $\sigma'_x$, $\sigma''_x$ being Hermitian operators)

\begin{align*}
|u_+\rangle \langle u_+| &= \frac{I + \sigma'_z}{2}, \\
|v_+\rangle \langle v_+| &= \frac{I + \sigma''_z}{2}, \\
|u_-\rangle \langle u_-| &= \frac{I - \sigma'_z}{2}, \\
|v_-\rangle \langle v_-| &= \frac{I - \sigma''_z}{2}, \\
|u_+\rangle \langle u_-| &= \frac{\sigma'_y + i\sigma'_x}{2}, \\
|v_+\rangle \langle v_-| &= \frac{\sigma''_y + i\sigma''_x}{2}, \\
|u_-\rangle \langle u_+| &= \frac{\sigma'_y - i\sigma'_x}{2}, \\
|v_-\rangle \langle v_+| &= \frac{\sigma''_y - i\sigma''_x}{2},
\end{align*}
we can easily show that, for $H_2(\gamma)$,

$$
\begin{align*}
R' |x'\rangle &= |x'\rangle, \\
R' |y'\rangle &= -|y'\rangle,
\end{align*}
$$

$$
\begin{align*}
P' \left[ \frac{|x'\rangle \pm |y'\rangle}{\sqrt{2}} \right] &= \pm \frac{|x'\rangle \pm |y'\rangle}{\sqrt{2}}, \\
Q' \left[ \frac{|x'\rangle \pm i|y'\rangle}{\sqrt{2}} \right] &= \pm \frac{|x'\rangle \pm i|y'\rangle}{\sqrt{2}},
\end{align*}
$$

$$
\begin{align*}
P' \left[ \frac{|x'\rangle \pm |y'\rangle}{\sqrt{2}} \right] &= \pm \frac{|x'\rangle \pm |y'\rangle}{\sqrt{2}}, \\
Q' \left[ \frac{|x'\rangle \pm i|y'\rangle}{\sqrt{2}} \right] &= \pm \frac{|x'\rangle \pm i|y'\rangle}{\sqrt{2}},
\end{align*}
$$

and, for $H_2(e)$,

$$
\begin{align*}
\sigma'_z |u_{\pm}\rangle &= \pm |u_{\pm}\rangle, \\
\sigma'_x \left[ \frac{|u_{+}\rangle \pm |u_{-}\rangle}{\sqrt{2}} \right] &= \pm \frac{|u_{+}\rangle \pm |u_{-}\rangle}{\sqrt{2}}, \\
\sigma'_y \left[ \frac{|u_{+}\rangle \pm i|u_{-}\rangle}{\sqrt{2}} \right] &= \pm \frac{|u_{+}\rangle \pm i|u_{-}\rangle}{\sqrt{2}},
\end{align*}
$$

$$
\begin{align*}
\sigma''_z |v_{\pm}\rangle &= \pm |v_{\pm}\rangle, \\
\sigma''_x \left[ \frac{|v_{+}\rangle \pm |v_{-}\rangle}{\sqrt{2}} \right] &= \pm \frac{|v_{+}\rangle \pm |v_{-}\rangle}{\sqrt{2}}, \\
\sigma''_y \left[ \frac{|v_{+}\rangle \pm i|v_{-}\rangle}{\sqrt{2}} \right] &= \pm \frac{|v_{+}\rangle \pm i|v_{-}\rangle}{\sqrt{2}}.
\end{align*}
$$

Therefore, the only eigenvalues of $P'$, $Q'$, $R'$ are $\pm 1$ and their eigenstates are linear-polarization states along $x$ and $y$ (for $R'$), linear-polarization states along $45^\circ$ and $-45^\circ$ (for $P'$) and circular-polarization states clockwise and anti-clockwise (for $Q'$). Similar interpretations hold for $P''$, $Q''$, $R''$. Likewise it is obvious that $\sigma'_z$, $\sigma'_x$, $\sigma'_y$, $\sigma''_z$, $\sigma''_x$, $\sigma''_y$ are the components along the $x$, $y$, $z$ axes of the spin of the first and the second electron, respectively.

Relations (3), (4) and their physical meaning are the starting point for the physical interpretation of the observables built by means of our theorem. Furthermore, we emphasize that, on account of the strict formal analogy between (3) and (4), we can develop our analysis of the sensitive observables only for $H_2(e)$, it being clear that the analysis for $H_2(\gamma)$ is obtained simply by substituting $|u_{\pm}\rangle$, $|u_{\pm}\rangle$, $|v_{\pm}\rangle$ by $|x'\rangle$, $|y'\rangle$, $|x''\rangle$, $|y''\rangle$ and $\sigma'_z$, $\sigma'_x$, $\sigma'_y$, $\sigma''_z$, $\sigma''_x$, $\sigma''_y$ by $P'$, $Q'$, $R'$, $P''$, $Q''$, $R''$. Hence in this paper we shall consider in detail only the problem of two correlated electrons with spin and we shall obtain the results for polarized photons by merely formal substitutions.

3. – Sensitive observables for the singlet state.

We begin this section by the construction of the most general sensitive observable for the singlet state, namely for a state described by the $\Pi$-type vector $[|\varphi_s\rangle - |\varphi_s\rangle]/\sqrt{2}$ belonging to $H_2(e)$. 
In order to attain this aim, let us consider an orthonormal base for \( H_x(e) \) different from the base \( \{|\varphi_k\rangle, \ K = 1, 2, 3, 4, \rangle \) of the previous section; if we put

\[
\begin{align*}
|\chi_1\rangle &= \frac{1}{\sqrt{2}} \left[ |\varphi_2\rangle - |\varphi_3\rangle \right] \quad \text{(singlet state)}, \\
|\chi_2\rangle &= \frac{1}{\sqrt{2}} \left[ |\varphi_2\rangle + |\varphi_3\rangle \right] \quad \text{(triplet state)}, \\
|\chi_3\rangle &= \frac{1}{\sqrt{2}} \left[ |\varphi_4\rangle - |\varphi_5\rangle \right], \\
|\chi_4\rangle &= \frac{1}{\sqrt{2}} \left[ |\varphi_4\rangle + |\varphi_5\rangle \right],
\end{align*}
\]

(5)

it is obvious that the following vectors

\[
\begin{align*}
|\psi_1\rangle &= |\chi_1\rangle, \\
|\psi_2\rangle &= a_1|\chi_2\rangle + a_2|\chi_3\rangle + a_3|\chi_4\rangle, \\
|\psi_3\rangle &= b_1|\chi_2\rangle + b_2|\chi_3\rangle + b_3|\chi_4\rangle, \\
|\psi_4\rangle &= c_1|\chi_2\rangle + c_2|\chi_3\rangle + c_3|\chi_4\rangle,
\end{align*}
\]

(6)

for every choice of the numbers \( a_k, b_k, c_k, k = 1, 2, 3 \), which fulfills the orthonormalization conditions

\[
\begin{align*}
|a_1|^2 + |a_2|^2 + |a_3|^2 &= 1, \\
|b_1|^2 + |b_2|^2 + |b_3|^2 &= 1, \\
|c_1|^2 + |c_2|^2 + |c_3|^2 &= 1, \\
\langle \psi_2 | \psi_2 \rangle &= \bar{a}_1 b_1 + \bar{a}_2 b_2 + \bar{a}_3 b_3 = 0, \\
\langle \psi_3 | \psi_3 \rangle &= \bar{b}_1 c_1 + \bar{b}_2 c_2 + \bar{b}_3 c_3 = 0, \\
\langle \psi_4 | \psi_4 \rangle &= \bar{c}_1 a_1 + \bar{c}_2 a_2 + \bar{c}_3 a_3 = 0,
\end{align*}
\]

(7)

constitute an orthonormal base for \( H_x(e) \); furthermore, it is the most general orthonormal base which contains the singlet state as an element.

Hence, by varying \( a_k, b_k, c_k \), the base (6) allows one to construct, on the ground of our theorem (*), the sentive observables \( I_\sigma \) for the singlet state. In fact, we have

\[
I_\sigma = \lambda |\psi_1\rangle \langle \psi_1| + \mu |\psi_2\rangle \langle \psi_2| + \nu |\psi_3\rangle \langle \psi_3| + \sigma |\psi_4\rangle \langle \psi_4|,
\]

(8)

(*) For greater convenience, we recall here the statement of our theorem (see also ref. (1)): let us consider a given II-type vector \( |\Psi\rangle \), an arbitrary complete orthonormal system \( \{|\varphi_n\rangle \} \) of elements of \( H_x \) such that \( |\varphi_n\rangle = |\Psi\rangle \) and a sequence of real numbers \( \{\lambda_n\} \) such that \( \lambda_N > \lambda_n, \ \forall \ n \neq N \) (or \( \lambda_N < \lambda_n, \ \forall \ n \neq N \)); we have that \( \Gamma = \sum \lambda_n |\varphi_n\rangle \langle \varphi_n| \) is a sensitive observable for the mixture of II type described by \( |\Psi\rangle \).
where \( \lambda, \mu, \varrho, \sigma \) are four real numbers (which here act as eigenvalues of \( I_k^* \)) with \( \lambda > \mu, \varrho, \sigma \) (or \( \lambda < \mu, \varrho, \sigma \)). Since from (2) and (5) we can express \( |\psi_k\rangle \), \( k = 1, 2, 3, 4 \), by means of \( |u_\pm\rangle, \psi_\pm\rangle \), from (4) we can also express \( I_k^* \) by means of the spin observables. A laborious, but straightforward calculation shows that

\[
4I_k^* = (\lambda + \mu + \varrho + \sigma)I + \\
+ 2 \Re \left[ \mu a_0 \bar{a}_3 + \varrho b_0 \bar{b}_3 + \sigma c_0 \bar{c}_3 \right] (\sigma'_3 + \sigma''_3) + \\
+ 2 \Re \left[ \mu a_1 \bar{a}_3 + \varrho b_1 \bar{b}_3 + \sigma c_1 \bar{c}_3 \right] (\sigma'_3 + \sigma''_3) + \\
+ 2 \Im \left[ \mu a_2 \bar{a}_3 + \varrho b_2 \bar{b}_3 + \sigma c_2 \bar{c}_3 \right] (\sigma'_3 + \sigma''_3) + \\
+ \left[ \mu(-|a_1|^2 + |a_2|^2 + |a_3|^2) + \varrho(-|b_1|^2 + |b_2|^2 + |b_3|^2) + \right. \\
\left. + \sigma(-|c_1|^2 + |c_2|^2 + |c_3|^2) - \lambda \right] \sigma'_3 \sigma''_3 + \\
+ \left[ \mu(|a_1|^2 - |a_2|^2 + |a_3|^2) + \varrho(|b_1|^2 - |b_2|^2 + |b_3|^2) + \right. \\
\left. + \sigma(|c_1|^2 - |c_2|^2 + |c_3|^2) - \lambda \right] \sigma'_3 \sigma''_3 + \\
+ \left[ \mu(|a_1|^2 + |a_2|^2 - |a_3|^2) + \varrho(|b_1|^2 + |b_2|^2 - |b_3|^2) + \right. \\
\left. + \sigma(|c_1|^2 + |c_2|^2 - |c_3|^2) - \lambda \right] \sigma'_3 \sigma''_3 + \\
+ 2 \Re \left[ \mu a_1 \bar{a}_3 + \varrho b_1 \bar{b}_3 + \sigma c_1 \bar{c}_3 \right] (\sigma'_3 + \sigma''_3) + \\
+ 2 \Im \left[ \mu a_2 \bar{a}_3 + \varrho b_2 \bar{b}_3 + \sigma c_2 \bar{c}_3 \right] (\sigma'_3 + \sigma''_3).
\]

All the \( I_k^* \) (obtained by varying \( \lambda, \mu, \varrho, \sigma \) and \( a_k, b_k, c_k \) under conditions (7)) are observables which have the singlet state \( |\psi_k\rangle \) as an eigenstate for the eigenvalue \( \lambda \). If we introduce the representation of \( I_k^* \) on the orthonormal set \( |\chi_k\rangle \), \( k = 1, 2, 3, 4 \), we have (we recall here that \( I_k^* \) is diagonal only for the set of its eigenstates \( |\psi_k\rangle \) with \( \lambda, \mu, \varrho, \sigma \) as diagonal elements)

\[
4I_k^* = \text{Tr} \left( I_k^* \right) \cdot \left[ I - \sigma' \cdot \sigma'' \right] + \left[ I_3^* + I_4^* \right] (\sigma'_3 + \sigma''_3) - i[\left[ I_2^* - I_3^* \right] (\sigma'_3 + \sigma''_3) + \\
+ [I_1^* + I_4^*] (\sigma'_3 + \sigma''_3) + 2[I_2^* + I_3^*] \sigma'_3 \sigma''_3 + 2[I_2^* + I_3^*] \sigma'_3 \sigma''_3 + 2[I_3^* + I_4^*] \sigma'_3 \sigma''_3 + \\
+ [I_2^* + I_3^*] (\sigma'_3 \sigma'_3 + \sigma''_3 \sigma''_3) - i[\left[ I_2^* - I_3^* \right] (\sigma'_3 \sigma'_3 + \sigma''_3 \sigma''_3) - i[\left[ I_2^* - I_3^* \right] (\sigma'_3 \sigma'_3 + \sigma''_3 \sigma''_3) \\
\right] \text{with} \\
\text{Tr} \left( I_k^* \right) = \lambda + \mu + \varrho + \sigma.
\]
and

\[
\begin{aligned}
\Gamma_m^m &= \langle \chi_m | \hat{I}_m | \chi_n \rangle, & m, n = 1, 2, 3, 4, \\
\Gamma_1^1 &= \lambda, \\
\Gamma_2^2 &= \Gamma_1^1 = \Gamma_3^3 = \Gamma_4^4 = \Gamma_2^1 = \Gamma_3^3 = \Gamma_4^4 = 0, \\
\Gamma_2^2 &= \mu |a_1|^2 + \varrho |b_1|^2 + \sigma |c_1|^2, \\
\Gamma_3^3 &= \mu |a_2|^2 + \varrho |b_2|^2 + \sigma |c_2|^2, \\
\Gamma_4^4 &= \mu |a_3|^2 + \varrho |b_3|^2 + \sigma |c_3|^2, \\
\Gamma_3^3 &= \mu \alpha_1 \alpha_2 + b_1 \tilde{b}_1 + \sigma \alpha_1 \tilde{c}_1, \\
\Gamma_4^4 &= \mu \alpha_2 \alpha_2 + \varrho \beta_1 \tilde{b}_2 + \sigma \alpha_2 \tilde{c}_2, \\
\Gamma_4^4 &= \mu \alpha_3 \alpha_3 + \varrho \beta_2 \tilde{b}_3 + \sigma \alpha_3 \tilde{c}_3.
\end{aligned}
\]

It is easy to show that formally similar relations hold for correlated photons if we start from (3) in $H_2(y)$.

4. - Sensitive observables for the singlet state. Examples.

A particular case of sensitive observable for the singlet state is obtained from (9) when we put

\[
a_z = a_3 = 0, \quad b_1 = b_3 = 0, \quad c_1 = c_3 = 0,
\]

namely when $|\psi_k\rangle = |\chi_k\rangle$, $k = 1, 2, 3, 4$. Now we have

\[
\Gamma^{(1)} = \frac{1}{2} \left[ \lambda + \mu + \varrho - \mu + \varrho + \sigma \right] \sigma_1^s \sigma_2^s + \\
+ \frac{1}{2} \left[ -\lambda + \mu + \varrho - \sigma \right] \sigma_1^s \sigma_2^s + \frac{1}{2} \left[ -\lambda + \mu + \varrho + \sigma \right] \sigma_1^s \sigma_2^s = \\
= \lambda |\chi_1\rangle \langle \chi_1 | + \mu |\chi_2\rangle \langle \chi_2 | + \varrho |\chi_3\rangle \langle \chi_3 | + \sigma |\chi_4\rangle \langle \chi_4 |.
\]

Now we shall show that $\Gamma^{(1)}$ is really a sensitive observable for the singlet state: in this singlet state $|\chi_1\rangle$ the expectation value of our observable is

\[
\langle \chi_1 | \Gamma^{(1)} | \chi_1 \rangle = \lambda.
\]

Given an arbitrary I-type vector $|\eta\rangle$, we have from (2) and (5)

\[
|\eta\rangle = [\alpha |\psi_+\rangle + \beta |\psi_-angle] \cdot [\gamma |v_+\rangle + \delta |v_-\rangle] = \\
= \alpha \gamma |\psi_1\rangle + \alpha \delta |\psi_2\rangle + \beta \gamma |\psi_3\rangle + \beta \delta |\psi_4\rangle = \\
= \frac{\alpha \delta - \beta \gamma}{\sqrt{2}} |\chi_1\rangle + \frac{\alpha \delta + \beta \gamma}{\sqrt{2}} |\chi_2\rangle + \frac{\alpha \gamma - \beta \delta}{\sqrt{2}} |\chi_3\rangle + \frac{\alpha \gamma + \beta \delta}{\sqrt{2}} |\chi_4\rangle.
\]
For the expectation value of $R_\alpha^{(1)}$ in $|\eta\rangle$ it follows that

\[
\langle \eta | R_\alpha^{(1)} | \eta \rangle = \lambda \left( \frac{|x\delta - \beta\gamma|^2}{2} + \mu \frac{|x\delta + \beta\gamma|^2}{2} + \rho \frac{|xy - \beta\delta|^2}{2} + \sigma \frac{|xy + \beta\delta|^2}{2} \right) = \\
= \frac{\lambda + \mu}{2} \sin^2 (\varphi + \psi) + \frac{\rho + \sigma}{2} \cos^2 (\varphi - \psi) - \\
- \frac{\sin 2\varphi \cdot \sin 2\psi}{2} \left[ \lambda \cos^2 \frac{\Phi}{2} + \mu \sin^2 \frac{\Phi}{2} + \rho \cos^2 \frac{\Phi'}{2} + \sigma \sin^2 \frac{\Phi'}{2} \right]
\]

with

\[
\begin{cases}
|x| = \cos \varphi, & \varphi \in \left[ 0, \frac{\pi}{2} \right], \\
|\beta| = \sin \varphi, & \varphi \in \left[ 0, \frac{\pi}{2} \right], \\
|\gamma| = \cos \psi, & \psi \in \left[ 0, \frac{\pi}{2} \right], \\
|\delta| = \sin \psi, & \psi \in \left[ 0, \frac{\pi}{2} \right], \\
\frac{\alpha \beta \gamma \delta}{|x\beta\gamma\delta|} = \exp [i\Phi], & \frac{\alpha \beta \gamma \delta}{|x\beta\gamma\delta|} = \exp [i\Phi'].
\end{cases}
\]

By means of differential calculus, we can prove that the extremal values of this function are

\[
\frac{\lambda + \mu}{2}, \quad \frac{\lambda + \rho}{2}, \quad \frac{\lambda + \sigma}{2}, \quad \frac{\mu + \rho}{2}, \quad \frac{\mu + \sigma}{2}, \quad \frac{\rho + \sigma}{2}.
\]

Now, if $\lambda > \mu, \rho, \sigma$, we have obviously that

\[
\lambda > \frac{\lambda + \mu}{2}, \quad \frac{\lambda + \rho}{2}, \quad \frac{\lambda + \sigma}{2}, \quad \frac{\mu + \rho}{2}, \quad \frac{\mu + \sigma}{2}, \quad \frac{\rho + \sigma}{2},
\]

namely $R_\alpha^{(1)}$ is sensitive for $|\chi_\lambda\rangle$. It is easy to show that this result is obtained also for $\lambda < \mu, \rho, \sigma$.

Moreover, since we have

\[
\max \left\{ \frac{\lambda + \mu}{2}, \frac{\lambda + \rho}{2}, \frac{\lambda + \sigma}{2}, \frac{\mu + \rho}{2}, \frac{\mu + \sigma}{2}, \frac{\rho + \sigma}{2} \right\} = \frac{\lambda + \max \{\mu, \rho, \sigma\}}{2},
\]

the difference between the expectation value in $|\chi_\lambda\rangle$ and in every I-type state vector is

\[
\delta_{\lambda} > \frac{\lambda - \max \{\mu, \rho, \sigma\}}{2}.
\]

(*) It is possible to demonstrate that $R_\alpha^{(1)}$ is a sensitive observable also for the triplet state $|\chi_\lambda\rangle$ if $\mu > \lambda, \rho, \sigma$ (or $\mu < \lambda, \rho, \sigma$).
Finally, if we eliminate from $I^{(1)}_s$ the part $((\lambda + \mu + \varrho + \sigma)/4) I$ without meaning for the measurements, the observable

$$
\gamma^{(1)}_s = I^{(1)}_s - \frac{\lambda + \mu + \varrho + \sigma}{4} I
$$

is also sensitive with

$$
\begin{align*}
\langle \chi_1 | \gamma^{(1)}_s | \chi_1 \rangle &= \lambda - \frac{\lambda + \mu + \varrho + \sigma}{4}, \\
\langle \eta | \gamma^{(1)}_s | \eta \rangle &\leq \frac{\lambda + \max \{\mu, \varrho, \sigma\}}{2} - \frac{\lambda + \mu + \varrho + \sigma}{4}
\end{align*}
$$

and hence

$$
\delta_1 \geq \frac{\lambda - \max \{\mu, \varrho, \sigma\}}{2}.
$$

In this case, for the relative difference we have

$$
\Delta_1 = \frac{\delta_1}{\langle \chi_1 | \gamma^{(1)}_s | \chi_1 \rangle} \geq \frac{2}{3} \frac{\lambda - \max \{\mu, \varrho, \sigma\}}{\lambda - (\mu + \varrho + \sigma)/3}
$$

and hence, since

$$
0 \leq \frac{\lambda - \max \{\mu, \varrho, \sigma\}}{\lambda - (\mu + \varrho + \sigma)/3} < 1
$$

with

$$
\frac{\lambda - \max \{\mu, \varrho, \sigma\}}{\lambda - (\mu + \varrho + \sigma)/3} = 1,
$$

for

$$
\max \{\mu, \varrho, \sigma\} = \frac{\mu + \varrho + \sigma}{3},
$$

namely for $\mu = \varrho = \sigma$, we have that the most favourable condition for our aim leads to a relative difference

$$
\Delta_1 \geq \frac{2}{3}
$$

and corresponds to the observable

$$
\gamma^{(1)}_s = -\frac{\lambda - \varrho}{4} \sigma^\prime \cdot \sigma^\prime = K \sigma^\prime \cdot \sigma^\prime,
$$

(see also ref. (9)).
Another example, which partly generalizes the first example, is obtained if we put in (9)

\[
\begin{align*}
(a_1 &= 1, \quad b_1 = 0, \quad c_1 = 0, \\
(a_2 &= 0, \quad b_2 = -i \cos \theta \exp[i\omega], \quad c_2 = i \sin \theta \exp[i\omega'], \quad \theta \in [0, \frac{\pi}{2}], \\
(a_3 &= 0, \quad b_3 = \sin \theta \exp[i\omega], \quad c_3 = \cos \theta \exp[i\omega']).
\end{align*}
\]

It is easy to show that the vectors \(|\psi_\theta\rangle\) obtained from (6) and (24) are orthonormal and that now the sensitive observable \(I^{(2)}_s\) is (*) (with \(\text{Tr}(I^{(2)}_s) = \lambda + \mu + \varrho + \sigma\) and \(\lambda > \mu, \varrho, \sigma\))

\[
4I^{(2)}_s = \text{Tr}(I^{(2)}_s) \cdot I +
+ \left[-\lambda + \mu - (\varrho - \sigma) \cos 2\theta\right] \sigma_x^1 \sigma_x^2 + \left[-\lambda + \mu + (\varrho - \sigma) \cos 2\theta\right] \sigma_x^3 \sigma_x^4 +
\]

\[
\left[-\lambda - \mu + \varrho + \sigma\right] \sigma_y^1 \sigma_y^2 + (\varrho - \sigma) \sin 2\theta \left(\sigma_y^3 \sigma_y^4 + \sigma_z^3 \sigma_z^4\right) =
\]

\[
\lambda |\chi_1\rangle \langle \chi_1| + \mu |\chi_2\rangle \langle \chi_2| + (\varrho \cos^2 \theta + \sigma \sin^2 \theta) |\chi_3\rangle \rangle
\]

\[
\langle \chi_n| I^{(2)}_s |\chi_n\rangle = \lambda
\]

(*) We remark here that, for the simmetry of (25) in \(\lambda\) and \(\mu\), we can consider \(I^{(2)}_s\) also as a sensitive observable for the triplet state \(|\chi_2\rangle\) if we put \(\mu > \lambda, \varrho, \sigma\) (or vice versa).
with $\varphi$, $\psi$, $\Phi$, $\Phi'$ defined as in (15). A straightforward calculation leads to extremal values which are again given by (16), and hence the analysis of the sensitivity of this observable coincides with the analysis for $I_4''$.

5. — Friedberg observables.

Another particular class of sensitive observables has been indicated by FRIEDBERG (*) and it allows one to use our theorem in the analysis of a given observable rather than in the construction of these observables from a given II-type state vector. The suggested observable is (*)

\begin{equation}
I_y = (n_1 \cdot \sigma')(n_2 \cdot \sigma'^*) + (n_2 \cdot \sigma')(n_3 \cdot \sigma'^*) + (n_3 \cdot \sigma')(n_1 \cdot \sigma'^*) ,
\end{equation}

where $n_1$, $n_2$, $n_3$ are three arbitrary unit vectors and $\sigma'$, $\sigma'^*$ the spin observables for two correlated electrons. The matrix representation of $I_y$ in the base $|\varphi_k\rangle$ is (see also (2), (4))

\begin{equation}
\begin{aligned}
G_m &= \langle \varphi_m | I_y | \varphi_n \rangle , \\
G_1 &= -G_2^a = -G_3^b = G_4^c = C \cos \theta_k \cos \theta_1 , \\
G_1^* &= \overline{G_2} = -G_3^* = -G_4^* = C \cos \theta_k \sin \theta_1 \exp[-i\varphi_1] , \\
G_1^* &= \overline{G_2} = -G_3^* = -G_4^* = C \sin \theta_k \cos \theta_1 \exp[-i\varphi_k] , \\
G_1^* &= \overline{G_2} = -G_3^* = -G_4^* = C \sin \theta_k \sin \theta_1 \exp[-i(\varphi_k + \varphi_1)] , \\
G_1^* &= \overline{G_2} = -G_3^* = -G_4^* = C \sin \theta_k \sin \theta_1 \exp[-i(\varphi_k - \varphi_1)] , 
\end{aligned}
\end{equation}

where $(\theta_k, \varphi_k)$, $k = 1, 2, 3$, are the angles of $n_k$ with respect to the $(x, y, z)$-system, namely

\begin{align*}
(n_k)_x &= \sin \theta_k \sin \varphi_k , \\
(n_k)_y &= \sin \theta_k \cos \varphi_k , \\
(n_k)_z &= \cos \theta_k ,
\end{align*}

and $C_{k,i} a_k b_i$ is defined as a sum with cyclic permutations of indices:

\begin{equation}
C_{k,i} a_k b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 .
\end{equation}

(*) Of course, it is quite easy to find also the corresponding observable for polarization of two correlated photons in the way pointed out in sect. 2.
Moreover, it is easy to convince ourselves that the class of sensitive observables defined by (27) is not coincident with class (9). This means that among observables (27) some may have the singlet state \( |\chi_1\rangle \) as an eigenstate, but some do not. Here we shall not analyse the eigenstates and the eigenvectors of the most general \( I \) in order to demonstrate, by our theorem, that \( I \) is a sensitive observable, but we shall restrict ourselves to some examples of more direct physical interest.

If we choose \( n_1, n_2, n_3 \) as unit vectors direct along the axes \( x, y, z \), namely if we put

\[
\begin{align*}
(n_1)_x &= 1, & (n_2)_x &= 0, & (n_3)_x &= 0, \\
(n_1)_y &= 0, & (n_2)_y &= 1, & (n_3)_y &= 0, \\
(n_1)_z &= 0, & (n_2)_z &= 0, & (n_3)_z &= 1,
\end{align*}
\]

we have

\[
(29) \quad I^n = \sigma'_x \sigma''_x + \sigma'_y \sigma''_y + \sigma'_z \sigma''_z
\]

and hence, from (4),

\[
(30) \quad I^n = -i |\varphi_3\rangle \langle \varphi_4| + i |\varphi_2\rangle \langle \varphi_3|-i |\varphi_2\rangle \langle \varphi_3| + i |\varphi_1\rangle \langle \varphi_2| -
\]

\[
- i |\varphi_1\rangle \langle \varphi_2| + i |\varphi_2\rangle \langle \varphi_3| + i |\varphi_1\rangle \langle \varphi_3| - i |\varphi_4\rangle \langle \varphi_3| +
\]

\[
+ |\varphi_2\rangle \langle \varphi_2| + |\varphi_3\rangle \langle \varphi_3| - |\varphi_4\rangle \langle \varphi_4| - |\varphi_4\rangle \langle \varphi_3|.
\]

Now the matrix representation is

\[
(31) \quad \langle \varphi_m | I^n | \varphi_n \rangle,
\]

\[
\begin{pmatrix}
0 & 1 & -i & -i \\
1 & 0 & i & i \\
i & -i & 0 & -1 \\
i & -i & -1 & 0
\end{pmatrix}
\]

Moreover, we can show, by direct calculation, that (29) is not deducible from (9) (that is the observable (29) does not have the singlet state as an eigenstate). Starting from (31) we can also develop the eigenvalue analysis: it is well known that the four eigenvalues \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) of \( I^n \) are the roots of the equation

\[
\det \left[ \langle \varphi_m | I^n | \varphi_n \rangle - \lambda \delta_m^n \right] =
\]

\[
\begin{vmatrix}
-\lambda & 1 & -i & -i \\
1 & -\lambda & i & i \\
i & -i & -\lambda & -1 \\
i & -i & -1 & -\lambda
\end{vmatrix} = -(1 - \lambda)^3(3 + \lambda) = 0,
\]
and hence we have \( \lambda_1 = \lambda_2 = \lambda_3 = 1 \) and \( \lambda_4 = -3 \). Furthermore, we can also show that the eigenvector for \( \lambda_4 = -3 \) is

\[
|\Psi_4\rangle = \frac{1}{2} [i|\varphi_1\rangle - i|\varphi_2\rangle + |\varphi_3\rangle + |\varphi_4\rangle] =
\]

\[
= -\sqrt{2} \left[ \frac{|u_+\rangle + i|u_-\rangle}{\sqrt{2}} |v_+\rangle + \frac{|u_+\rangle - i|u_-\rangle}{\sqrt{2}} |v_-\rangle \right]
\]

(we recall here that \( (|u_+\rangle \pm i|u_-\rangle)/\sqrt{2} \) are the eigenstates of \( \sigma_y \), see also sect. 2). Moreover, we can choose as orthonormal eigenstates for \( \lambda_1 = \lambda_2 = \lambda_3 = 1 \)

\[
|\Psi_1\rangle = \frac{1}{\sqrt{2}} [i|\varphi_1\rangle - i|\varphi_2\rangle],
\]

\[
|\Psi_2\rangle = \frac{1}{\sqrt{2}} [i|\varphi_1\rangle + i|\varphi_2\rangle],
\]

\[
|\Psi_3\rangle = \frac{1}{2} [i|\varphi_1\rangle + |\varphi_2\rangle + i|\varphi_3\rangle - |\varphi_4\rangle] =
\]

\[
= \frac{1}{\sqrt{2}} \left[ \frac{|u_+\rangle + i|u_-\rangle}{\sqrt{2}} |v_+\rangle + \frac{|u_+\rangle - i|u_-\rangle}{\sqrt{2}} |v_-\rangle \right].
\]

Now the set \( |\Psi_n\rangle \) is an orthonormal base which diagonalizes \( I^n_F \):

\[
\langle \Psi_m|I^n_F|\Psi_n\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -3
\end{pmatrix},
\]

Moreover, from the standpoint of our theorem, it is possible to show that \( I^n_F \) must be sensitive because a \( \Pi \)-type vector \( |\Psi_4\rangle \) corresponds to the minimum eigenvalue \( \lambda_4 = -3 \).

In order to calculate the sentivity of \( I^n_F \), we put the arbitrary vector \( |\Phi\rangle \) in the form (*)

\[
|\Phi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle + c|\Psi_3\rangle + d|\Psi_4\rangle,
\]

\[
|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1,
\]

\[
(32)
\]

(*) We emphasize here that, if we choose

\[
a = \frac{1 - i}{2}, \quad b = 0, \quad c = \frac{i + 1}{2 \sqrt{2}}, \quad d = \frac{i - 1}{2 \sqrt{2}},
\]

we have a singlet state, and if we choose

\[
a = 0, \quad b = 0, \quad c = \frac{1}{\sqrt{2}}, \quad d = \frac{i}{\sqrt{2}},
\]

we have a triplet state.
and we deduce that

\[ \langle \Phi | \Gamma^0_P | \Phi \rangle = |a|^2 + |b|^2 + |c|^2 - 3\bar{d}|^2 = 1 - 4|\bar{d}|^2 , \]

namely

\[ -3 < \langle \Phi | \Gamma^0_P | \Phi \rangle < 1 . \]

Obviously \( \Gamma^0_P \) takes the value \(-3\) for and only for (*) \( |\Phi\rangle = |\Psi_4\rangle \). Now let us consider a I-type vector

\[ |\eta\rangle = \left( a_1 |u_+\rangle + a_2 |u_-\rangle \right) \left( b_1 |v_+\rangle + b_2 |v_-\rangle \right) , \]

\[ |a_1|^2 + |a_2|^2 = |b_1|^2 + |b_2|^2 = 1 . \]

In this case we have easily, from (30),

\[ \langle \eta | \Gamma^0_P | \eta \rangle = \sin \alpha \cos \beta \sin \phi + \cos \alpha \sin \beta \cos \phi' + \sin \alpha \sin \beta \cos \phi \sin \phi' \]

with

\[ \cos \alpha = |a_1|^2 - |a_2|^2 , \]

\[ \cos \beta = |b_1|^2 - |b_2|^2 , \]

\[ \frac{a_1 a_2}{|a_1 a_2|} = \exp [i\phi] , \quad \frac{b_1 b_2}{|b_1 b_2|} = \exp [i\phi'] . \]

An analysis of the extremal values shows that

\[ -1 < \langle \eta | \Gamma^0_P | \eta \rangle < 1 . \]

This result confirms that \( \Gamma^0_P \) is really a sensitive observable and points out that absolute and relative gaps between the expectation values are

\[ \delta_3 = \langle \Psi_4 | \Gamma^0_P | \Psi_4 \rangle - \langle \eta | \Gamma^0_P | \eta \rangle < -2 , \]

\[ A_3 = \frac{\delta_3}{-3} > \frac{2}{3} , \]

which shows that \( \Gamma^0_P \) is sensitive also for vectors \( |\Phi\rangle \) (different from the vector considered up to now):

\[ -3 < \langle \Phi | \Gamma^0_P | \Phi \rangle < -1 . \]

(*) Indeed, if \( \langle \Phi | \Gamma^0_P | \Phi \rangle = -3 \), we must have, from (33), \(|\bar{d}|^2 = 1\) and hence \(|a|^2 = |b|^2 = |c|^2 = 0\), namely \( |\Phi\rangle = |\Psi_4\rangle \).
namely for (*)

\[ \frac{1}{2} < |d|^2 < 1. \]

6. — Conclusions.

We will summarize here the results of our analysis of particular cases. Firstly for correlated electrons and photons we have, respectively, that

(39a) \[ \Gamma_{s}^{(1)} = \frac{1}{4} (\lambda + \mu + \varrho + \sigma) I + \frac{1}{4} (\lambda - \mu + \varrho + \sigma) \sigma_x^2 \sigma_x^2 + \]
\[ + \frac{1}{4} (-\lambda + \mu + \varrho + \sigma) \sigma_y^2 \sigma_y^2 + \frac{1}{4} (-\lambda + \mu + \varrho - \sigma) \sigma_x^2 \sigma_y^2, \]

(39b) \[ S^{(1)} = \frac{1}{4} (\lambda + \mu + \varrho + \sigma) I + \frac{1}{4} (-\lambda - \mu + \varrho + \sigma) \mathcal{R} \mathcal{R}^* + \]
\[ + \frac{1}{4} (-\lambda - \mu - \varrho + \sigma) \mathcal{P} \mathcal{P}^* + \frac{1}{4} (-\lambda + \mu + \varrho - \sigma) \mathcal{Q} \mathcal{Q}^*. \]

(see also (3), (4)) are sensitive observables (with \( \lambda > \mu, \varrho, \sigma \)): \( \Gamma_{s}^{(1)} \) for the singlet state and \( S^{(1)} \) for photons both polarized along \( x \) or \( y \) axes. Moreover, because the expectation values of the left-hand sides of (39a) and (39b) are the expectation values of the right-hand sides, we emphasize here that a measurement of \( \Gamma_{s}^{(1)} \) or of \( S^{(1)} \) is always a measurement of the correlation functions \( \langle \sigma_x^2 \sigma_x^2 \rangle, \langle \sigma_y^2 \sigma_y^2 \rangle, \langle \sigma_x^2 \sigma_y^2 \rangle \rangle \) or of \( \langle \mathcal{P}^2 \mathcal{P}^* \rangle, \langle \mathcal{Q}^2 \mathcal{Q}^* \rangle, \langle \mathcal{R} \mathcal{R}^* \rangle \).

The relative gap between measurements in states described by I- and II-type vectors is in both situations (21), and in the most favourable condition one has \( \Delta_i > \frac{1}{2} \). For an experimentalist this is like saying that 1 is to be distinguished from 3.

A generalization of (39a), (39b) is given by

(40a) \[ \Gamma_{s}^{(2)} = \frac{1}{4} [\lambda + \mu + \varrho + \sigma] I + \]
\[ + \frac{1}{4} [(-\lambda - \mu) \cos 2\theta] \sigma_x^2 \sigma_x^2 + \frac{1}{4} [(-\lambda + \mu + \varrho - \sigma) \cos 2\theta] \sigma_y^2 \sigma_y^2 + \]
\[ + \frac{1}{4} [(-\lambda - \mu + \varrho + \sigma) \cos 2\theta] \sigma_x^2 \sigma_y^2 + \frac{1}{4} (\sigma - \varrho) \sin 2\theta [\sigma_x^2 \sigma_y^2 + \sigma_y^2 \sigma_x^2] , \]

(40b) \[ S^{(2)} = \frac{1}{4} [\lambda + \mu + \varrho + \sigma] I + \]
\[ + \frac{1}{4} [(-\lambda + \mu + \varrho - \sigma) \cos 2\theta] \mathcal{P} \mathcal{P}^* + \frac{1}{4} [(-\lambda + \mu + \varrho - \sigma) \cos 2\theta] \mathcal{Q} \mathcal{Q}^* + \]
\[ + \frac{1}{4} [(-\lambda + \mu + \varrho + \sigma) \cos 2\theta] \mathcal{R} \mathcal{R}^* + \frac{1}{4} (\sigma - \varrho) \sin 2\theta [\mathcal{P}^2 \mathcal{P}^* + \mathcal{Q} \mathcal{Q}^*]. \]

(*) From this result and from the footnote on p. 393 we have by direct calculation that \( \Gamma_{s}^{(n)} \) is not sensitive for singlet and triplet states.
A measurement of the observable \((40a)\), obviously, consists of a measurement not only of the correlation functions \(\langle \sigma_x' \sigma_y' \rangle, \langle \sigma_y' \sigma_z' \rangle, \langle \sigma_z' \sigma_x' \rangle\), but of the correlation functions \(\langle \sigma_x' \sigma_y' \rangle, \langle \sigma_y' \sigma_z' \rangle\) too (similar considerations hold for the photons).

However, the analysis of the sensitivity is the same as for \(I^{(1)}_{\alpha}, S^{(0)}\). Finally we have

\[
I^{(0)}_{\alpha} = \sigma_x' \sigma_y' + \sigma_y' \sigma_z' + \sigma_z' \sigma_x',
\]

\[
S^{(0)} = P'Q' + Q'R' + R'P',
\]

which are sensitive in

\[
|\Psi_4\rangle = -\frac{i}{\sqrt{2}} \left[ \frac{|u_+\rangle - i|u_-\rangle}{\sqrt{2}} |\nu_+\rangle + \frac{|u_+\rangle + i|u_-\rangle}{\sqrt{2}} |\nu_-\rangle \right],
\]

\[
|\zeta_4\rangle = -\frac{i}{\sqrt{2}} \left[ \frac{|x'\rangle - i|y'\rangle}{\sqrt{2}} |x''\rangle + \frac{|x'\rangle + i|y'\rangle}{\sqrt{2}} |y''\rangle \right],
\]

respectively, for electrons and photons.

It is interesting to notice that state \((42b)\) can be obtained from the \(J = 0\) two-photon state by passing the first photon through a quarter-wave plate. Similarly \((42a)\) can be obtained by rotating the spin of the first electron by \(\pi/2\) in the \((z, y)\)-plane.

Therefore, states \((42a)\) and \((42b)\) can be produced in the laboratory for experimental analysis.

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**RIASSUNTO**

Lo scopo del lavoro è costruire alcune osservabili che possono distinguere, per mezzo di misure di funzioni di correlazione, se un sistema quantico è descritto da un vettore di stato del I tipo (miscela propria) o da un vettore di stato del II tipo (miscela impropria). Si applica un teorema dimostrato in un lavoro precedente allo stato di «singolotto» di due particelle di spin \(\frac{1}{2}\). Si costruisce la più ampia classe di osservabili sensibili e si mostra che questa dipende da quattro parametri reali. Si studia in dettaglio una particolare sottoclasse; la sua sensibilità è tale da dover distinguere fra 1 e 3 nel caso più significativo. Un'applicazione inversa del teorema all'osservabile di Friedberg-Jammer conduce alla determinazione dello stato per il quale quest'osservabile è sensibile. Questo stato è una sovrapposizione di prodotti di stati di polarizzazione lineari delle due particelle lungo assi ortogonali. Anche in questo caso si deve distinguere sperimentalmente fra 1 e 3.
О наблюдаемых различиях между собственным и несобственным смешиванием.

Резюме (*). — Цель этой статьи — сконструировать некоторые наблюдаемые, которые можно различить с помощью измерений корреляционных функций, в зависимости от того, описывается ли квантовая система вектором состояния первого типа (собственное смешивание) или вектором состояния второго типа (нестабильное смешивание). Мы применяем теорему, доказанную в предыдущей статье к «синглетному» состоянию двух частиц со спином половина. Конструируется широкий класс чувствительных наблюдаемых и показывается, что они зависят от четырех вещественных параметров. Подробно исследуется специальный подкласс: чувствительность этого подкласса является такой, что возможно экспериментально различить 1 и 3 в случае, имеющем глубокий физический смысл. Обратное применение нашей теоремы к наблюдаемой Фридберга-Джаммера приводит к определению состояния, для которого эта наблюдаемая является чувствительной. Это состояние представляет суперпозицию произведений линейных состояний поляризации системы двух частиц вдоль ортогональных осей. В этом случае также возможно экспериментально различить 1 и 3.

(*) Переведено редакцией.