STOCHASTIC DERIVATION OF PROCA'S EQUATION IN TERMS OF A FLUID OF WEYSSENHOFF TOPS ENDOWED WITH RANDOM FLUCTUATIONS AT THE VELOCITY OF LIGHT

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If one analyzes the stochastic behaviour of classical Weyssenhoff particles imbedded in a relativistic thermostat one obtains (for random jumps at the velocity of light) a probability distribution corresponding to Proca's equations for nonzero-mass spin-one particles.

Recent research [1,2] on possible stochastic interpretations of the Klein–Gordon equation has shown that for a non-point-like, spinless, scalar, extended particle imbedded in a random thermostat

(a) Nelson's stochastic equations naturally result from the assumption that its random stochastic jumps occur at the velocity of light so that as a consequence

(b) the Klein–Gordon equation correctly describes its stochastic distribution.

These new results can, of course, be considered as an encouraging step along the long line of attempts to interpret quantum mechanics in terms of various models of realistic random subquantum behaviour [2].

One difficulty remains however, i.e., the imbedding of spin into this type of models. Clearly this implies the introduction of new properties into stochastic models. The aim of the present letter is to present the consequences of such a modification i.e., to show that if one assumes that our stochastic thermostat is built from the classical extended spinning particles introduced by Weyssenhoff and Raabe [4] (and later fully analysed by Halbwachs [5]), one immediately recovers the particular form of the Proca equation utilised by de Broglie and Vigier [6] to interpret electromagnetic phenomena.

To do this we start from the above mentioned [1] demonstration. We consider in a first step our particle as submitted to stochastic fluctuations described by Nelson's equation for free particles, i.e.,

$$(DD - \delta D \delta D)x^\mu = 0\,, \tag{1}$$

where $x^\mu(\tau)$ is the position of the particle in Minkowski space–time and (in the notation of Guerra–Ruggiero [1]) D and δD represent, respectively, the total derivative with respect to the proper time τ and the stochastic derivative, namely

$$D = \partial/\partial\tau + b^\mu\partial_\mu\,, \quad \delta D = \delta b^\mu \partial_\mu - (\hbar/2m)\Box\,,$$

where $b^\mu = Dx^\mu$ is the drift velocity and $\delta b^\mu = \delta Dx^\mu$ the stochastic velocity. Moreover, it is in general possible to show that the stochastic velocity is given by

$$\delta b^\mu = -(\hbar/m)\partial^\mu \log(\rho^{1/2})\,,$$

where ρ is a density which satisfies the continuity equation

$$\partial\rho/\partial\tau = -\partial_\mu(\rho b^\mu) . \quad (2)$$

At this stage we make the physical assumption [1] that the drift current is irrotational so that we can write: $b^\mu = (1/m)\partial^\mu\Phi$, where Φ is a scalar function containing x^μ and τ as independent variables.

In a second step we introduce as stochastic elements the classical spinning particles of Weyssenhoff and Raabe [4]. As one knows [5] we can represent each individual particle of that type by a space-like vector R_μ (representing the distance between the spinning particle's center of mass and the center of matter density) and thus substitute for the scalar field density ρ of the spin-zero particles the new density $\rho = a_\nu a^\nu$, where $a^\nu = \rho^{1/2} R^\nu$ is the real amplitude of a more general complex vector field whose phase factor disappears in making the scalar product for ρ. Now a straightforward calculation [1] shows that, if we choose $\sigma\rho/\sigma\tau = 0$ (because we are only interested, for the time being, in the time-independent wave equations) Nelson's equation and the continuity equation become, respectively:

$$\partial^\mu\left(\frac{T^{\alpha\nu\beta}T_{\alpha\nu\beta}}{2\rho^2} + \frac{a^\nu\Box a_\nu}{\rho} - \frac{\partial^\nu\Phi\partial_\nu\Phi}{\hbar^2} - \frac{2m}{\hbar^2}\frac{\partial\Phi}{\partial\tau}\right) = 0 , \quad (3)$$

$$a^\mu(2\partial^\nu a_\mu \cdot \partial_\nu\Phi + a_\mu\Box\Phi) = 0 , \quad (4)$$

where

$$T^{\alpha\nu\beta} = a^\alpha\partial^\nu a^\beta - a^\beta\partial^\nu a^\alpha$$

is a tensor equal to zero in the case of a scalar field.

We can now easily derive Proca's equations. Indeed, if we recall that $a^\mu = \rho^{1/2}R^\mu$ with $R^\mu R_\mu = 1$, it is easy to show that

$$T^{\alpha\nu\beta}T_{\alpha\nu\beta}/2\rho^2 = R^\nu\Box R_\nu .$$

So that, if we further complete our model by the hypothesis that the constant vector R^ν satisfies an equation like

$$\Box R^\nu = c_0 R^\nu , \quad (5)$$

where c_0 is a constant, we can reduce eq. (3) to

$$\frac{a^\mu\Box a_\mu}{\rho} - \frac{\partial^\nu\Phi\partial_\nu\Phi}{h^2} - \frac{2m}{h^2}\frac{\partial\Phi}{\partial\tau} = E ,$$

where E is a constant that we can make equal to zero by rearranging the energy scale. Eq. (3) then becomes

$$a^\mu\left[\Box a_\mu - \left(\frac{\partial^\nu\Phi\partial_\nu\Phi}{\hbar^2} + \frac{2m}{\hbar^2}\frac{\partial\Phi}{\partial\tau}\right)a_\mu\right] = 0. \quad (6)$$

As said in the beginning, in order to have eq. (5) it is sufficient to adopt the classical Weyssenhoff model. As shown by Halbwachs [5], in each particle's rest frame eq. (5) becomes $\ddot{R}^\nu = c_0 R^\nu$, which yields exactly Weyssenhoff's equation for the free spinning particle. Moreover, in our model R^ν (the constant vector which joins the center of matter to the center of gravity of the Weyssenhoff particle) is also perpendicular to the velocity $b^\mu = \partial^\mu\Phi/m$, so that we have [5] $a_\mu\partial\Phi = 0$. Now if we consider eqs. (4) and (6) as representing, respectively, the imaginary and real part of a total equation, it can easily be shown that (with $B_\mu = a_\mu \times \exp(i\Phi/\hbar)$ we get

$$B_\mu^*((\hbar^2/2m)\Box + i\hbar\partial/\partial\tau)B^\mu = 0 ,$$

an equation which reduces to the following set of equations:

$$-(\hbar^2/2m)\Box B_\mu = i\hbar\partial B_\mu/\partial\tau , \quad (7)$$

by successive Lorentz transformations which allow only one B^μ to be different from zero.

Finally, if we follow the tentative suggestion of Feynman [7] in the way proposed by Guerra and Ruggiero [1] and assume for B_μ the proper-time dependence

$$B_\mu(x,\tau) = \exp\left(\tfrac{1}{2}i(mc^2/\hbar)\,\tau\right)A_\mu(x) ,$$

we find the classical set of Proca equations:

$$(\Box - m^2c^2/\hbar^2)A_\mu = 0 .$$

Beyond the importance that this first stochastic derivation of a wave equation for spinning particles can have in general, the authors want to stress in conclusion that this proposed model for spin-one bosons paves the way to possible interpretations for Aspect's forthcoming experiments [7], experiments first devised to verify Bell's inequalities, but which now raise the problem of the existence of space-like interactions between quantum mechanical independent measurements [9]. Indeed, preliminary experiments seem to favor the quantum mechanical predictions and hence new features for transmission of information in a space-like direction. An analysis of this phenomenon was given by the authors in previous papers [2,9] only for spinless particles. It is clear that an adequate interpretation must concern the analysis of correlated spin-one bosons, namely photons, which are the particles utilized in these experiments.

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