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Baryon Octet Magnetic Moments in an Integer-Charged-Quark Oscillator Model.

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Since the origin of the SU_3 fractionally charged-quark model, the problem of the numerical determination of the baryon octet masses, magnetic moments and the decay modes has been considered as a crucial test, both for quark models themselves—from SU_3 to SU_6—and for the quark coupling forces.

Despite important successes in unravelling the resonance zoology, the SU_3 quark theory, after a promising start, due essentially to DE RÚJULA *et al.* (1) and LIPKIN (2), has apparently reached an impasse. As shown by TEESE and SETTLES (3), its theoretical predictions of hadronic magnetic moments disagree significantly with experimental measurement. As we see from the first part of our table III (3), they lie well outside the experimental error bars, so that their confidence level falls below 1%.

The aim of the present letter is to attack the baryon octet problem along an alternative line, namely that of the integer-charged-quark model, proposed by SALAM (4)

(1) A. DE RÚJULA, H. GEORGI and S. L. GLASHOW: *Phys. Rev. D*, **12**, 147 (1975).
(2) H. J. LIPKIN: *Phys. Lett. B*, **74**, 399 (1978), and Fermilab-Conf. 79/60 THV.
(3) R. B. TEESE and R. SETTLES: *Phys. Lett. B*, **87**, 3 (1979).
(4) A. SALAM: in *Elementary Particle Physics*, edited by N. SVARTHOLM (1968).

and PATI (5), and to use to that effect the unification of quarks and leptons within the stochastic oscillator model, proposed by GUERET *et al.* (6) and developed by the present authors (7). Following SALAM (8), qualitatively we shall assume

a) that quarks and leptons are not different forms of matter, but that they are associated within basic representations, representing quantized excited internal states of motion;

b) that quarks and leptons are fermions, endowed with integer charges, and surrounded by short range ($\sim\frac{1}{3}$ fm) clouds of heavy Yang-Mills bosons, which break the internal symmetry and give different masses to quarks and leptons in the low-energy limit; according to SALAM (4), bound quarks in hadrons are stable (the so-called Archimedes effect), have reduced masses, but decay strongly into leptons in the free state;

c) that our integer charged quarks, *i.e.* an extended core plus a boson cloud, behave as free nonrelativistic spin-$\frac{1}{2}$ objects, moving in a scalar collective potential, which can be approximated by a harmonic-oscillator potential (fig. 1). We shall neglect

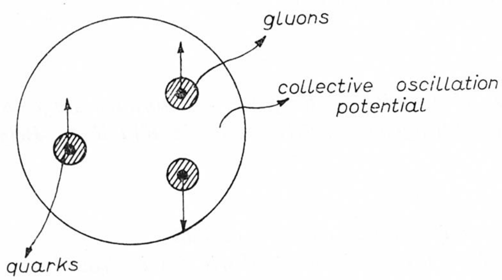

Fig. 1.

the contribution to the magnetic moments coming from the nonrelativistic orbital motions. Following LIPKIN and TAVKHELIDZE (9), we shall consider the magnetic moments as a vector sum of individual quark magnetic moments, using « effective » quark masses, corresponding to the levels that the quarks occupy in the harmonic-oscillator potential.

The heuristic picture exposed above is supported by the recent experiments concerning high-energy polarized proton-proton collisions. The experiments performed by KRISCH (10,11) at the Michigan and Argonne storage ring on polarized (parallel and antiparallel) proton-proton scattering have yielded the following—rather surprizing!—results:

1) the proton appears as a compound object, built with three substructures, $\sim\frac{1}{3}$ fm radius each;

(5) J. C. PATI and A. SALAM: *Phys. Rev. D*, **8**, 1240 (1973); *Phys. Rev. Lett.*, **31**, 661 (1973); *Phys. Rev. D*, **10**, 275 (1974).
(6) P. GUERET, P. MERAT, M. MOLES and J. P. VIGIER: *Lett. Math. Phys.*, **3**, 47 (1979).
(7) N. CUFARO PETRONI, Z. MARIĆ, DJ. ŽIVANOVIĆ and J. P. VIGIER: *Stable states of a relativistic bilocal stochastic ossillator: a new quark-lepton model*, to be published *J. Phys. A* (1980).
(8) A. SALAM: IC 176/21 (1976).
(9) H. J. LIPKIN and A. TAVKHELIDZE: *Phys. Lett.*, **17**, 331 (1965).
(10) A. D. KRISCH: Brookhaven National Laboratory Report B.N.L.5+947 (1979).
(11) A. D. KRISCH: *Sci. Am.* (May 1979).

2) these « partons » (or quarks) scatter two by two, the proton scattering cross-section at great angles being equal to the sum of all quark-quark scattering cross-sections;

3) in the low-energy collisions the final state does not depend much on the spins of colliding protons (with $\sigma(\uparrow\uparrow) > \sigma(\uparrow\downarrow)$), while in violent high-energy collisions the experiment suggests that the spin-spin contributions become dominant; for angles $> 60°$, $\sigma(\uparrow\uparrow) \to 4\sigma(\uparrow\downarrow)$, and everything goes as if the orbital rotations were practically negligible.

These properties suggest a first test, in order to compare the standard fractionally charged-quark model with the integer-charge model proposed here. Indeed, if one assumes that each proton is built with three quarks, there will be in proton-proton scattering nine quark-quark scattering cross-sections, which are different according to whether the spins of quarks are $\uparrow\uparrow$ or $\uparrow\downarrow$. Since the experiment shows that $\sigma(\uparrow\uparrow)/\sigma(\uparrow\downarrow) = 4$, the sum of all possible quark-quark cross-sections should be in the same ratio.

As was indicated in ([10,11]), the standard SU_3 model leads to an impossible conclusion: $\sigma(\uparrow\downarrow) = -(11/16)\,\sigma(\uparrow\uparrow)$. In our model, however, the situation is different, as there is no reason to assume that quark-quark scattering cross-sections are approximately equal. A detailed analysis shows that the afore-mentioned absurd conclusion can be avoided.

Our baryon octet model is now constructed in three steps.

I) We assume quarks and leptons to be excited quantized levels of an extended material structure, that can be (according to YUKAWA ([12]), TAKABAYASI ([13]), BOHM and VIGIER ([14])) approximated by a bilocal oscillator model. If we now extend to this model essential concepts of the stochastic interpretation of quantum mechanics ([15-17]), *i.e.* if we assume that these structures contain an internal random subquantal thermostat, corresponding to Dirac's aether ([18]), then we must consider excited states of a relativistic stochastic oscillator ([6,7]), invariant under the internal dynamic group $U_1 \otimes SO_{6,2}$, that commutes ([7]) with the external Poincaré group. As shown in ([7]), this implies that the random oscillator contains two basic $J = \frac{1}{2}^+$ quark-lepton representations, summarized in table I.

II) We construct the baryon octet with three quarks as building blocks. Here we remark that our integer-charge quark model is different from the usual scheme. In the SU_3 scheme baryons are built with qqq and bosons with $q\bar{q}$ combinations. With the usual u ($Q = \frac{2}{3}$), d ($Q = -\frac{1}{3}$) and s ($Q = -\frac{1}{3}$) quarks, we have the following quark content: (uud) and (ddu) for the proton and the neutron, respectively, (uus), (uds) and (dds) for Σ^+, Σ^0 or Λ, and Σ^-, and finally (ssu) and (ssd) for Ξ^0 and Ξ^-. Fitting the quark masses to: $m_u = 338$ MeV, $m_d = 322$ MeV, $m_s = 512$ MeV, one can calculate the magnetic moments ([2,3]); the results are presented in the first part of table III. Denoting the three fundamental objects by a, b and c, we see that there are two independent coupling schemes for three fundamental $\frac{1}{2}$ angular momenta to the total $\frac{1}{2}$ angular momentum, *i.e.* when the states of a and b are coupled to spin zero ($S = 0$)

([12]) H. YUKAWA: *Phys. Rep.*, **77**, 219 (1950); **80**, 1047 (1950); **91**, 416 (1953).
([13]) T. TAKABAYASI: *Prog. Theor. Phys.*, **39**, 424 (1955); *Prog. Theor. Phys. Suppl.*, **67**, 1 (1979).
([14]) D. BOHM and J. P. VIGIER: *Phys. Rev.*, **96**, 208 (1954); **109**, 882 (1958).
([15]) E. NELSON: *Phys. Rev.*, **150**, 1079 (1966).
([16]) J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 258, 262 (1979).
([17]) N. CUFARO PETRONI and J. P. VIGIER: *Int. J. Theor. Phys.*, **18**, 807 (1979).
([18]) P. A. M. DIRAC: *Nature (London)*, **168**, 906 (1951).

TABLE I.

T_3	Y	Z	s	c	Q	Particle	SU_3	SU_2
0	0	$+\frac{3}{4}$	$-\frac{1}{2}$	0	0	c^y	singlet	doublet $H_3=-\frac{1}{2}$
0	$+\frac{2}{3}$	$-\frac{1}{4}$	$+\frac{1}{2}$	$+1$	$+1$	s^y	triplet	
$+\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$+1$	$+1$	u^y		doublet $H_3=+\frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$+1$	0	d^y		
0	0	$-\frac{3}{4}$	$+\frac{1}{2}$	0	0	ν_τ	singlet	doublet $H_3=+\frac{1}{3}$
0	$-\frac{2}{3}$	$+\frac{1}{4}$	$-\frac{1}{2}$	-1	-1	τ^-	triplet	
$-\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	-1	-1	e^-		doublet $H_3=-\frac{1}{2}$
$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	-1	0	ν_e		
0	0	$-\frac{3}{4}$	$+\frac{1}{2}$	$+\frac{3}{2}$	$+1$	c^b	singlet	doublet $H_3=+\frac{1}{2}$
0	$-\frac{2}{3}$	$+\frac{1}{4}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	s^b	triplet	
$-\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	$+\frac{1}{2}$	0	d^b		doublet $H_3=-\frac{1}{2}$
$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{4}$	0	$+\frac{1}{2}$	$+1$	u^b		
0	0	$+\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{3}{2}$	-1	M^-	singlet	doublet $H_3=-\frac{1}{2}$
0	$+\frac{2}{3}$	$-\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	M^0	triplet	
$+\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	ν_μ		doublet $H_3=+\frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	-1	μ^-		

and to spin one ($S=1$), respectively. For these two cases the magnetic moments are given by

(1a) $$\mu_0=\mu[(ab)_{S=0},\, c]_{S=\frac{1}{2}}=\mu_c\,,$$

(1b) $$\mu_1=\mu[(ab)_{S=1},\, c]_{S=\frac{1}{2}}=2/3(\mu_a+\mu_b)-1/3\mu_c\,.$$

In the usual scheme, *i.e.* the SU_3 quark model, this leads to the numerical values given in the first part of table III, together with the $(1/m_{\mathrm{B}})^2$ correction [3]. In our oscillator model the situation is different: the baryon octet is obtained by multiplying the singlet c by the boson octet of the type $q\bar{q}$, where q are the SU_3 yellow-quark triplet from table I. So, we have $|p=cs\bar{d}$, $|\eta\rangle=cs\bar{u}$, $|\Lambda\rangle=c(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$, $|\Sigma^+\rangle=cu\bar{d}$, $|\Sigma^0\rangle=c(u\bar{u}-d\bar{d})/\sqrt{2}$, $|\Sigma\rangle^-=c\bar{u}d$, $|\Xi^0\rangle=c\bar{s}u$, $|\Xi^-\rangle=c\bar{s}d$. In our scheme, however, a choice of one of these combinations is not sufficient to determine both the mass and the magnetic moments; since we imbedded our integer-charged quarks into a nonrelativistic potential (representing their collective interaction), each quark can be in a different excited oscillator level. A new step is thus needed.

III) In our model, the assumptions discussed (in the framework of the standard SU_3 model. *i.e.* without our quark identification) by FAIMAN and HENDRY [19], describe our octet in the simplest possible shell model. Assuming our quarks to move (without interacting) in a spherically symmetric harmonic-oscillator potential (nonrelativistic

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[19] D. FAIMAN and A. W. HENDRY: *Phys. Rev.*, **173**, 1720 (1968).

in the first approximation), we have the energy (*i.e.* mass) levels $[\hbar = c = 1]$:

$$m_{n+1}(\mathrm{q}) = m_0(\mathrm{q}) + (n + \tfrac{3}{2})\,\omega\,, \tag{2}$$

where $m_0(q)$ are the so-called bare masses of quarks in the collective potential, and $n = l + 2k$ is the total quantum number (taking the values $n = 0, 1, 2, \ldots$) with l = angular momentum and k = number of nodes. Here we suppose that quarks can tunnel outside the potential well, *i.e.* that the bare masses m_0 correspond to the masses of the free quarks: the harmonic-oscillator potential is so an approximation valid only for the lowest bound states.

According to the standard nuclear practice, we should now try all possible combinations for the masses and the magnetic moments, assuming that each quartet (u, d, s, c) is the basis of ground states for quarks (since there are no lower states), and that formula (2) applies. In order to calculate bare masses of our quarks, we must remember that m_1 (obtained for $n = 0$) is determined by the values given in table II. Namely, in our scheme, the masses are separated approximately by multiples of 90 MeV, and $\frac{3}{2}\cdot 90 \simeq 135$ MeV, so that we can build up a spectrum of harmonic-oscillator states with the following values: $\omega \sim 87$, and bare masses $m_0(\mathrm{u}) = 2$, $m_0(\mathrm{d}) = 0$, $m_0(\mathrm{s}) = 93$ and $m_0(\mathrm{c}) = 452$. That gives the following oscillator level table.

TABLE II.

	$m_1(\mathrm{q})$	$m_2(\mathrm{q})$	$m_3(\mathrm{q})$	$m_4(\mathrm{q})$
u	133	220	307	394
d	131	218	305	392
s	224	311	398	485
c	583	670	757	844

The calculated values for the baryon octet masses and their magnetic moments, in the framework of our model, are given in the lower part of table III.

In this table we have recalled the SU_3 results given by TEESE and SETTLES and have given our results calculated, according to the following expressions for masses and magnetic moments:

$$m = \langle B|\sum_i m_i|B\rangle\,, \tag{3}$$

$$\boldsymbol{\mu} = \langle B|\sum_i \boldsymbol{\mu}_i\,|B\rangle\,. \tag{4}$$

The simple additive formula for the baryon masses is used here, as we suppose that the essential part of the quark-quark interaction has already been taken into account through our self-consistent harmonic-oscillator potential.

However, the mass formula can be corrected by adding to expression (3) the corrective term Δm, which does not change the magnetic moments, due to the vectorial character of the two-body forces, originating from isospin, strangeness and hypercharge quark-quark (antiquark) interactions:

$$\Delta m = \langle B|\Big(\alpha\sum_{i\neq j}\boldsymbol{t}_i\boldsymbol{t}_j + \beta\sum_{i\neq j}\boldsymbol{s}_i\boldsymbol{s}_j + \gamma\sum_{i\neq j}y_i y_j\Big)|B\rangle \tag{5}$$

TABLE III.

	p	n	Λ	Σ+	Σ0	Σ−	Ξ0	Ξ−
μ_{SU_3}	2.79	−1.91	−0.611	2.67	1.63	−1.09	−1.43	−0.49
SU_3 (1/m cor.)	2.79	−1.91	−0.612	2.39	1.45	−0.95	−1.27	−0.48
μ_{exp}	2.79	−1.91	−0.613 ± ±0.005	2.33 ± ±0.13	$1.82^{+0.25}_{-0.18}$	−1.40 ± ±0.25	−1.20 ± ±0.06	−1.85 ± ±0.75
m_{exp}	938	940	1116	1189	1192	1197	1315	1321
μ_{osc}	2.79	−1.91	−0.61	2.38	1.82	−1.59	−1.20	− 2.01
m_{osc}	938	940	1143	1195	1195	1195	1288	1286
$m_{osc}+\Delta m$	938	940	1116	1195	1195	1195	1314	1312

(we recall that in our SU_4 scheme the strangeness quantum number has a vectorial character). Masses calculated with the corrective term Δm (for $\alpha = 13.5$; $\beta = 9.0$; $\gamma = 30.3$) are given in the last row of table III.

Results are calculated from the following internal-structure assignment:

$$|p\rangle \;= [(s_1\bar{d}_1)_{S=1};\, c_1]_{S=\frac{1}{2}}\,,$$

$$|n\rangle \;= [(s_1\bar{u}_1)_{S=1};\, c_1]_{S=\frac{1}{2}}\,,$$

$$|\Lambda\rangle \;= \frac{1}{\sqrt{6}}\{[(\bar{u}_4 u_2)_{S=1};\, c_1]_{S=\frac{1}{2}};\, (\bar{d}_1 d_1)_{S=0};\, c_1|_{S=\frac{1}{2}} - 2([\bar{s}_1 s_3)_{S=1};\, c_1]_{S=\frac{1}{2}}\}\,,$$

$$|\Sigma^+\rangle = [(c_1\bar{d}_2)_{S=0};\, u_4]_{S=\frac{1}{2}}\,,$$

$$|\Sigma^0\rangle \;= \frac{1}{\sqrt{2}}\{[(c_1 u_2)_{S=1};\, \bar{u}_4]_{S=\frac{1}{2}} - |(c_1 d_3)_{S=1};\, \bar{d}_3|_{S=\frac{1}{2}}\}\,,$$

$$|\Sigma^-\rangle = [(c_1\bar{u}_4)_{S=1};\, d_2]_{S=\frac{1}{2}}\,,$$

$$|\Xi^0\rangle = [(u_4\bar{s}_1)_{S=1};\, c_2]_{S=\frac{1}{2}}\,,$$

$$|\Xi^-\rangle = [(d_3\bar{s})_{S=1};\, c_2]_{S=\frac{1}{2}}\,.$$

Table III now sumarizes all results.

One sees immediately that our magnetic moments fall well inside experimental error bars. Moreover, the SU_3 sum rule $\sum \mu = 0$ is pretty well satisfied by experimental values ($\sum \mu = -0.03$) and by our model ($\sum \mu = -0.33$), but not at all by the fractional-charge model.

We conclude the present discussion with the following remarks. The first: that in our scheme there is no need to introduce parastatistics, or to assume various coloured quark multiplets, which (in our opinion) unduly multiply the number of basic objects. The second: that only a further work on the resonance spectra can lead to disentangling the two possible types of resonances in the framework of our model, namely: 1) the appearance of $q\bar{q}$ excited pairs (which probably explain part of the boson mass intervals observed between fermionic levels) and: 2) that of possible excited internal orbital motions, which could explain another part of the resonance tower.

The third: that the use, at least for the lower states, of the simplest possible potential, *i.e.* the nonrelativistic harmonic oscillator, can be nothing else but an approximation, justified by results. Our model can and should be extended and complexified.

The fact, however, remains: that for the first time (to our knowledge) the integer-charged-quark model of Salam *et al.*, coupled with a « naive » nuclear model of elementary particles, brings the theoretical predictions *within* the observed error bars of the magnetic moments. This justifies further efforts into this unorthodox line of research.

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