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Action-at-a-Distance and Causality in the Stochastic Interpretation of Quantum Mechanics.

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Since the results of the first stage of Aspect's experiment (without the switches) (1) have 1) strongly confirmed quantum-mechanical predictions, 2) disproved Bell's inequalities (and thus eliminated the possibility of local hidden variables), it is now probable that the second stage (with switches) will also confirm the existence of a nonlocal correlation between distant polarizers (practically separated by 12 meters) measuring the relative polarization of photon pairs emitted in the calcium $4p^2\,{}^1S_0$-$4s4p\,{}^1P_1$-$4s^2\,{}^1S_0$ transitions. It is thus to be expected that the same result will appear in any concrete realization of the EPR gedanken-experiment (2) proposed by BARTELL (3) to test (with Furry microscopes) the initial version of the EPR paradox on correlated p^μ and q^μ values in pairs of noninteracting scalar particles.

In brief we are now confronted with the very probable result that the forthcoming experiment of Aspect and Rapisarda et al. (1) *will confirm in the near future the nonlocality predicted by quantum mechanics.*

This is an important event, since many people still believe in the antagonistic character of nonlocality and causality. This is understandable, since one knows that

(1) A. ASPECT, PH. GRANGIER and Q. ROGER: *Experimental test of realistic local theories via Bell's theorem*, Orsay preprint (1981); A. ASPECT: *Phys. Lett. A*, **67**, 117 (1975); *Prog. Sci. Cult.*, **1**, 439 (1976); *Phys. Rev. D*, **14**, 1944 (1978).
(2) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).
(3) L. S. BARTELL: *Phys. Rev. D*, **22**, 1352 (1980).

unless one imposes particular restrictions on possible superluminal interactions one faces causal paradoxes tied with possible retroaction in time [4].

The aim of the present letter is to discuss the relation of these two concepts in the particular case of two identical noninteracting quantum particles in order to interpret causally the corresponding EPR situation analysed (in the nonrelativistic limit) by BOHM and HILEY [5].

We first define what we mean with the word «causality» by three properties:

a) the system of our two particles can be solved in the forward (or backward) time direction in the sense of the Cauchy problem;

b) the paths of all material particles must be timelike;

c) the formalism must be invariant under the Poincaré group $P = T \otimes \mathscr{L}\uparrow$.

As shown by one of us (PDV) [6], one can have action at a distance between two identical particles and preserve causality in the following case: we start with the two free Hamiltonians $H_{01} = p_1^2/2 = m_1^2/2$, and $H = p_2^2/2 = m_2^2/2$, completed with additive interaction terms V_1 and V_2 which are nonlocal potentials (*). Note that

I) we shall call V_1, V_2 potentials for convenience, although they have not the dimensions of energy: they must have the dimensions of squared masses;

II) the Hamiltonians are not directly related with energy but related with half the squared masses.

We thus get

$$H_1 = H_{01} + V_1 \,, \qquad H_2 = H_{02} + V_2 \tag{1}$$

now defined in the sixteen-dimensional phase space $q_1^\mu, q_2^\mu, p_1^\mu, p_2^\mu$. One sees immediately that the potentials cannot be chosen arbitrarly, since the existence of world-lines requires for identical particles, the vanishing of Poisson's brackets $\{H_1, H_2\}$ [6]. The phase space has 16 dimensions and the standard brackets are assumed among q_1^μ, q_2^μ and unconstrained p_1^μ, p_2^μ.

We now perform the following separation of internal and external variables:

$$\begin{cases} P^\mu = p_1^\mu + p_2^\mu \,, & y^\mu = \frac{1}{2}(p_1^\mu - p_2^\mu) \,, \\ Q^\mu = \frac{1}{2}(q_1^\mu + q_2^\mu) \,, & z^\mu = q_1^\mu - q_2^\mu \,, \end{cases} \tag{2}$$

so that, in the case that $V_1 = V_2 = V$, and $m_1 = m_2 = m$, we have

$$\begin{cases} H_1 + H_2 = 4P^2 + y^2 + 2V \,, \\ H_1 - H_2 = yP \,. \end{cases} \tag{3}$$

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[4] C. MØLLER: *The Theory of Relativity* (Oxford, 1962), p. 52.

[5] D. BOHM and B. HILEY: in *Quantum Mechanics a Half Century Later*, edited by J. LEITE-LOPES and M. PATY (1975).

[6] PH. DROZ-VINCENT: *Ann. Inst. Henry Poincaré*, **27**, 407 (1977); *Phys. Rev. D*, **19**, 702 (1979) and references quoted therein.

(*) Relativistic action-at-a-distance dynamics is generally nonlocal. However, it has been exhaustively shown that also conventional local field theories (electromagnetism and gravitation) can be cast into this scheme: L. BEL, A. SANS and J. M. SANCHEZ: *Phys. Rev. D*, **7**, 1099 (1973); L. BEL and J. MARTIN: *Phys. Rev.*, **8**, 4347 (1973).

The condition for the existence of causal timelike world-lines then reduces to the relation

(4) $$\{yP, V\} = 0 .$$

If we define the projector $\Pi^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - P^{\mu}P_{\nu}/P^2$ and $\tilde{z}^{\mu} = \Pi^{\mu}{}_{\nu}z^{\nu}$, $\tilde{y}^{\mu} = \Pi^{\mu}{}_{\nu}y^{\nu}$, relation (4) implies that V depends on $\tilde{z}^2$, P^2, $\tilde{y}^2$, $\tilde{z}\tilde{y}$, yP, but does not depend on zP which is, in the rest frame of the system, the relative time co-ordinate up to a factor $|P|$. Moreover, one finds $\{P^{\mu}, H_1\} = \{P^{\mu}, H_2\} = 0$, so that the centre-of-mass momentum P^{μ} is constant and one can slice space-time with 3 planes orthogonal to P^{μ} and connect the two particles by spacelike lines in these hyperplanes.

We now come to the description of two quantum noninteracting particles. For a system of two classical relativistic particles interacting at distance the evolution, in our multitemporal formalism (6), is described by two parameters: τ_1, τ_2, *i.e.* the proper times of the two particles. The movement is generated in the phase space $T(M_4) \times T(M_4)$ in a symplectic way by the covariant Hamiltonians H_1 and H_2 analysed in the first part of this letter. Of course we can build the canonical transformation theory in this covariant framework (7). The transformation which solves the motion equation is generated by Jacobi's principal function S, but it is simpler to consider the covariant Hamiltonian-Jacobi characteristic function $W = S - (m^2/2)(\tau_1 + \tau_2)$ which is determined by the Hamiltonian-Jacobi *system*:

(5) $$H_1\left(q_1^{\mu}, q_2^{\mu}; \frac{\partial W}{\partial q_1^{\mu}}, \frac{\partial W}{\partial q_2^{\mu}}\right) = \frac{m^2}{2}, \qquad H_2\left(q_1^{\mu}, q_2^{\mu}; \frac{\partial W}{\partial q_1^{\mu}}, \frac{\partial W}{\partial q_2^{\mu}}\right) = \frac{m^2}{2}.$$

One remarks here (in accordance with the well-known no-interaction theorem (8) that the canonical variables q_1^{μ}, q_2^{μ} are not coincident with the positions x_1^{μ}, x_2^{μ} except when the interaction vanishes.

By straightforward quantization of this multitemporal canonical formalism we obtain, for a system of two *free particles*, the Klein-Gordon system (for $\hbar = c = 1$)

(6) $$-\Box_1\psi(x_1, x_2) = m^2\psi(x_1, x_2) \quad \text{and} \quad -\Box_2\psi(x_1, x_2) = m^2\psi(x_1, x_2) ,$$

where ψ is a two-point–dependent function. Out of (6) we can extract the usual main equation

(7) $$(\Box_1 + \Box_2)\psi(x_1, x_2) = 2m^2\psi(x_1, x_2)$$

completed by the so-called « subsidiary » condition:

(8) $$(\Box_1 - \Box_2)\psi = 0 .$$

We now introduce in a relativistic way the concept of quantum potential (9). Following the original de Broglie's method, we set $\psi = \exp[R + iW]$, where R, W are

(7) D. Hirondel: Thesis, Paris (1977).

(8) D. G. Currie: *J. Math. Phys.* (*N. Y.*), **4**, 1470 (1963); *Phys. Rev.*, **142**, 817 (1966).

(9) L. de Broglie: *Une interprétation causale et non linéaire de la mécanique ondulatoire* (Paris, 1972). D. Bohm and J. P. Vigier: *Phys. Rev.*, **96**, 208 (1954); **109**, 1882 (1958).

real functions. Separating eq. (7) into the real and the imaginary part, we get for the real part

$$\text{(9)}\qquad \begin{cases} \frac{1}{2}(\partial_{1\mu} W \partial_1^\mu W) + U_1 = \frac{1}{2} m^2 \,, \\ \frac{1}{2}(\partial_{2\mu} W \partial_2^\mu W) + U_2 = \frac{1}{2} m^2 \,, \end{cases}$$

where we have

$$\text{(10)}\qquad \begin{cases} U_1 = -\frac{1}{2}(\Box_1 R + \partial_1^\mu R \partial_{1\mu} R) \,, \\ U_2 = -\frac{1}{2}(\Box_2 R + \partial_2^\mu R \partial_{2\mu} R) \,. \end{cases}$$

In spite of an obvious analogy (*) the system (9) cannot be immediately identified with eq. (5). To be more specific, we will consider the case of a ψ eigenstate of $P^\mu = i(\partial_1^\mu + \partial_2^\mu)$:

$$\psi = \exp\left[i\left(K_\mu \frac{x_1^\mu + x_2^\mu}{2}\right)\right] \varphi(z_\mu) \,;$$

where K_μ is a constant timelike vector: so we have

$$\text{(11)}\qquad (\partial_1^\mu + \partial_2^\mu) R = 0 \,, \qquad (\partial_1^\mu + \partial_2^\mu) W = K_\mu \,.$$

Moreover, since the difference of eq. (9) gives

$$\text{(12)}\qquad K^\mu \frac{\partial}{\partial z^\mu} R = 0 \,,$$

we see that R only depends on $z^\mu = x_1^\mu - x_2^\mu$ and more precisely only through its spatial part with respect to K^μ, namely $z_\perp^\mu = z^\mu - (z_\nu K^\nu) K^\mu / K^2$. In this case from (11) we have $U_1 = U_2 = U = f(z_\perp^\mu)$. But, as seen before, U has not a suitable expression because it depends only on $z_\perp$ and it cannot satisfy the condition $\{yP, V\} = 0$. In fact this process gives U as a function of z^μ and K^μ and not of z^μ, P^μ.

Making the substitution

$$\text{(13)}\qquad z_\perp^\mu \to \tilde{z}^\mu$$

in U, we get finally $V = f(\tilde{z})$ which depends on P^μ in a correct way, so that we can interpret it as a relativistic potential. Equations

$$\text{(14)}\qquad \frac{1}{2}(\partial_{1\mu} W \partial_1^\mu W) + V(\tilde{z}) = \frac{m^2}{2} \,, \qquad \frac{1}{2}(\partial_{2\mu} W \partial_2^\mu W) + V(\tilde{z}) = \frac{m^2}{2} \,,$$

(*) Of course eqs. (5) are written in terms of q_1^μ, q_2^μ, while eq. (9) involves x_1^μ, x_2^μ; but for the original *free* system the position variables are canonical, so that we can write without problems $q_1^\mu = x_1^\mu$, $q_2^\mu = x_2^\mu$ which makes the analogy between (5) and (9) manifest.

are now coincident with (5) if $q^\mu = x^\mu$, *i.e.*

$$H_1 = \frac{P_1^2}{2} + V\,, \qquad H_2 = \frac{P_2^2}{2} + V\,. \tag{15}$$

We remark here that the variables $x_1^\mu = q_1^\mu$ and $x_2^\mu = q_2^\mu$ are canonical for the free quantum system as well as for the classical interacting system. Moreover, they are also position variables for the quantum free system, but they do not represent the positions in the classical interacting system except in the particular rest frame system where we recover the Hamilton-Jacobi equations for a classical system in interaction through the potential V.

At this stage of our work, as was the case for the old de Broglie's derivation, we have only exhibited a mathematical analogy between a system of two quantum free particles and a system of « fictitious », but causally interacting particles. We are going now to recall and summarize the physical interpretation of this fictitious system (in the framework of the stochastic interpretation of quantum mechanics) in two points.

A) We can give a physical basis to our quantum potential only if we consider the ψ-field of a quantum particle not as a pure mathematical tool but as a real wave field on a subquantal medium ([10]). Indeed it is well known, since Dirac's pioneer work ([11]), that Einstein's relativity theory (and Michelson's experiment) are perfectly compatible with an underlying relativistic stochastic aether model, so that quantum statistic will reflect the real random fluctuations of a particle embedded in this aether ([12]). More precisely the quantum potential, introduced at the beginning of this paper on the basis of a pure formal analogy, is now interpreted as a real interaction among the particles and the subquantal fluid polarized by the presence of the particles ([13]). The quantum potential now represents a true stochastic potential. In this sense we can also understand how, starting from classical free particles, we have obtained, through quantization, two classical interacting (at a distance) particles. In fact the quantization procedure, which brought (5) into (6), is equivalent, in our aether interpretation, to add to our original free system (described by eq. (5)) the action of the subquantal medium so that finally the « free » quantum system (6) is equivalent to a system of classical interacting (via Dirac's aether) particles described by (14).

B) One has shown that the existence of the quantum aether allows one to deduce ([12]) the relativistic quantum equations for single free particles ([12,14]) and for systems of two particles ([13]) as describing the stochastic motion of classical particles in interaction with the aether, if the random jumps are made at the velocity of light ([12]).

We conclude with the remark that the causality implied in our model is absolute in the sense that the measuring processes themselves (and the observers) satisfy the same causal laws and are real physical processes with antecedents in time. The measuring process (observer plus apparatus plus observed particles) is a set of particles which are part of an overall causal process. In this scheme the intervention of a measuring process contains no supranatural « free will » or « observer consciousness », since quantum

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([10]) J. P. VIGIER: *Lett. Nuovo Cimento*, **29**, 467 (1980).
([11]) P. A. M. DIRAC: *Nature (London)*, **168**, 906 (1951).
([12]) W. LEHR and J. PARK: *J. Math. Phys. (N. Y.)*, **18**, 1235 (1977); F. GUERRA and P. RUGGIERO: *Lett. Nuovo Cimento*, **23**, 529 (1978); J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 265 (1979).
([13]) N. CUFARO PETRONI and J. P. VIGIER: *Lett. Nuovo Cimento*, **26**, 149 (1979).
([14]) N. CUFARO PETRONI and J. P. VIGIER: *Phys. Lett. A*, **73**, 289 (1979); **81**, 12 (1981).

measuring devices act as spectral analysers which split into subpackets the real de Broglie's waves associated with particles (which behave as planes flying at Mach 1 within their own sound waves): the particle entering into one of them according to its random causal motion (15). In that scheme there is no « free will » signal production and thus no possible causal paradoxes (12): nothing exists beyond the motion and interactions of material particles in a random stochastic aether.

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(15) M. CINI, M. DE MARIA, G. MATTIOLI and F. NICOLO: *Found. Phys.*, **9**, 479 (1979).