STOCHASTIC MODEL FOR THE MOTION OF CORRELATED PHOTON PAIRS

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If one analyzes the stochastic behaviour of two massive ($m_\gamma \neq 0$) photons imbedded in Dirac's vacuum one obtains (with stochastic jumps at velocity of light) the two-particle Proca equations which one can use to interpret (i) the first results of the Aspect experiment; (ii) the future issues of the complete Aspect and Rapisarda experiments, if they will violate Bell's inequality.

The aim of the present letter is to extend to the two-photon case the relativistic stochastic treatment used by Cufaro Petroni and Vigier [1] and Namsrai [2] in the two-scalar-particle case. As one knows this treatment rests upon

(i) the generalization to an eight-dimensional configuration space of the stochastic formalism given by Guerra and Ruggiero [3];

(ii) a perfectly causal action-at-a-distance description of the relativistic stochastic quantum potential which appears in the said formalism [4,5]. This is necessary, if one wants to establish a realistic causal explanation [6] of quantum nonseparability. No experiments have yet been performed to test whether the latter is a real phenomenon, but a series of related experiments, culminating in the recent one of Aspect et al. [7], suggests to us that, in this as in all previous tests, the predictions of quantum theory will be confirmed, and, in particular, that they will be confirmed in the photon-correlation experiments proposed by Aspect [8] and Rapisarda [9]. Of course, if the experiments, instead of confirming the phenomenon of quantum nonseparability, should turn out to give a result consistent with Bell's inequality, then it would be necessary to examine "modifications" of quantum theory rather than new "interpretations", because of the failure of a key prediction of that theory.

Our demonstration is based upon the introduction of an eight-dimensional configuration space where a pair position is defined by an eight component vector X^i ($i = 1, \ldots, 8$) where:

$$\{X^i\}_{i=1,\ldots,8} \equiv \{x_1^\mu; x_2^\nu\}_{\mu,\nu=0,\ldots,3}\,, \tag{1}$$

with x_1^μ, x_2^ν four-vectors of the position of each body [1,2], so that

$$X^2 = X_i X^i = (x_1)^2 + (x_2)^2\,. \tag{2}$$

If $x_1^\mu(\tau_1)$, $x_2^\nu(\tau_2)$ are the trajectories for the two photons, the trajectory in the configuration space will be $X^i(\tau_1, \tau_2)$. As a consequence of Nelson's equations [10] we can now generalize the differential operators defined by Guerra and Ruggiero [3] (which we have already used to demonstrate Proca's and Dirac's equations for isolated particles [11]) to a system of two photons. We thus write:

$$D = \partial/\partial\tau_1 + \partial/\partial\tau_2 + b_i\partial^i\,,$$

$$\delta D = -(2m_\gamma)^{-1}\Box + \delta b^i \partial_i\,, \quad \partial^i = \partial/\partial X_i\,,$$

$$\Box = \partial_i\partial^i = \Box_1 + \Box_2\,, \quad b_i = DX_i\,, \quad \delta b_i = \delta D X_i \tag{3}$$

(with $\hbar = c = 1$).

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We now suppose (as we have done for the Feynman and Gell-Mann equation [11]) that the drift and stochastic velocities b_i and δb_i can be derived from:

$$b^i = \partial^i\Phi/m_\gamma ,$$

$$\delta b^i = -\partial^i \ln \rho^2/2m_\gamma = -m_\gamma^{-1}(\partial^i\rho)/\rho , \tag{4}$$

where $\Phi(X^i, \tau_1, \tau_2)$ and $\rho(X^i)$ are real tensors with two indices, which can be represented by 4 × 4 matrices, the indices μ, ν taking the values 0, ..., 3. We also require that ρ is nonsingular, that ρ^2 is a trace-invariant matrix (which plays the role of a density matrix), and finally that Φ and ρ are commuting matrices.

The classical covariant stochastic equation and the continuity equations for our photon pair are

$$(DD - \delta D\delta D)X^i = 0 ,$$

$$\partial\rho/\partial\tau_1 = -\partial_{1\mu}(\rho b_1^\mu) , \quad \partial\rho/\partial\tau_2 = -\partial_{2\nu}(\rho b_2^\nu) . \tag{5}$$

Of course these equations are two-indices (4 × 4) tensor equations. In our model, as a generalization of the assumption [3] that ρ is independent of the proper time in the one-body case, we make the physical hypothesis that the total number of particles is conserved, and thus we write

$$\partial\rho/\partial\tau_1 + \partial\rho/\partial\tau_2 = 0 , \tag{6}$$

so that our continuity equation in the configuration space is

$$\partial_i(b^i\rho) = 0 . \tag{7}$$

From eqs. (3) and (4) we have for the stochastic covariant equation (5)

$$\partial_i\left(\frac{1}{2m_\gamma}\frac{\Box\rho}{\rho} - \frac{\partial\Phi}{\partial\tau_1} - \frac{\partial\Phi}{\partial\tau_2} - \frac{1}{2m_\gamma}\partial_j\Phi\partial^j\Phi\right) = 0 , \tag{8}$$

which shows that the term in the parentheses is constant. If we pose this constant equal to zero, by rearranging the energy scale, eq. (8) takes the following form

$$\left(\Box - 2m_\gamma\frac{\partial\Phi}{\partial\tau_1} - 2m_\gamma\frac{\partial\Phi}{\partial\tau_2} - \partial_j\Phi\partial^j\Phi\right)\rho = 0 . \tag{9}$$

Similarly for the continuity equation (5) we have from eqs. (3) and (4)

$$(2\partial_j\Phi\partial^j + \Box\Phi)\rho = 0 . \tag{10}$$

If we now make a linear combination of eqs. (9) and (10) with coefficient 1 and i, we have after multiplication by $\exp(i\phi)$:

$$\left[\left(\Box + 2i\partial^j\Phi\partial_j + i\Box\Phi - \partial_j\Phi\partial^j\Phi - 2m_\gamma\frac{\partial\Phi}{\partial\tau_1} - 2m_\gamma\frac{\partial\Phi}{\partial\tau_2}\right)\rho\right]\exp(i\Phi) = 0 , \tag{11}$$

which takes the following synthetic form

$$\left[\Box + 2im_\gamma\frac{\partial}{\partial\tau_1} + 2im_\gamma\frac{\partial}{\partial\tau_2}\right]\rho\exp(i\Phi) = 0 . \tag{12}$$

With the notations

$$\Phi(X^j;\tau_1,\tau_2) = S(X^j) - m_\gamma(\tau_1+\tau_2) ;$$

$$A = \rho\exp(i\Phi) , \tag{13}$$

we have finally (A is a two-indices tensor)

$$[\Box + 2m_\gamma^2]A = 0 . \tag{14}$$

This is the wave equation that generalizes Proca's equation to correlated photon pairs represented (as suggested by Landau and Lifshitz [12]) by a tensor field formalism. As Landau and Lifshitz have shown, the singlet corresponds to the antisymmetric part of the $A_{\mu\nu}$ field, the triplet is described by the symmetric component.

We conclude with two remarks. The first is that the preceding calculations represent real physical collective motions on the top of a covariant stochastic "vacuum" (i.e. Dirac's aether [13]). The pair's motion along correlated world lines in real space–time are represented by single world lines in configuration space. The nature of their correlations results from the model since the presence of the particle 1 disturbs the aether i.e. the motion of the particle 2 and vice versa. Physically this amounts (in the hydrodynamical model of Bohm and Vigier [14]) to the superposition in the space–time of two interacting fluids which undergo lightlike internal stochastic motions, particle–antiparticle transitions and possible number-preserving transfers from one fluid to another ... so that we have a conserved tensor fluid particle density in configuration space.

The second remark is that eq. (14) extends the quantum potential action-at-a-distance [5] to spin–spin correlations as will be shown in a subsequent letter. These actions-at-a-distance are carried by intervening stochastic extended vacuum particles which transmits nonlocal interactions in their interior [13].

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