CAUSAL STOCHASTIC INTERPRETATION OF FERMI–DIRAC STATISTICS IN TERMS OF DISTINGUISHABLE NON-LOCALLY CORRELATED PARTICLES

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Like in the Bose–Einstein case, Fermi–Dirac statistics are shown to correspond to subquantal, causal, space–time behaviour of distinguishable particles correlated by actions-at-a-distance. This justifies the introduction of the non-local hidden parameters introduced by Bohm and Vigier in the stochastic interpretation of quantum mechanics.

The aim of this letter is to generalize to the case of Fermi–Dirac (FD) statistics the demonstration already given by Tersoff and Bayer [3] and Kyprianidis et al. [4] for Bose–Einstein (BE) statistics, i.e., to show that FD statistics can also be interpreted in terms of real subquantal motions of distinguishable particles correlated by the non-local actions-at-a-distance corresponding to a many-body quantum potential [‡1].

This of course (as in the BE case) implies a reevaluation of the usual assumptions about probabilities. Starting with N particles distributed among M discrete states instead of assuming that the particles will occupy each available state with an equal probability weighting $w_i = 1/M$ one introduces a random probability weight w_i of the set of occupancies $\{n_i\}$ (which is simply proportional to the distinct configurations corresponding to $\{n_i\}$) satisfying the restriction $0 \leqslant w_i \leqslant 1$ with

$$\sum_{i=1}^{M} w_i = 1 .$$

The BE statistical distribution then immediately results [3] from taking the average over all possible w_i without any assumption on undistinguishability. This astonishing result can be immediately justified [4] by assuming, following an assumption of Einstein himself [5] that one is not dealing with independent particles [between local stochastic collisions like in Maxwell–Boltzmann (MB) statistics] but that BE statistics "expresses indirectly a certain hypothesis of a mutual influence, which, for the moment, is of a quite mysterious nature". Of course Einstein himself never developed this suggestion for an obvious reason. For him these "mysterious influences" could not re-

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[‡1] This justifies the introduction of the non-local hidden parameters introduced by Bohm [1] and Vigier [2] in the stochastic interpretation of quantum mechanics.

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flect any form of local interaction since these could only lead to MB statistics [6]. In his opinion, they could not be non-local either since he believed they would then destroy causality. He did not know then that under certain conditions (satisfied by the many-body quantum potential [7]) actions-at-a-distance do indeed satisfy his own definition of relativistic causality.

To recover FD statistics with distinguishable particles one must evidently:

(a) Assume that $n_i = 0$ or 1. This can be either assumed a priori (as an ad hoc starting point) or be justified within a reasonable physical model. For example one can either assume directly that half-integer FD particles with parallel spins strongly repell when brought together at any given point, or suppose that this property results from repulsive local gauge fields between identical (spin included) particles.

As a possible model of such a justification the authors propose to utilize the Pati–Salam basic ground state spin 1/2 particles. This classification can be deduced as the ground states of a relativistic oscillator model invariant under the internal $U(1) \times SO(6,2)$ group [8]. Their e.m. charge Q satisfies the relation

$$Q = T_3 + (Y/2) - (2Z/3) - (H_4/2),$$

which implies that the corresponding Yang–Mills gauge vectors (with or without mass) induce repulsions of identical particles with parallel spins.

(b) Assume that a set of fermions, exactly like bosons, are also correlated by causal many-body non-local quantum potentials and quantum torques [9]. We leave the detailed justification of this to a forthcoming publication and limit ourselves here to the statement that the existence of such potentials for isolated particles has already been deduced by two of us (C.-P. and J.P.V.) from the second-order Feynman–Gell-Mann form [10,11] of Dirac's linear equation. In our model the many-body forces resulting from the quantum potential thus represent a concrete model of Einstein's "mysterious influences". Of course if our N particles are not independent one should in a random process attribute arbitrary weights w_i to each of our M cells associated with the distribution

$$P\{n_i\} = N!/(n_1! \dots n_M!)(w_1)^{n_1} \dots (w_M)^{n_M}, \qquad (1)$$

and perform an average over w_i with the restriction $0 \leqslant w_i \leqslant 1$ and $\Sigma_{i=1}^{M} w_i = 1$.

In general this yields the usual BE result

$$P_{\mathrm{BE}}\{n_i\} = N!(M-1)!/(N+M-1)! . \qquad (2)$$

Since $P_{\mathrm{BE}}\{n_i\}$ is independent of $\{n_i\}$ then due to the equiprobability of each configuration one can write $P_{\mathrm{BE}}\{n_i\} = P^*_{\mathrm{FD}}\{n_i\}$. We have written $P^*_{\mathrm{FD}}\{n_i\}$ instead of $P_{\mathrm{FD}}\{n_i\}$ since any set of states with $n_i > 1$ is not occupied due to our assumption (a). Our $P^*_{\mathrm{FD}}\{n_i\}$ thus only represent a relative frequency of FD states *because out of the total number of available states i.e.*

$$N_{\mathrm{tot}} = (N+M-1)!/N!(M-1)! ,$$

all states with $n_i > 1$ have to be excluded.

The exact number of such rejected states can now be calculated. There are $\binom{N-1}{k-1}\binom{M}{k}$ such states with k cells occupied, which yield a total number of states with $n_i > 1$ equal to

$$\mathcal{N}_{n_i>1} = \sum_{k=1}^{N-1} \binom{N-1}{k-1}\binom{M}{k}. \qquad (3)$$

With the help of the well-known mathematical identity [12]

$$\sum_{k=0}^{p} \binom{n}{k}\binom{m}{p-k} = \binom{n+m}{p} \qquad (4)$$

one immediately obtains the relation

$$\mathcal{N}_{n_i>1} = \sum_{k=1}^{N} \binom{N-1}{k-1}\binom{M}{k} - \binom{M}{N}$$

$$= \frac{(M+N-1)!}{N!(M-1)!} - \frac{M!}{N!(M-N)!}, \qquad (5)$$

from which one deduces the total sum of the relative frequencies of the FD states to be

$$(P^*_{\mathrm{FD}})_{\mathrm{tot}} = 1 - \mathcal{N}_{n_i>1} P^*_{\mathrm{FD}} = \frac{M!(M-1)!}{(N+M-1)!\,(M-N)!}. \qquad (6)$$

The true probability of states in the Fermi–Dirac case can now be calculated by renormalizing the relative frequency of states P^*_{FD} with respect to $(P^*_{\mathrm{FD}})_{\mathrm{tot}}$. This finally yields:

$$P_{\mathrm{FD}}\{n_i\} = P^*_{\mathrm{FD}}/(P^*_{\mathrm{FD}})_{\mathrm{tot}} = \frac{N!(M-1)!}{(N+M-1)!}$$

$$\times \left(\frac{M!(M-1)!}{(N+M-1)!(M-N)!}\right)^{-1} = \frac{N!(M-N)!}{M!}, \qquad (7)$$

which is exactly the Fermi–Dirac prediction.

This extension of the Tersoff–Bayer BE result to the FD case shows that there exists a unique fundamental qualitative difference between classical and quantum statistics. To paraphrase a famous sentence "God does not play dices. In classical statistics he plays with independent dices determined by local hidden parameters. In quantum statistics he plays with correlated dices permanently tied by the actions-at-a-distance causally determined by non-local hidden parameters".

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