Quantum Uncertainties

Recent and Future Experiments and Interpretations

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TOWARD A CAUSAL INTERPRETATION OF THE RELATIVISTIC QUANTUM MECHANICS OF A SPINNING PARTICLE

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ABSTRACT

As a first step in the direction of a causal interpretation, we analyze the features of the second order wave equation for spin 12/ fields. It is shown that this equation allows a coherent statistical interpretation by means of a conserved density which is positive definite. On this basis we can also construct all the usual Hilbert space formalism of the relativistic quantum mechanics. The relations with the fields ruled by the first order Dirac equation are also discussed. Finally, the perspectives of the definition of a relativistic spin-dependent quantum potential, of the connection with stochastic processes and the extension to the case of two correlated particles are briefly discussed.

In this note we will briefly sketch an outline of a research[1], still in progress, that will bring us to a complete deterministic interpretation of the non local quantum interactions of the E.P.R. type [2], for the Bohm-Aharonov [3] case of spinning correlated particles. We hope so also on the basis of the fact that an analogous previous work on spinless relativistic particles gives encouraging results[4].

Our interest in the second-order relativistic wave equation for spin 1/2 particles is based on the following considerations:

a) it is , in some sense, astonishing the fact that spin 1/2 particles should be described by means of a first order equation, whereas the integer spin fields are usually ruled by second order equations, which are the most natural quantum analog of the relativistic energy-momentum relations;

b) if we are looking for a causal interpretation of the relativistic quantum equations, a classical analogy is more easy found starting from second order wave equations.

Here we want to sketch the main lines followed and the first results obtained in this research. By introducing the symbols

$$D_\mu = \frac{1}{mc}\,(i\hbar\partial_\mu - \frac{e}{c}\,A_\mu), \quad \not{D} = \gamma_\mu D^\mu$$

the second order equation we are talking about takes the from

$$(I - \not{D}^2) \quad \psi(x) = 0, \tag{1}$$

ore the ore familiar one

$$[(i\hbar\partial_\mu - \frac{e}{c} A_\mu)\ (i\hbar\partial^\mu - \frac{e}{c} A^\mu) - \frac{e\hbar}{2c} F_{\mu\nu}\sigma^{\mu\nu} - m^2c^2]\ \psi(x) = 0.$$

Along with it we will consider the two first order equations

$$(I - \not{D}) \quad \psi(x) = 0 \tag{2}$$

$$(I + \not{D}) \quad \psi(x) = 0 \tag{3}$$

We will denote by means of F, D_-, D_+ the set of spinors solution respectively of (1), (2), (3). Now we can easy prove the following propositions:

1. the only spinor common to D_+ and D_- is the identically zero spinor;
2. D_+ and D_- are vector subspaces of the vector space F ;
3. there is a one-to-one correspondence between D_+ and D_- realized by means of the matrix γ_5;
4. F is the direct sum D_+ and D_-.

Among others, from the property 3. it follows that, while eq.(2) is the usual Dirac equation, eq. (3) is satisfied by $\psi_+ = \gamma_5\psi_-$ if ψ_- is solution of (2): it is the well-known correspondence between a positron wave function and an electron wave function moving backward in space-time with the sign of energy inverted.

All the theory can be derived from the Lagrangian density

$$L = \overline{\not{D}\psi}\ \ \not{D}\psi \ - \ \bar{\psi}\psi \quad . \tag{4}$$

If we derive the usual current density from it

$$J_\mu(x) = 1/2\ (\overline{\not{D}\psi}\ \gamma_\mu\psi + \bar{\psi}\gamma_\mu\ \not{D}\psi) \tag{5}$$

we find that its zero component

$$J_o(x) = \mathrm{Re}\ (\psi^+\not{D}\psi) \tag{6}$$

is not positive definite. That is not an unexpected result: as everybody knows it is a feature common to all the second order wave equations like the Klein-Gordon equation. As a consequence, $J_o(x)$ can not be interpreted as a probability density and a coherent statistical interpretation of the formalism is forbidden on this basis. Moreover in a quantum theory we can always pose the following questions: Are the subspaces D_+ and D_- mutually orthogonal? Are the elements of F normalizable wave functions? How can we approximate the solution of a physical problem in F? To what extent we can reproduce in F the outcomes of the Dirac theory like, for example, the hydrogen atom spectrum? All these questions and others, along with that about the statistical interpretation, can find an answer only if we succeed in building a Hilbert space structure on F, based on a positive definite norm. Of course, non positive norms are conceivables, but how can we mantain the most usual results and what about their physical meaning?
The fundamental achievement of Dirac can be read, from this standpoint, as a restriction to the subspace D_- as the set of the physically acceptable states, where $J_\mu(x) = \bar{\psi}\gamma_\mu\psi$ and hence $J_o(x) = \psi^+\psi > 0$.

Despite the impressive importance of this Dirac position, we want to stress here two remarks:

A) the Dirac restriction is too much severe: it eliminates, as unphysical a lot of states with positive density;

B) the Dirac restriction is unnecessary: a solution of all problems can be found in the whole space F .

In order to prove A) Let us take the free case ($A_\mu = 0$), namely

$$(\Box + \frac{m^2c^2}{\hbar^2}) \quad \psi(x) = 0 \tag{7}$$

whose plane wave solutions are

$$\begin{aligned} \psi_p(x) &= N\, e^{ipx/\hbar} \quad u \\ p_\mu p^\mu &= m^2c^2; \qquad E = \pm\, mc^2 \sqrt{1 + (\frac{\vec{p}}{mc})^2} \end{aligned} \tag{8}$$

where u is an arbitrary four-spinor, constant in the space-time. For the free Dirac solutions there is a further restriction on u:

$$(\not{p} \pm mc)\ u = 0 \tag{9}$$

with the sign dependent on the energy sign. We can now find plane waves (8), solutions of (7), with positive conserved density, without obeying the condition (9). In fact, if we label the u with a parameter $\varepsilon = \pm 1$, so that

$$u_\varepsilon = \begin{bmatrix} H_\varepsilon(\alpha)\,\xi \\ H_{-\varepsilon}(\alpha)\,\eta \end{bmatrix}$$

$$H_\varepsilon(\alpha) = \frac{e^\alpha + \varepsilon e^{-\alpha}}{2} \quad , \ \alpha \in [0,\infty]$$

with ξ,η two spinors such that $\xi^+\xi = \eta^+\eta = 1$, we will have $\bar{u}_\varepsilon u_\varepsilon = \varepsilon = \pm 1$.
Now the current (5) becomes

$$J_\mu = \mathrm{Re}(\bar{\psi}\gamma_\mu \frac{i\hbar}{mc} \not{\partial}\psi) = \varepsilon N^2 \frac{P_\mu}{mc}$$

and hence

$$J_o = \frac{N^2}{mc^2} \ \varepsilon\, E.$$

Consequently the most general positivity conditions is $\varepsilon = \mathrm{sgn}(E)$ and the spinors

$$\psi_p(x) = N e^{ipx/\hbar} \ u_{\varepsilon\, =\, \mathrm{sgn}(E)}$$

always lead to positive J_o.
Hence, a solution at rest will be

$$\psi(x) = \begin{cases} N\, e^{-imc^2t/\hbar} \begin{bmatrix} \cosh\alpha\ \xi \\ \sinh\alpha\ \eta \end{bmatrix} & ;\ E = mc^2 \\ N\, e^{imc^2t/\hbar} \begin{bmatrix} \sinh\alpha\ \xi \\ \cosh\alpha\ \eta \end{bmatrix} & ;\ E = -\,mc^2 \end{cases}$$

In other words, only $\alpha = 0$ is possible, under the Dirac condition (9), that, hence, must be considered too much restrictive.

To prove the statement B) it is enough to show, by direct calculations, that besides the current (5), the vector

$$j_\mu(x) = 1/2\ (\bar{\psi}\ \gamma_\mu \psi + \overline{\not{D}\psi}\gamma_\mu\ \not{D}\psi) \tag{10}$$

is another conserved current density, whose zero component

$$j_o(x) = 1/2[\psi^+\psi + (\not{D}\psi)^+\ \not{D}\ \psi\ ]\ > 0 \tag{11}$$

is always positive. We did not find it at first because it is not derived from our initial Lagrangian density. Anyway, $j_\mu(x)$ allows one to define a coherent scalar product, and all the machinery of the Hilbert space connected to it, and, of course, a statistical interpretation.

More light can be cast on this breakthrough by considering what the word "state" means here. If we look for the state of a system at a given time x^o, we can not consider it as completely determined by the knowledge of $\psi|_{x^o}$, because we are dealing with a second order equation, and hence $\psi|_{x^o}$ is not enough to determine $\psi(x)$ in all the space-time. In fact, we need here two initial conditions: $\psi|_{x^o}$ and $\partial_o\psi|_{x^o}$, or, in a covariant form

$$\psi\Big|_{x^o} = \phi_1(\vec{r})\ ,\quad \not{D}\psi\Big|_{x^o} = \phi_2(\vec{r}),$$

where ϕ_1, ϕ_2 are two arbitrary four-spinors, which, along with eq.(1), completely determine the spinor field $\psi(x)$ in all the space-time. Hence the vector space of the states at a given time x^o is the space of the eight-component double-spinors

$$\Psi\Big|_{x^o} = 1/\sqrt{2}\ \begin{Bmatrix}\phi_1(\vec{r})\\ \phi_2(\vec{r})\end{Bmatrix}$$

and the complete space-time dependence of such a double spinor is

$$\Psi(x) = 1/\sqrt{2}\ \begin{Bmatrix}\psi(x)\\ \not{D}\psi(x)\end{Bmatrix} \tag{12}$$

where $\psi(x)$ is are element of F . if now H is the vector space of these Ψ (x), we can show that it coincides with the space of the doble spinors solutions of

$$(C_\mu\ D^\mu - C)\quad \Psi(x) = 0 \tag{13}$$

where

$$C_\mu = \begin{bmatrix}\gamma_\mu & 0\\ 0 & \gamma_\mu\end{bmatrix}\ ,\qquad C = \begin{bmatrix}0 & I\\ I & 0\end{bmatrix}$$

and that it is a Hilbert space whose scalar product is defined by means of the positive density $j_o(x)$:

$$<\Psi|\Phi> = 1/2\int d^3\vec{r}\ (\bar{\psi}\gamma_o\phi + \overline{\not{D}\psi}\ \gamma_o\not{D}\phi).$$

In other words (13) is perfectly equivalent to (1) and F is in a one to one correspondence with H . It is in H that we can build the operator algebra of the observables. As a consequence, in this framework, it can be shown that the energy spectrum of a quantum system obtained from (13) [or equivalently from (1)] is always the same as that derived from the Dirac equation (2); only the number of states for each eigenvalue is doubled, because now we must take solutions in D_- and D_+ .

As a conclusion we will sketch the further steps required for a deterministic interpretation of eq. (1). First of all let us point out that in the non relativistic, spinless case, the position $\psi = R\, e^{iS/\hbar}$ directly leads to a splitting of the Schrödinger equation into a continuity and a dynamical equation. The latter can be interpreted either as a generalized Hamilton-Jacobi equation or as a velocity potential equation for a Madelung fluid. For spinning particles only the second interpretation can be retained: in fact, for a classical spinning body, the Hamilton function S should depend on space-time coordinates and Euler angles. But a spinor $\psi(x)$ shows no dependence on angular variables at all. On the other hand, in fluidodynamics, a spinning fluid can be described by means of a number of fields, all of them functions of the space-time only, that can be accomodated into the spinor components. In this case a velocity field $V_\mu(x)$ will be described by means of a velocity potential s(x) plus a couple of Clebsch parameters $\chi(x)$, $\omega(x)$, in the following way

$$v_\mu(x) = \partial_\mu s(x) + \omega(x)\, \partial_\mu \chi(x) \; . \qquad (14)$$

Hence the forthcoming step will be the decomposition of the four-spinors $\psi(x)$ in terms of velocity potentials and Clebsch parameters, all connected with complex Euler angles. In fact, the complexification of these parameters rests on the basis of the well-known isomorfism between the group of the Lorentz transformation and that of the rotations in a there dimensional complex Euclidean space. In the representation where

$$\gamma_o = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \quad \gamma_k = \begin{bmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{bmatrix} \; , \quad \gamma_5 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

a tentative, but not yet final, decomposition is

$$\psi(x) = R(x)\, e^{is(x)/\hbar}\, \phi(x)$$

where R is the 4x4 matrix

$$R(x) = p(x) + iq(x)\, \gamma_5$$

S is a real velocity potential and

$$\phi(x) = 1/\sqrt{2} \begin{bmatrix} \cos u\, e^{iw} \\ i \sin u\, e^{-iw} \\ \cos u^*\, e^{iw^*} \\ i \sin u^*\, e^{-iw^*} \end{bmatrix}$$

is a spinor satisfying the relations

$$\bar{\phi}\phi = 1, \qquad \bar{\phi}\gamma_5\, \phi = 0.$$

That leads to forms like (14) for the velocity field. The final step is, of course, the derivation, from (1), of the equations for all the parameters involved, that would lead to the definition of a relativistic, spin-dependent quantum potential.

Furthermore, the generalization to the case of two particles of the eq. (1), would enable one to decide if the resulting non local quantum potential can satisfy the compatibility conditions of the relativistic predictive mechanics[6]. If it will be so, we could realistically hope to build up a coherent, deterministic, relativistic, non local theory of the quantum phenomena. Finally we will remark that it seems likely that our starting point of a second order wave equation will allow a coherent connection with subquantum stochastic processes, as previous works have already shown [7].

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