The Nature of Quantum Paradoxes

Italian Studies in the Foundations and Philosophy of Modern Physics

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CONTENTS

PERSPECTIVES OF PHYSICAL DETERMINISM

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ABSTRACT

A short review of the problems posed to the physical determinism by quantum mechanics and relativity is presented, along with the suggested ways to circumvent them.

1. INTRODUCTION

Generally speaking the so called "impossibility theorems" cannot be considered as absolute impossibility statements. In fact, it is trivial to remark that, if we look at the hypotheses on the ground of which these theorems are proved, they fix rather the price we must pay in order to get some result, than the fact that this result cannot be reached at all.

In that sense we think that the von Neumann theorem and the no-interaction theorem can be considered as impossibility theorems about the problem of the construction of a causal interpretation of the quantum mechanics. In fact, they were for a long time considered as a crucial obstacle on the way to a definition of a deterministic theory of quantum phenomena.

However, recent and less recent developments have shown that it was not completely impossible to get such a deterministic description. Of course, the theorems are correct, but it is possible, by looking at their hypotheses, to find a way to circumvent them. The aim of this paper is to sketch a possible path between the obstacles represented by the von Neumann and the no-interaction theorems toward the final result of a causal description of nature.

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2. VON NEUMANN'S THEOREM AND BEYOND IT

It is well known that the first difficulty for a causal interpretation of the quantum theory was posed by the existence of the uncertainty relations. They could be viewed as a consequence of the formal property of the Fourier transform characterizing the wave function of a microscopic system, and thus, in some sense, as an inherent behavior of the microscopic world, in which the observables are not always well defined (so that sometime they can lack their physical meaning). However, in another sense, the uncertainty principle could be considered as the effect only of the disturbance induced by a measurement of the observables of the physical system. Of course, in this case the observables always mantain their meaning, but the description of the system given by the wave function must be regarded, in some sense, as an incomplete one.

It is in this context that we can pose the problem of the existence of the "dispersion free" states, namely the states (or wave functions) for which all the observables have a well defined value without dispersion.

Regarding this question, it is well known that the von Neumann theorem(1) states the impossibility of the existence of dispersion free states in the framework of a theory reproducing the results of the quantum mechanics. We do not enter here into a discussion of this theorem, whose analysis can be found in an already vast and well-known literature. We, however, allow ourselves the remark that, despite the proof of von Neumann, dispersion free states were explicitely constructed in a causal model proposed by Bohm(2) which reproduces all the features and the results of non-relativistic quantum mechanics.

This causal theory, based on earlier works of Madelung and de Broglie,(3) considers the wave function as a physical field and, by means of a separation of the real and imaginary parts of the Schrödinger equation, furnishes a continuity equation and a gereralized Hamilton-Jacobi equation, describing the dynamics of a classical particle subjected to a field of force exerted by the wave function. For such a classical particle all the physical observables are well defined.

The field of force of ψ was represented by the socalled quantum potential defined as

$$U(\vec{r}) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\ ; \qquad R^2 = |\psi|^2\ . \tag{2.1}$$

But how was it possible to get around the von Neumann theorem?

We must remark that, among the hypotheses of this theorem, there is the statement, directly generalized from ordinary probability theory, that the expectation values of observables are linear, in the sense that

$$E(aA + bB) = aE(A) + bE(B) \tag{2.2}$$

for A, B observables and a, b real numbers. This statement is revealed as an excessively restrictive hypothesis which could be relaxed if one takes into account the existence in quantum mechanics of incompatible (noncommuting) observabes. In this case, following a remark of Bell,(4) we can immediately see that in the Bohm model, for example, the relation (2.2) is not always verified when A and B are incompatible observables.

Thus the Bohm model and its subsequent analysis indicated that a causal interpretation of quantum mechanics could not be considered absolutely impossible on the basis of the von Neumánn theorem. This important remark opened the way to a number of investigations on this argument: modifications, generalizations of the original model, stochastic interpretations, etc.

3. QUANTUM POTENTIAL AND CAUSAL ANOMALIES

However, if the road to building a causal interpretation of the quantum formalism appeared open from the standpoint of the von Neumann theorem, another important difficulty lurked in the idea of quantum potential itself.

If we consider two particles 1 and 2, described by a nonfactorizable ψ (1,2), and if we decompose the wave equation into its real and imaginary parts, we find that the two particles are subjected to quantum potentials respectively of the form

$$U_1(1,2) = -\frac{\hbar^2}{2\mu}\frac{\nabla_1^2 R}{R}, \quad U_2(1,2) = -\frac{\hbar^2}{2\mu}\frac{\nabla_2^2 R}{R}\ ; \quad R^2 = |\psi|^2 \ . \tag{3.1}$$

It is evident that these are two nonlocal potentials, as Bohm himself pointed out immediately[(2)], in the sense that the force acting on 1 (or on 2) instantaneously depends on the position of 2 (or of 1). Of course there was no problem in a nonrelativistic theory, but it is possible to carry out the same analysis on the relativistic Klein-Gordon equation; one obtains as relativistic nonlocal quantum potentials,

$$U_1(1,2) = -\frac{\hbar^2}{2\mu}\frac{\Box_1 R}{R}, \quad U_2(1,2) = -\frac{\hbar^2}{2\mu}\frac{\Box_2 R}{R} \ . \tag{3.2}$$

On the other hand, it is well known that the nonlocal potentials between directly interacting particles, such as (3.2), are generally viewed as carriers of instantaneous signals between the two particles and that this possibility introduces the socalled "causal anomalies." Classical examples of these (signals travelling backward in time, effects preceding causes, etc.) can be found in the standard literature on relativity theory.[(5)]

It thus seems that the idea of a quantum potential, introduced in order to get a coherent causal interpretation of quantum mechanics, directly implies the breakdown of causality in another sense. Indeed, the existence of signals (or particles) travelng faster than light is not in direct contradiction with the formalism of the theory of the relativity: a number of investigations in this field (and about tachyons) are based on this possibility. But the difficulty aries when, besides the relativity principle, we admit a suitable idea of causality. In this sense, the existence of nonlocal quantum potentials could be incompatible with a causal theory of the microscopic phenomena.

In fact, a suitable Lorentz transformation applied to a superluminal signal, could allow it to propagate backward in time, so that the succession of causes and effects will depend on the choice of the reference frame and, as a direct consequence, we could influence our past history.

First of all we must say that there exists a general opinion today that it is impossible to send macroscopic superluminal signals by means of quantum potentials or, for example with an EPR apparatus.[6] That will depend on the "uncontrollable" (macroscopic) character of this subquantum nonlocality, which manifests itself in the fact that we never can get any signal from an EPR apparatus if we look only at one polarizer. We must always consider coincidences of measurements in order to appreciate changes or signals in the apparatus. Accordingly, we must in any case wait for the knowledge of the results of both polarizers and never can receive a signal by looking at only one of them.

However, going beyond this preliminary remarks, we need here a deeper discussion about the idea of causality.[7] In some sense we must distinguish between two concepts of causality, depending on whether we are dealing with closed or non closed systems:

(a) The first concept is a development of the ideas of the classical mechanics: For a closed systems we speak of causal or predictive description when the knowledge of the initial conditions, at t_0, determines the evolution in time by means of the equations of motion (Cauchy problem).

(b) For nonclosed systems we speak of the propagation of a signal from one event (cause) to another (effect) by means of an interaction of the given system with an external perturbation: In this case "causality" deals with statements about an invariant succession of causes and effects such that anomalies can be avoided.

These two concepts are not equivalent, and the causality idea discussed in connection with the principle of relativity and the quantum potentials is of type (b). Hence, from the contradiction between (b) and the existence of superluminal signals, we cannot deduce anything about the relation between (a) and the nonlocal quantum interactions.

In this sense, the possibility of a theory of a direct nonlocal interaction, in the scheme of a relativistic predictive mechanics, is not influenced by the preceding analysis of the causal anomalies, if we clearly state that we always are dealing with closed dynamical systems.

As an example,[8] let us consider the case of two particles 1 and 2 interacting via a nonlocal potential. We can solve our dynamical equations and determine their world lines L_1 and L_2. Until now, no causal anomalies can arise. However, we can perturb L_1 so that the disturbance will be propagated instantaneously via the nonlocal interaction. Of course, in order to do this we must use something external to the system 1 + 2: for example a particle 3 acting on 1. As long as 3 remains an "external" agent with the ability "to be or not to be ," there is place for causal anomalies. But, if the world is a unique set of events in which the particle 3 is (and cannot "choose" anything else), we need only consider the enlarged closed system 1 + 2 + 3 in order to get a new perfectly causal system in a predictive sense, completely free of any causal anomaly.

4. THE NO-INTERACTION THEOREM ...

Following the analysis of the preceding section, we now consider the possibility of existence of a predictive theory of closed systems in the presence of a direct nonlocal interaction.

To this end we must start with a more clear discussion about the idea of covariance.[9] We can distinguish two meanings of this expression: The first is the socalled "manifest covariance" which deals with the use of a mathematical apparatus (tensor calculus) that exploits the spacetime symmetry. Another, nonequivalent meaning concerns the group of inhomogeneous Lorentz transformations. In this sense relativistic covariance requires that the linear space of the states of our physical system be a representation space for the Lorentz group. This approach was first developed for the relativistic quantum theories, where the ambiguity of the concept of the "position" of a particle in spacetime makes it difficult to directly use the scheme of manifest covariance. In fact, this idea of relativistic covariance is concerned only with the Lorentz group as expressing the relationship between physical phenomena as viewed by different observers. Accordingly, the fundamental demand is that the vector space for the states of our system (defined, for example, by means of a homogeneous linear differential equation) be the ground on which we can define a linear representation of the Lorentz group.

In this sense the problem of defining the dynamics of our system is that of identifying the ten fundamental infinitesimal generators of the group that satisfy the Lie bracket relationship characteristic of their algebra:

$$\begin{aligned}
&[P_i,P_j] = 0, \quad [P_i,H] = 0, \quad [J_i,H] = 0,\\
&[J_i,J_j] = \varepsilon_{ijk}J_k, \quad [J_i,P_j] = \varepsilon_{ijk}P_k,\\
&[J_i,K_j] = \varepsilon_{ijk}K_k, \quad [K_j,H] = P_j,\\
&[K_i,K_j] = -\varepsilon_{ijk}J_k, \quad [K_i,P_j] = P_j;\\
&\quad i,j,k = 1,2,3 \ .
\end{aligned} \tag{4.1}$$

This program, developed in relativistic quantum theories, has an extension to classical mechanics that was pioneered by Dirac.[10] In fact, the Lorentz symmetry in a classical Hamiltonian theory of a fixed number of particles can be introduced by means of functions of their canonical variables. This can be done by requiring that the generators of the canonical transformations connected to the Lorentz group satisfy the usual relations characteristic of the group structure, i.e., as (4.1) in the quantum case. Of course, they being the generator functions of the canonical variables, the Lie brackets will be the classical anolog of the quantum commutators, viz. the Poisson brackets.

Until now nothing has been said about the particular way in which some specific physical quantities transform in the theory. If, for example, we require[9,10] that the spacetime coordinate Q_j of a particle transform, in the familiar manner, according the Lorentz transformation formula, we find that they will satisfy a set of Poisson bracket equations with the ten generators of the group:

$$[Q_j,P_j] = \delta_{ij}, \quad [Q_i,J_j] = \varepsilon_{ijk}Q_k,$$
$$[Q_i,K_j] = Q_j[Q_i,H] \; ; \quad (i,j,k = 1,2,3). \tag{4.2}$$

These relations, which must be postulated in addition to the algebraic relations (4.1) among the generators, are connected with the canonical character of the Q_j and with the question of the invariance property of the world lines.

It is in this context that the no-interaction theorem was developed.[11] This theorem shows that, by combining the demand of relativistic symmetry (as expressed by the relations between the generators of the group) with the requirement of the manifest covariance for the space time positions of the particles of our system, we get a very restrictive set of conditions on the form of the generators. More precisley, it can be proved that the above conditions are compatible only if we are dealing with a set of noninteracting particles. If indeed we calculate the acceleration of each particle, we find that it always is zero, and hence all the velocities must be constant.

In this sense, in a Lorentz symmetric, classical, Hamiltonian theory, the requirement of the ordinary manifest covariance of the space time positions rules out any interaction. This important theorem was considered for a long time as a proof of the impossibility of constructing a classical theory of directly interacting particles and, consequently, as the main reason for introducing a quantized field theory in order to describe interacting systems.

5. . . . AND BEYOND IT

In a precise sense, the fact that, in a relativistic Hamiltonian theory, one cannot use covariant particle coordinates as canonical variables, indicates the formal "cost" of a relativistic theory with action at a distance, rather that the impossibility of it. Indeed, a few forms of a relativistic dynamics of directly interacting particles have emerged during last years.[7,12] Of course, all these approach must take into account the above mentioned results of the no-interaction theorem, so that in general we will deal with noncanonical positions or noninvariant world lines, and so on. However, these attempts show that, beyond some conpatibility condition imposed on the form of the interaction, a theory of relativistic particles interaction via action at a distance is possible and works. Thus the problem, outlined at the end of Sec. 3, of the construction of a covariant dynamics for interacting closed systems admits solutions, even if not the more simple conceivable ones.

Here we will briefly sketch one of these approaches, based on the socalled constraint formalism for classical mechanics,[13] that present, in a new suggestive form, the essential results of the preceding predictive theories.

In a phase space with simplectic form (Poisson's brackets) there is a way to define trajectories without reference to any time parameter. Thus these trajectories will be manifestly covariant. The theory will also be Lorentz symmetric if it constitutes a linear representation of the Lorentz group. To accomplish this we consider k functions $K_i(q,p)$ $(i = 1,\dots,k)$ satisfying the relations

$$\{K_i, K_j\} = \sum_m \lambda^m_{ij} K_m, \quad \lambda^m_{ij} \text{ const.} \tag{5.1}$$

The set of equations $K_i = 0$ will constitute a constraint which defines a hypersurface in phase space. If now we consider canonical transformations generated by K_i, it is evident that the constraint hypersurface will be left invariant. Thus, starting with an arbitrary point on the hypersurface, the iteration of the canonical transformation generated by K_i defines a trajectory lying on the hypersurface.

Now it is possible to define generalized Hamiltonian equations of motion by establishing the functional dependence between the q^o_j and the remaining canonical variables. However, this passage is not necessary in the constraint formalism, which is totally algebraic: No explicit time dependence of variables needs to enter into its statements, so that it will remain covariant and also manifestly covariant. In this sense the Hamiltonian formalism and the constraint formalism are closely related but not identical.

In this formalism we can also introduce direct interactions, for example, in the case of two particles, by means of a scalar potential $V(r)$, where r, a scalar, is the spatial separation of the two particles in the rest frame of the center of mass. Thus we get a well defined covariant system of directly interacting particles. Of course, not all forms of $V(r)$ are permitted.

However, the price to be payed for the no-interaction theorem is that we need in this theory a deeper analysis of the idea of observable. Formally we can introduce in phase space the socalled "syntactic" observables, as functions of p_i and q_i, which commute with the K_i on the constraint hypersurface: They are constant along each phase space trajectory. But these observables are not coincident with the physical "semantic" observables, namely: times, distances, velocities, etc. as measured by rulers and clocks in some frame of reference. The main difficulty of the constraint formalism lies in the connection between syntactic and semantic observables. In fact, in order to obtain the spacetime orbits, we must associate semantic observables with the phase space coordinates. But this association is ambiguous, in the sense that it is unique only in a preferred frame of reference: the center of mass rest frame. In this frame, the association of the q^o_i with the semantic time gives rise to auxiliary conditions and to a unique identification of Lorentz covariant orbits in spacetime.

6. CONCLUSIONS

The preceding brief discussion shows that the road to a causal interpretation of quantum theory cannot be considered closed on the ground of the von Neumann theorem and of the no-interaction theorem. In fact, these theorems state what we can do and what we cannot do in order to build a deterministic theory of the microscopic phenomena. On this question preliminary works exist[14] showing that, at least for the scalar-particle case, the quantum potential between two correlated particles satisfies the compatibility conditions for a predictive mechanics, so that it can be used in the construction of a "classical" mechanics for these systems. Of course, this is only a first step along the way leading to the definition of the description of a deterministic world. However, nothing has been said till now about the problem, outlined in Sec. 5, of the canonicity of the coordinates or of the identification of the physical observables, which items will constitute nothing less than an exciting program for the future.

A final remark: In order to avoid any possible causal anomaly, the world must be treated in a nonlocal theory as a unique, closed, predictive system, in the sense sketched in the Sec. 3. This implies that we must accept, in some sense, a completely deterministic point of view about the evolution of the world. Of course, such a choise could not be made without a discussion concerning fields and arguments (epistemology, philosophy, ethics, etc.) not directly connected with physics. We cannot eneter here in to a debate on these questions; we recall only that in the past centuries the idea of a deterministic universe was widely considered a real possibility and that a long chain of thinkers and physicists, from Spinoza through Diderot and Laplace up to Einstein himself, was in various forms concerned with this "Weltanschauung." Perhaps the time has come to reopen the debate on these problems.

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