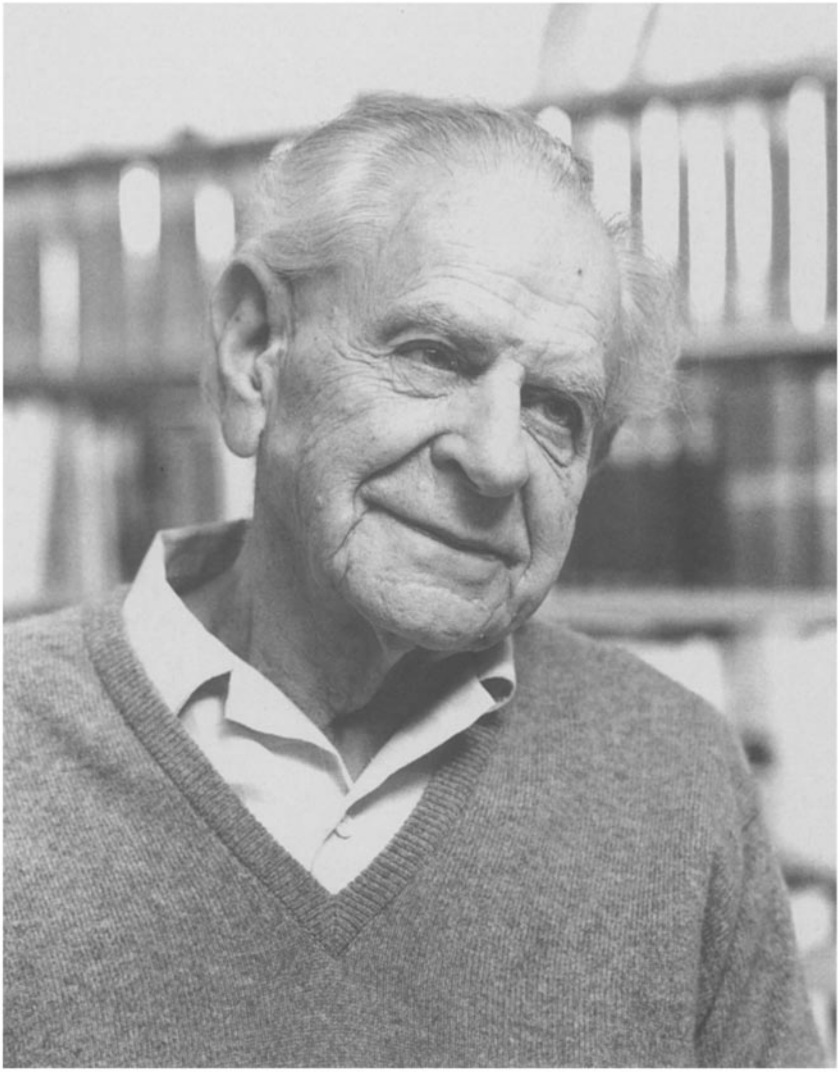

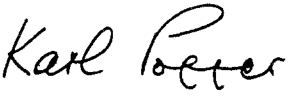

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Single-Particle Trajectories and Interferences in Quantum Mechanics[1]

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In this paper some topics concerning the possibility of describing phenomena of quantum interference in terms of individual particle spacetime trajectories are reviewed. We focus our attention, on the one hand, on the recent experimental advances in neutron and photon interferometry and, on the other hand, on a theoretical analysis of the description of these experiments allowed by stochastic mechanics. It is argued that, even if no conclusive argument is yet at hand in both the theoretical and the experimental fields, the researches of the last 10 *years now seem to favor Einstein's and de Broglie's realistic spacetime description and interpretation of quantum mechanics.*

1. INTRODUCTION

The aim of this paper is to review the more recent aspects of the old discussion about the idea of spacetime trajectories of quantum particles. We all know that such trajectories apparently exist when we directly observe them (in a bubble chamber, for example), but here we are interested in a particular type of experiments where the wavelike nature of these particles also manifests itself: we think, for example, of the interference in a Young two-

[1] This paper, written in honor of Sir Karl Popper's birthday, evidently supports his realistic view on the physical meaning of quantum mechanics. It is derived from a talk given at the Sochi Conference (USSR) in April 1991 by one of the authors (N.C.P.) and the common work of the other (J.P.V.) with Professor Helmut Rauch. The ideas and the proposals contained in the Secs. 4 and 6 reflect only the opinions of one of the authors (J.P.V.)

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slit experiment. In these cases we were all taught that not only can we not observe both the trajectory of the particles and the interference pattern at once, but that in a quantum framework it is even impossible to think that a particle has a trajectory between the source and the screen.

A clearcut account of this situation was given by Feynman [see, for example, *Rev. Mod. Phys.* **20**, 267 (1948)]: "... consider an imaginary experiment in which we can make three measurements successive in time: first of a quantity A, then of B, and then C ... it will do just as well if the example of three successive position measurements is kept in mind. Suppose that a is one of a number of possible results which could come from measurement A, b is a result that could arise from B, and c is a result possible from the third measurement C." If P_{ab}, P_{bc}, P_{ac} are the conditional probabilities that respectively B give b if A give a and so on, and if "the events between a and b are independent of those between b and c," we expect the classical relation

$$P_{ac} = \sum_b P_{ab} P_{bc}$$

However, in quantum physics we should consider the complex numbers ϕ_{ab}, ϕ_{bc}, ϕ_{ac} such that their square modulus gives the correct conditional probabilities, and the quoted classical relation must be replaced by

$$\phi_{ac} = \sum_b \phi_{ab} \phi_{bc}$$

But if this is correct, ordinarily the former is incorrect. "The logical error made in deducing [the first formula] consisted, of course, in *assuming that to get from a to c the system had to go through a condition such that B had to have some definite value, b.*" However, this point was never completely convincing for a number of physicists who rather see, in this situation, an indication of the incomplete character of the quantum mechanics, so that the discussion about the existence of trajectories in interference experiments goes on until today. In the following sections we will consider some experimental and theoretical researches that have been carried out in the 80's about this problem. In Sects. 2 and 3 we will focus our attention on a type of interference experiments made possible by recent developments in neutron interferometry. These experiments have the advantage, with respect to that based on light interference, for example, that the neutrons are massive particles with evident corpuscular properties that make it particularly hard to believe that they have no trajectory. In Sec. 4 one of the authors (J.P.V.) will discuss recent experimental proposals with photons, and in Sec. 5 we will review some considerations on the interference experiments made in the framework of the stochastic mechanics, a theory inherently based on the idea that particles follow trajectories and that nevertheless reproduces the quantum mechanical predictions. In Sec. 6 one of the authors (J.P.V.) will argue about some particular connections with other fields of research. Of course we cannot claim today that these researches can be considered conclusive; we think, however, that recent progress in experimental techniques, both in neutron and photon interferometry, will tell us whether quantum particles travel in spacetime or not, i.e., shed new light on these crucial problems.

2. 1963–83: FIRST PHASE OF NEUTRON INTERFEROMETRY

The story begins with a proposed *Gedanken* experiment aimed to show the validity of the *superposition principle* in a test case on spin states. The idea comes from a discussion[1] about the measurement theory: Quantum mechanics describes a measurement process as an interaction between an object (O) and a measuring apparatus (A). Let us consider for the sake of simplicity only the case of *sharp* states $|\phi_n\rangle$ for O, with eigenvalues λ_n. Let $|\Phi\rangle$ be the state of A in the general case and $|\Phi_n\rangle$ the state of A with the pointer indicating λ_n. If we start with O in $|\phi_n\rangle$, the evolution is

$$|\Phi\rangle\,|\phi_n\rangle \rightarrow |\Phi_n\rangle\,|\phi_n\rangle \tag{1}$$

If the initial state of O is $\sum a_n\,|\phi_n\rangle$, the linearity of the quantum equations (a consequence on the superposition principle) imposes that the initial state evolves in a final pure state, *and not in a mixture*:

$$|\Phi\rangle\left(\sum a_n\,|\phi_n\rangle\right) \rightarrow \sum a_n\,|\Phi_n\rangle\,|\phi_n\rangle \tag{2}$$

We see in (2) that we get a "correlation between the state of the object and that of the apparatus," so that "the state of the object can be ascertained by an observation on the apparatus": no terms $|\Phi_n\rangle\,|\phi_m\rangle$, $n\neq m$, are present. Of course, when we ascertain (look at) which of the state vectors is present, we get a *wave packet collapse*, namely the state of the total system abruptly changes in $|\Phi_n\rangle\,|\phi_n\rangle$ if we find λ_n.

The alternative, analyzed by Wigner, to this ortodox theory is based on the idea that the evolution induced by the interaction object/apparatus is

$$|\Phi\rangle\left(\sum a_n\,|\phi_n\rangle\right) \rightarrow \text{a mixture of } |\Phi_n\rangle\,|\phi_n\rangle\text{'s} \tag{3}$$

namely: only *one* of the state vectors $|\Phi_n\rangle\,|\phi_n\rangle$ is present, and this will emerge from the interaction with a probability $|a_n|^2$. In this case, when we ascertain the result of the measurement, the state of the total system does not change: "one would merely ascertain which of various possibilities has occurred. In other words, the final observation only increases our knowledge of the system: it does not change anything." The basic point here is that "the state represented by vector (2) has properties which neither of the states (3) has."[1]

To see this last point, we perform a modified Stern–Gerlach experiment without observing the result of the interaction between O and A in order to avoid the wave packet collapse. The aim of this *Gedanken* experiment was to show that in the interaction O/A, if we do not look at the results (so that in fact we do not perform the *measurement*), the final state is a vector superposition of eigenvectors and not a mixture. In this experiment (see Fig. 1) "the projection of the spin of an incident beam of particles, into the direction which is perpendicular to the plane of the drawing, is measured ... The *apparatus* is that positional coordinate of the particle which is also perpendicular to the plane of the drawing ... The ordinary use of the experiment is to obtain the spin direction, by observing the position, i.e., the location of the beam ... What is important for us, however, is the right side of the drawing ... If the two beams are brought together by the magnetic field due to the current in the cable indicated, the two beams will interfere and the spin will be vertical again. This could be verified by letting the united beam pass through a second magnet which is, however, not shown in the figure. If the state of the system corresponded to the beam toward us, its passage through the second magnet would show that it has equal probabilities to assume its initial and the opposite directions. The same is true of the second beam which was deflected away from us ... Hence, the properties of the system, object plus apparatus, is surely correctly represented by an expression of the form (2) and shows, *in this case*, properties which are different from those of *either* alternative (3)."[1]

Moreover, it can be shown that the change (3) is even not consistent

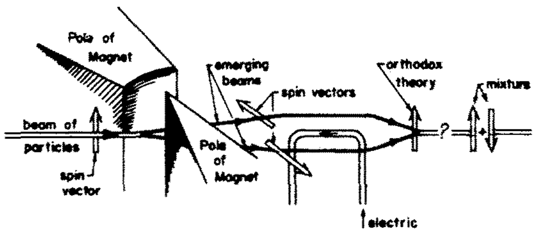

Fig. 1. Schematic representation of Wigner's *Gedanken* experiment.

with the principles of the quantum mechanics. More precisely: "it is not compatible with the equations of motion of quantum mechanics to assume that the state of object-plus-apparatus is, after a measurement, a mixture of states each with one definite position of the pointer. It must be concluded that *measurements which leave the system object-plus-apparatus in one of the states with a definite position of the pointer cannot be described by the linear laws of quantum mechanics.*"[1] We only remember here that this stated principle of linear superposition of probability *amplitudes* is one of the outstanding principles of quantum mechanics, and is the very foundation of the interference of the matter waves as well as of the (apparent) impossibility of thinking the microworld in terms of *trajectories*, as will be discussed later.

An interesting experimental verification of this *Gedanken* experiment came 20 years later when the progress in neutron interferometry made it possible. The experiences described here were initially designed to study the transformation properties of spin states of neutrons,[2] but they were even very soon utilized to verify the principle of linear superposition of probability amplitudes for neutron spin states.[3] The experimental setup is sketched in Fig. 2. Of course we will not analyze the technical aspects of this apparatus: we will only try to extract the conceptual part. The idea is to use a triple-Laue-case Si-crystal interferometer to obtain two neutron

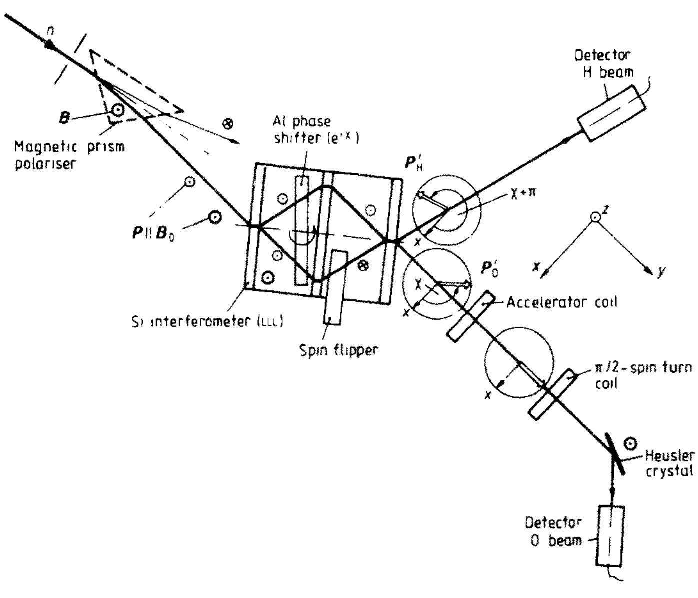

Fig. 2. Schematic arrangement of the spin state superposition experiment.

beams of opposite spin states that will be finally *superposed.* "Quantum theory predicts that the resulting beam would not be a mixture as one might intuitively visualize in a classical picture. Instead, one expects the final polarization vector to lie in a plane perpendicular to the initial polarization directions." Interference effects, produced by means of an added scalar phase shift χ, will be observed in order to verify this statement. The analogy with optical devices as Mach–Zehnder interferometer,[(4)] or the Young two slits, is evident. The incident beam of neutrons, propagating in the y direction, is polarized in the z direction, parallel to the direction of a static magnetic field **B** kept over the whole experimental arrangement, so that it is in the state $|\uparrow_z\rangle$ when it enters the interferometer. The first slab splits the beam (like a semitransparent mirror) into two subbeams: I and II. The beam I undergoes a phase shift $e^{i\chi}$ by means of a plane slab of Al.[4] When we superpose the two beams on the last slab we obtain, as output behind the interferometer, two beams: a forward O-beam and a deviated H-beam. Here for simplicity we will be interested only in the analysis of the O-beam. If we neglect here, and in the following, the propagation direction difference of the two interfering beams and omit all common phase factors, we obtain for the O-beam the vector

$$|O\rangle = \tfrac{1}{2}\,|\uparrow_z\rangle + \tfrac{1}{2}e^{i\chi}\,|\uparrow_z\rangle \tag{4}$$

so that the final intensity of the beam will be modulated by χ as follows:

$$\langle O\,|\,O\rangle = \tfrac{1}{2}(1+\cos\chi) \tag{5}$$

Of course (4) is not a normalized vector, but the conservation of the particle number is not violated since we should remember that there is also an H-beam that we are neglecting in our analysis.

However, in the actual experiments a further transformation on the beam I is carried out before the superposition: a rotation of π around the y axis flipping down the neutron spin to $|\downarrow_z\rangle$. This rotation is induced by a static dc spin flipper: "Since no explicitly time-dependent interaction is involved in such a flipping process, the total energy of the neutron is a constant of the motion. This means that the associated change of the Zeeman potential energy is exactly compensated by a corresponding inverse change of the kinetic energy, i.e., of the neutron wavelength."[(5)] Though this $\Delta\lambda$ is small, it is, however, sufficient to produce an appreciable decrease of the

[4] This phase shift is due to the nuclear index of refraction of the interposed material, and its value is given by $\chi = -N\lambda b_c \Delta D$, where λ is the neutron wave length, N is the number of nuclei per unit volume, b_c is the coherent neutron–nucleus scattering length, and ΔD is the effective thickness of the phase-shifter plate.

observable interference contrast in the experiment. This problem was solved later, as we will see in the following.[5] After this rotation, the final superposed state of the forward O-beam will have the form

$$|O\rangle = \tfrac{1}{2}\,|\uparrow_z\rangle + \tfrac{1}{2}e^{i\chi}\,|\downarrow_z\rangle \tag{6}$$

so that the final intensity will be constant. However, in this case, our interest is shifted to the behavior of the polarization vector: while the polarization of I and II before superposition is antiparallel and parallel to the z direction, namely:

$$\begin{aligned} \mathbf{P}_{\mathrm{I}} &= \langle\downarrow_z|\,\sigma\,|\downarrow_z\rangle = (0, 0, -1) \\ \mathbf{P}_{\mathrm{II}} &= \langle\uparrow_z|\,\sigma\,|\uparrow_z\rangle = (0, 0, +1) \end{aligned} \tag{7}$$

where σ are the Pauli spin matrices, after the superposition we have

$$\mathbf{P}_O = \langle O|\,\sigma\,|O\rangle = (\cos\chi, \sin\chi, 0) \tag{8}$$

so that there is no z component in the polarization vector which instead points in directions in the $x-y$ plane which depend on the phase shift. In fact, the final wave functions are no longer eigenstates in the static magnetic field $\mathbf{B}$. Of course the conservation of the angular momentum is not violated since we must take into account that the total wave function includes the deviated H-beam that we are not analyzing here. In the actual experiences it was exactly this situation that was verified with an analysis of the polarization vector which gave exactly the results indicated by standard quantum mechanics: no z component was found, and $\mathbf{P}$ rotates entirely in the $x-y$ plane although the constituent waves are polarized in the z direction. This feature cannot be explained if strict incoherence (i.e., mixture) of spin states is expected. A subsequent modification of this experimental setup was later introduced in order to answer the already quoted problem[5] of the change in wavelength of neutrons which rotate their spin by means of a static spin flipping device. A different physical situation arises indeed if a radio-frequency flipper is used to invert the spin state of one of the particle beams within the interferometer.[5,6] (see Fig. 3). Here the polarization of the neutrons is inverted by means of the time-dependent magnetic field $\mathbf{B}_{\mathrm{rf}}(t)$ produced by a radio-frequency coil, so that their total energy is no longer conserved because of an exchange of photons of energy $\hbar\omega_{\mathrm{rf}}$ between the neutron and the radio-frequency field. If $\mathbf{B}_{\mathrm{rf}}(t)$ rotates in a plane orthogonal to $\mathbf{B}$, this interaction has a resonant maximum when the photon energy equals the Zeeman energy difference $2\,|\mu|\,B$ between the two spin states of the neutron, that is, if $\hbar\omega_{\mathrm{rf}} = 2\,|\mu|\,B$. After passage through such a flipper, neutrons which were initially polarized

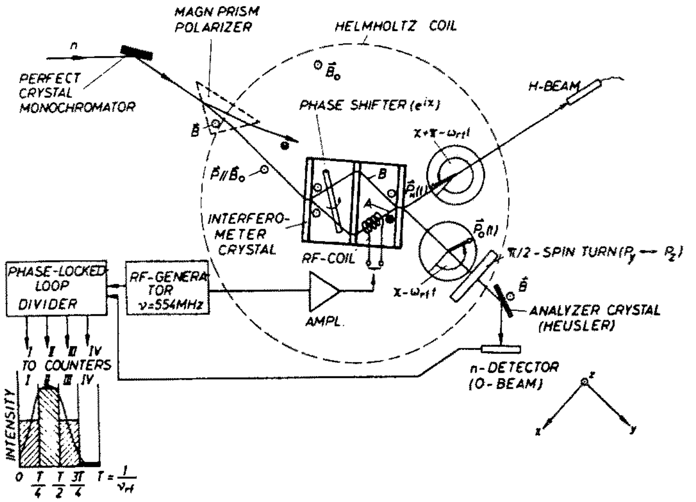

Fig. 3. Schematic representation of the spin superposition experiment and of the stroboscopic neutron registration.

parallel to the direction of **B** (z direction) and had an energy $E=\hbar^2k^2/2m+|\mu|\, B$ will have been flipped in the opposite direction and lost an amount of energy $2\,|\mu|\, B$ whereas they maintain their initial momentum and wavelength. However, in this case the final coherent superposition of states for the O-beam is not (6) but

$$|O\rangle=\tfrac{1}{2}\,|\uparrow_z\rangle+\tfrac{1}{2}e^{i(\chi-\omega_{\mathrm{rf}}t)}\,|\downarrow_z\rangle \tag{9}$$

Hence the forward beam is still in a spin state which is orthogonal to the spin states of both interfering beams, but unlike the static spin superposition experiment this state is not stationary in time and the polarization vector behind the interferometer will rotate, as can be seen immediately from

$$\mathbf{P}_O(t)=\langle O|\,\sigma\,|O\rangle=(\cos(\chi-\omega_{\mathrm{rf}}t),\sin(\chi-\omega_{\mathrm{rf}}t),0) \tag{10}$$

This time-dependent rotation of the polarization can be detected if a stroboscopic registration of the neutrons is applied synchronously with the phase of the radio-frequency field. Here again, as prescribed by the quantum mechanical formalism, no z component of the polarization was found in the experiment and, in the plane $x-y$, the coherent oscillations of the detected intensity as a function of χ were clearly observed.

Some remarks must be made here in order to introduce the discussion of the next section: "intuitively one might argue that, at least in principle, it should be possible to detect the passage of the neutron through the rf coil by detecting the change of the damping of the electronic resonance circuit which is caused by the emission or the absorption of a photon during the flipping process. Although this would allow one to find out over which of the two possible paths within the interferometer the neutrons have propagated, at first sight it looks as if the interference pattern could nevertheless be observed in that case."(5) However, the uncertainty principle seems to forbid once more this detection: The stroboscopic registration is submitted to the number-phase uncertainty relations,(7) so that (apparently) it is impossible to detect single-photon transitions simultaneously with the interference pattern. A similar argument holds if one were to try to find out which path the neutrons have taken by measuring the energy change $\hbar\omega_{rf}$.

3. 1984–91: SECOND PHASE OF NEUTRON INTERFEROMETRY. PROPOSED TEST OF EINSTEIN'S "EINWEG" ASSUMPTION

The last remarks of the preceding section about the possibility of detecting the path of the neutrons within the interferometer opened new perspectives to this sort of experiments. In a sequence of papers proposing several changes in the experimental setup,(8) some new features of the neutron interferometry were put in evidence. First of all, it was emphasized that all these neutron interferometric experiments belong to the regime of self-interference where at any moment of time a single neutron only, if any at all, is inside the interferometer. When a neutron is detected, the next one is usually still confined within the uranium nucleus of the reactor fuel.(9) This means that the wave function is related to single-particle systems and not only to statistical ensembles. That, of course, is well explained by the wave character of quantum particles, but it should also be considered that (differently from the case of photons, for example) also well-defined particle properties can be attributed to the neutron: mass, spin, magnetic moment, effective-mass radius, and internal structure consisting of quarks. This not only sharpens the old quantum paradox of particle self-interference (every neutron in the area of interference behaves as if it knows simultaneously what has happened in both paths), but invites one to try to find a way to show that somehow the neutrons traveled along a path within the interferometer even if they self-interfere. It is clear that to believe in the possibility of speaking of trajectories for quantum particles is the same as to believe in the incomplete character of the quantum mechanics, a point that has never been completely settled. We will discuss in a subsequent section the theoretical arguments against such a possibility, and for the time being we will limit ourselves to an exposition of the experimental part. In order to bypass the problems posed by the uncertainty principle and discussed at the end of the previous section, the following variation has been proposed: The perfect crystal interferometer contains now two radio-frequency spin flippers, one for each neutron beam (see Fig. 4, where a new skew symmetrically cut interferometer which facilitates the installation of devices is shown). The interest of the modification is the following: In the present configuration both the neutron beams are spin inverted at the end of the interferometer so that the superposed final state will have a form similar to (4) with the only difference that the spins are now downward. Hence the intensity will be given by (5) and the polarization reduces to the constant z-directed vector $\mathbf{P} = (0, 0, -1)$. As a consequence, we have no more time-dependent polarization to measure and no stroboscopic registration of the neutrons to do. Rather we will observe the interference pattern given by (5) with this difference, with respect to the initial configuration where no spin flipper at all was present in the interferometer: now every neutron has flipped its spin within the interferometer, more precisely within the two coils, and we can hope to use this fact to say something about their trajectories. In other words, we want to examine the possibilithy of gathering enough *indirect* evidence of the existence of neutron trajectories, short of a *direct* measurement on the position of the neutrons within the interferometer which would trigger the wave packet collapse and the disappearance of the interference. Of course it is always possible to argue, as the orthodox interpretation of quantum m chanics does, that only *direct* evidence can be conclusive. However, the experiment, with its subsequent modifications, "delimits quite precisely a point of disagreement between opponents and adherents of the completeness of quantum mechanics in the

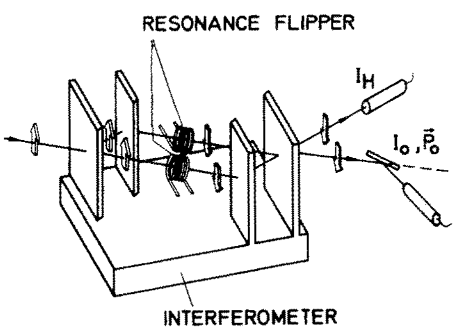

Fig. 4. Schematic arrangement of the radiofrequency flip coils within the skew-symmetric neutron interferometer in the double resonance experiment.

sense of an objective interpretation: it forces the latter to hold that there are physical situations where two quite distinct physical states of affairs like 'absorption of a photon by coil I' and 'absorption by coil II' can *coexist objectively*. In other words: if someone does *not* want to hold such a position, the experiment indeed forces him to believe in the incompleteness of the quantum mechanics."(10)

We begin by remarking that in the modified form of the experiment, since the stroboscopic registration is not necessary given the fact that the interference pattern is stationary, the referred argument about the number-phase uncertainty principle is no more relevant for us and we can hope in principle to detect neutron paths "due to single-photon energy transfer to the spin flipper while at the same time a time-independent constant intensity interference pattern can be observed."(8) However, even if no theoretical objection arises for a possible detection of single-photon transitions in the field of the radio-frequency flipper, such a single photon energy transfer is considered not detectable. The reason is primarily related to the width of the resonance curve and the energy uncertainties of the single photon, as well as with secondary technical difficulties. This apparently confirms in this setup only the fundamental impossibility of doing a *Welcherweg* experiment telling us *which way* the neutron has travelled while observing contemporarily interference patterns.[5] As a consequence, a shift in the conceptual structure of this experiment is needed if we want to use it in the debate on the completeness of quantum mechanics. Beside the said *Welcherweg* experiment, which would tell through which specific coil each individual particle has effectively gone, it is possible to design so-called *Einweg* experiments which should explicitly show that individual particles indeed go through one coil only without information on which one (namely without a direct measurement on the particular path chosen). This possibility was initially explored by means of the following suggestion(8): after having a passage of a sufficient number of neutrons through the interferometer, the energy transferred to each coil is summed up to an amount that is detectable. Then "the argument is as follows:

(i) A detection of an energy amount E_{det} by one of the rf circuits implies that this amount of energy has been transferred to the coil by the neutrons involved in the experiment.

[5] See, for example, the discussion of this point contained in M. O. Scully and H. Walther(4) and in T. Unnerstall(10): they argue that coils generate coherent states of photons and that there is no which-path information left in the coils after passage of the neutrons since the coherent photon distribution is essentially unchanged by the addition of a single-photon associated with spin flip.

(ii) A coil can absorb energy only at its resonance frequency ω_{rf}, i.e., the energy transfer has occurred as a series of single-energy transfers $\hbar\omega_{rf} = \Delta E$, i.e., as a series of energy transfers, corresponding to the Zeeman energy splitting. This implies that E_{det} is a sum of equal individual energy transfers corresponding to a spin-flip of each individual neutron, i.e., $E_{det} = n\Delta E$.

(iii) Consequently the energy E_{det} corresponds to a sum of n spin-flips, hence n neutrons have passed through the path containing the rf coil.

(iv) Therefore, if N neutrons are successively involved in the experiment, $N-n$ neutrons have passed through the other path.

(v) By means of this measurement, one cannot tell which neutron has gone through which path, but one establish the following: Out of N neutrons involved in the experiment, n neutrons pass through path II and $N-n$ through path I. Every neutron has a probability given by the transmission/reflection coefficient of the first incident plane in the interferometer of going I or II *but* it either goes through path I *or* through path II.

(vi) Since now neutron self-interference persists and shows that 'each neutron in the area of interference knows simultaneously what has happened in both paths' (Rauch, 1983), this implies that something which has a real physical existence independent of the particle travels along both paths and contributes to the forming of the interference.

This at least proves the incompleteness of the quantum mechanical Copenhagen description because the persistence of an interference pattern is combined with the existence of a definite trajectory for each particle: a fact forbidden in the Copenhagen interpretation."[8] In this sense, this sort of *Einweg* experiment strongly suggests the existence of neutron trajectories.

A first verification has been performed in 1985, but without any energy measurements on the two coils.[11] In this experiment several different modes of operation of the two spin-flip coils are allowed (resonance frequencies and initial oscillation phases can be chosen differently) and a new skewsymmetrically cut interferometer was used in order to facilitate the installation of devices which act on one of the subbeams only without influencing the other (see Fig. 4). The interference persists in all cases, indicating that "the spin-flip does not cause a collapse of the neutron wave function inside the interferometer. This fact is most remarkable in the case of a combination of polarized incident neutrons and polarization analysis of the final state. For in that case ... one knows with certainty that every neutron which reaches the detector must have undergone an energy transfer $|\Delta E| = \hbar\omega_{rf}$ ... Without doubt one would be free, at least on a *Gedanken* experiment level, to implement the whole interferometer setup within a device that measures energy transfers, as for instance a high-resolution neutron back-scattering spectrometer, without destroying the interference."[(11)] Even if it seems not possible to utilize the possibility of having two slightly different resonance frequencies to carry out the needed energy measurement as initially suggested,[(12)], these results allow some theoretical remarks[(13)]:

(1) We consider the operation mode with the two coils driven at equal frequencies. Moreover, we remember that this experimental setup never contains more than a neutron at a time.

(2) Each neutron enters the interferometer with its spin parallel to the constant magnetic field which bathes the whole setup.

(3) Each neutron leaves the setup with its spin antiparallel to the constant magnetic field. As a consequence, it has lost by resonance a quantum of energy $\Delta E = \hbar\omega_{rf}$. However, this individual neutron energy loss is still indirectly known: it is derived as a consequence of the spin-flip. But, if one assumes:

   a. that the neutron energy and momentum are always conserved in all microprocesses where there is an exchange of energy, i.e., that the quantum lost by the neutron must be absorbed by the coils;

   b. that all observable exchange of energy is tied with the particle aspect of matter[6];

   then one must accept that the neutron exchange of energy has necessarily happened in one or the other spin-flippers, but not in both simultaneously, since no half-quantum possible resonance frequencies exist anyway in the corresponding radio-frequency harmonic oscillators.

(4) Since the interference patterns are nevertheless observed, one can conclude that in the region where the waves interfere each neutron disposes of the information that there exist two associated paths whose phase difference obliges the neutron to interfere.

(5) As a consequence, between the source and the detectors each neutron manifests itself as a wave and as a particle

[6] See, for example, the Compton and the Compton–Simon scattering experiences.

simultaneously, so that the description given only in terms of the quantum probabilistic distributions is correct, but not complete, since it does not state that each neutron has manifested itself in one of the coils.

These conclusions, however, must be qualified with the following reservations. First of all we only known indirectly that all neutrons in the interference pattern have lost a quantum because they have flipped their spin. One can perfectly utilize the argument that until one has directly measured the passage of each neutron through one coil only, one does not create any wave packet collapse on the other path. In fact, such direct individual energy loss measurements seem practically impossible at present. The second remark is that all the referred experiments were performed with stationary neutron beams where the wave functions inside the interferometer follow from the solution of the time-independent Schrödinger equation. Hence the wave functions of the two separated beam paths remain connected at least at the beam splitter and at the place of beam superposition. As a consequence, it was argued that some information can be exchanged via these mesh points.[14]

These two remarks are the reasons for the more recent modifications of the experimental setup. First of all, the second problem was solved by means of the use of a fast neutron chopper[14] (see Fig. 5). "In fact in the case of chopped beams with burst lengths shorter than the dimension of the interferometer, completely unconnected wave packets exist inside of the interferometer ... The results show that the well-known interference phenomena exist as in the case of a stationary beam. The mean occupation number per burst was about 0.0024 neutrons per burst and, therefore, like any other interferometer experiment, these experiments belong to the regime of single-particle interference."

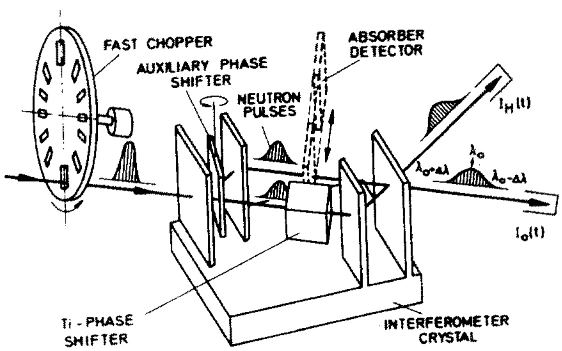

Fig. 5. Sketch of the arrangement for interferometer experiments with pulsed beams.

As for the first remark, addressed in the more recently proposed experiment,[15] let us remember that in the two-coil experiment "the situation exists that a photon with the Larmor frequency is exchanged between the neutron and the resonators. Because there are only photons with those Larmor frequencies, it is reasonable to accept that the exchange occurred only in one of both coils which suggests the existence of a trajectory, although which of both possible trajectories has been chosen remains undetermined. Here the energy exchange is not a measuring process; it rather shifts the system in a preparatory stage, permitting a forthcoming measurement of this energy exchange.[15] This argument, however, is evidently neither complete, nor absolutely convincing, from Bohr's point of view, due to the fact that the existence of a one-photon energy exchange has only been verified in those two separate experiments with different setups." Hence one can claim that the photon exchange has happened in coil I or in coil II only "if one can

(A) measure the energy shift $\Delta E = \hbar\omega_{\rm rf}$ in the same experiment where one observes interference;

(B) prove that this has occurred in one coil only by some process (i.e., modified setup) which does not destroy this interference."

Let us discuss now the proposed set up satisfying (A) and (B). First of all, it is recalled that in a first experiment[6,16] it has been demonstrated that an energy shift ΔE larger than the energy width δE can be achieved if a rather strong magnetic field **B** is applied. In a second experiment[11] it has been shown that the interference still persists in the case where a spin reversal and a related energy exchange for neutrons exist in both beam paths. Unfortunately in this experiment the energy exchange ΔE was considerably smaller than the energy width so that it could not be observed simultaneously with the interference pattern. However, in a modified experience we can choose to measure this ΔE on the neutrons at the end of the apparatus (and not as a change in energy of the coils), where the static magnetic field **B** is no longer present. Of course, since no energy measurements are now done on the two coils, no *Welcherweg* information can be extracted from this arrangement: but, if ΔE is larger than δE, we could simultaneously measure the interference pattern and the individual energy transfer to the neutrons *behind the static magnetic field* **B**. In fact, the kinetic energy of the neutrons changes, when it goes out of the region where $\mathbf{B} \neq 0$, by $2\mu B$. This can be measured by time-of-flight techniques or crystal spectroscopy. "In this case the initial momentum distribution $|A(k)|^2$ is shifted to $|A(k+\Delta k)|^2$, where $\Delta k = \mu Bm/\hbar^2 k$ and the beam intensity behind the interferometer reads as

$$I(k) \propto |A(k+\Delta k)|^2 \, (1+\cos\chi) \qquad (11)$$

whereby the momentum shift Δk and the interference appearing as a phase shift can be measured simultaneously ... Because the neutron–photon energy exchange can happen inside one resonator coil only, it can be assumed that the neutron has passed through one of these coils even if one does not know through which one it went. The absorption of half the photon energy in each coil is physically impossible because photons of that energy are not excited in either coil."[15]

The following last modification, proposed for this experiment, is finally aimed to make the *Einweg* statements even stronger (see Fig. 6). A stroboscopic chopping of the initial wave packet is introduced[14] along with a macroscopic spatial separation between the two coils. Hence the separate parts of the chopped wave packet cross through coil I or coil II at different times. Moreover, flippers can be switched on and off in such a way that there is only one coil working at the time. The frequency of the switching can be chosen such that when one packet I (II) goes through its coil no packet goes through coil II (I). Hence, in the time interval $l_1/v \leqslant t \leqslant l_2/v$ (when packet I has passed through coil I and packet II has not yet reached coil II) the wave function will have the form

$$A(r^{\mathrm{I}}, k+\Delta k)\, e^{i(\omega-\omega_{\mathrm{rf}})t}\, |\downarrow_z\rangle + A(r^{\mathrm{II}}, k)\, e^{i\omega t} e^{i\chi}\, |\uparrow_z\rangle \tag{12}$$

Behind coil II even the second part of this wave function changes as usual. Hence for the time interval mentioned the energy exchange occurred in

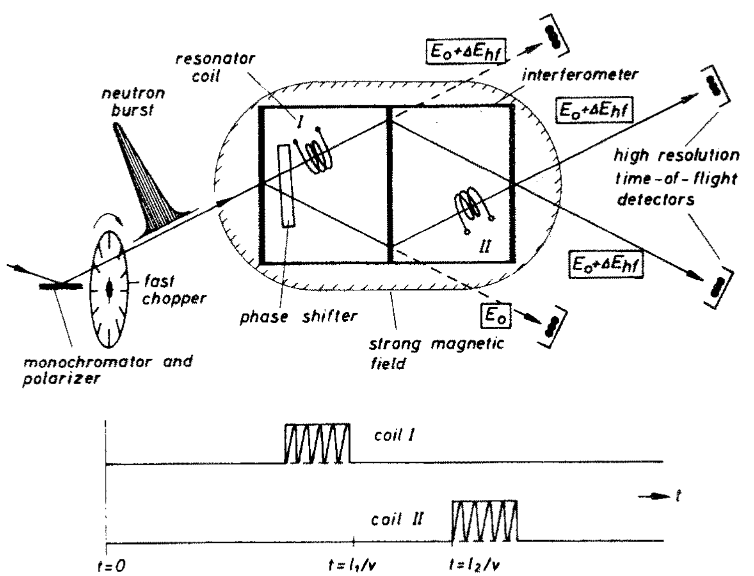

Fig. 6. Proposed experimental setup for a simultaneous detection of interference and energy exchange.

beam path I only. With a special arrangement it is also possible to measure this behavior directly. This provides an additional delayed choice option along with the possibility of answering some criticism as shown in the following.

Let us suppose now that this proposed experiment will confirm the predictions of quantum mechanics (a statement not put in discussion here) and let us make some final remarks about the conclusions that can be reached. The problem is: do these results compel us to say that the photon exchange necessarily happen in one coil or in the other? While it is true that the exchange cannot be divided up between the two coils in the sense that each one absorbs a photon of energy $\Delta E/2$, it might be that not only do we not know which coil has received the photon, but that it is in reality undecided which one has received it, as the believers in the orthodox interpretation can say.(10) If the quantum state vector gives a complete description of the reality, the two situations (absorption by coil I and by coil II) coexist with the same degree of reality. This is surely a quite peculiar state of affairs, but it is certainly not logically impossible, so that in this sense this experiment cannot be considered truly conclusive. However, as already recalled, this experiment at least forces the adherents of the completeness of the quantum mechanics "to hold that there are physical situations where two quite distinct physical states of affairs like 'absorption of a photon by coil I' and 'absorption by coil II' can *coexist objectively*. In other words: if someone does *not* want to hold such a position, the experiment indeed forces him to believe in the incompleteness of quantum mechanics".(16)

In the same spirit, a recent comment(17) by P. B. Lerner affirms that the statement *the (measurable) energy gain equal to $\hbar\omega$ suggests that the neutron has passed only through one coil during its flight; this suggests that the trajectory of the neutron inside the interferometer is definite, while quantum mechanics says nothing about it,* "is simply not true." He proposes to write the energy gain as

$$\Delta E = \hbar\omega_{rf} W_1 P_1 + \hbar\omega_{rf} W_2 P_2 \tag{13}$$

where the numbers 1 and 2 are assigned with respect to the coils, $W_{1,2}$ are the probabilities of localization of the neutron inside the two coils, and $P_{1,2}$ are the probabilities of the spin flip. In the ideal case when $P_{1,2} = 1$ and $W_{1,2} = 1/2$ the energy gain is simply $\Delta E = \hbar\omega_{rf}$. However, as Rauch argues in his response,(18) if the factorization (13) is possible, then additional arguments for the *Einweg* assumption arise. In fact, in the proposed setup with spatially separated coils (see Fig. 6), the probabilities of spin-flip are time dependent and $P_1 = 1$ in the interval $l_1/v - \Delta t \leqslant t \leqslant l_1/v + \Delta t$ and $P_1 = 0$ otherwise, while $P_2 = 1$ in the interval $l_2/v - \Delta t \leqslant t \leqslant l_2/v + \Delta t$ and $P_2 = 0$ otherwise, where l_1/v and l_2/v are the mean time of flight between the chopper and the center of the resonator coils and Δt is chosen so that the whole neutron burst is flipped inside the coils. Since in the proposed situation these two time intervals do not superpose, for the macroscopic time interval $l_1/v + \Delta t \leqslant t \leqslant l_2/v - \Delta t$, the situation arises that neutrons traveling along path 1 have surely flipped while neutrons traveling along path 2 have not yet reached the coil (and hence they have surely not flipped), so that in this time interval the energy transfer reads as $\Delta E = \hbar\omega_{\rm rf}/2$ which certainly is difficult to understand in terms of energy conservation because photons with half the resonance energy do not exist inside the resonance flippers. As a conclusion, in the words of Rauch himself[(18)]: "The proposed experiment ... does not disprove the Copenhagen view, but it demonstrates that other views may be more intuitively applicable to certain experimental situations."

4. POSSIBLE UTILIZATION OF A MACH–ZEHNDER INTERFEROMETER TO TEST EINSTEIN'S "*WELCHERWEG*" ASSUMPTION WITH SINGLE-PHOTON INTERFEROMETRY[7]

From the discussion of the preceding section it is reasonable to conclude that even if one considers that *Einweg* experiments are sufficient evidence to establish the reality of particle trajectories in quantum mechanics, we agree that *Welcherweg* experiments are in principle necessary to validate the Einstein–de Broglie wave-plus-particle model. Indeed the knowledge that a quantum particle experimentally goes through a definite slit in a double-slit experiment evidently transcends any knowledge derivable from the Copenhagen statistical interpretation of quantum mechanics and would prove that the present formalism is effectively incomplete, as suggested long ago by Einstein himself.

Any experimental effectively realizable *Welcherweg* proposal can thus be considered as a crucial test for any interpretation of quantum mechanics. The present progress in neutron detection and optical devices now allow open new possibilities in that direction as shown in a very recent paper.[(19)]: *Welcherweg* experiments for atoms have been proposed using recent advances in quantum optics, namely the existence of micromasers and the existence of laser cooling. The basic starting point is to build a double-slit experiment for individual excited atoms (see Fig. 7) in which the two wave beams pass into two micromaser cavities which corresponds to different excitations. These atoms emit photons when passing through the cavities which contain no photons. The destruction of the beam coherence in such process would destroy the interference pattern, but it can be restored in principle by a quantum erasure process described in Fig. 8. The setup proposed combines in the setup of Fig. 9 the following ideas:

[7] At present this section reflects only the opinions of one of the authors (J.P.V.).

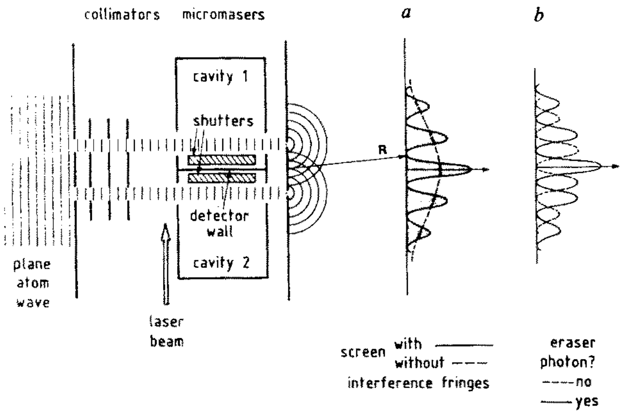

Fig. 7. (a) Quantum erasure configuration in which electro-optic shutters separate microwave photons in two cavities from the thin-film semiconductor (detector wall) which absorbs microwave photons and acts as a photodetector; (b) density of particles on the screen depending upon whether a photocount is observed in the detector wall ("yes") or not ("no"), demonstrating that correlations between the event on the screen and the eraser photocount are necessary to retrieve the interference pattern.

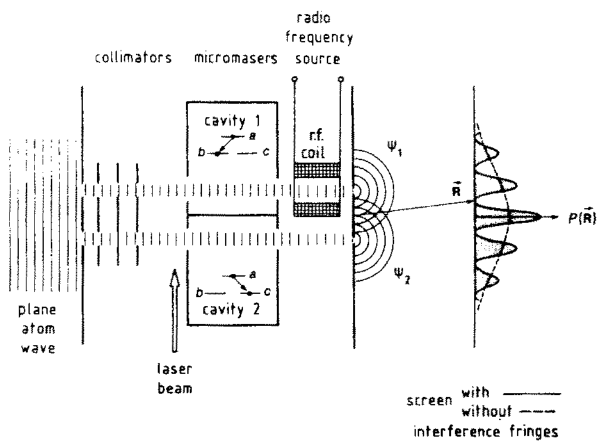

Fig. 8. Asymmetric setup in which cavity 1 induces the transition $a \rightarrow b$ and cavity 2 induces $a \rightarrow c$. Which-path information is erased by the radiofrequency in the coil where $b \rightarrow c$ happens.

(a) An idea of Elitzer and Vaidman[20] which illustrates the possibility of interaction-free measurements using Mach–Zehnder interferometers with a single photon source, such as the one utilized in Aspect's *et al.* experiments.[21] Their idea is simple: the interferometer is arranged so that, when both routes through it are open, no photons will arrive at the detector D_2 (se Fig. 9). On the other hand, if a totally absorbing obstacle is placed so that it blocks light through one route, i.e., on path I in Fig. 9, then it is possible for the photon to arrive at D_2. However, if the

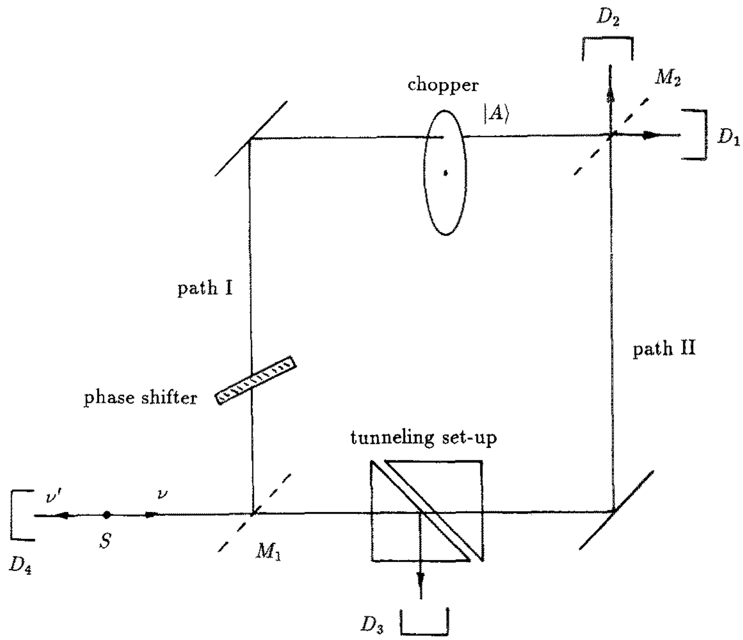

Fig. 9. Mach–Zehnder interferometer with single-photon source arranged so that when there are no completely absorbing atoms (denoted $|A\rangle$) along path I through it (but only the prisms on path II), no photons arrive at detector D_2. This is possible since it has been indeed shown[21] that the detection probabilities in the detectors D_1 and D_2 are oppositely modulated as a function of the path difference (induced by phase shifters) between the arms of the interferometer. The setup contains: (a) an absorbing atom layer on the tooths of a fast chopper on path I which ensures that any photon observed in D_2 must have gone through path II; (b) a tunnel device on path II (details in Fig. 10) such that any photon appearing in D_2 must have tunnelled through the gap, i.e., has utilized the wave property of light as discussed later in the text; (c) N photons ν' detected on D_4 yield $N\alpha T/2\nu$ photons on D_2 (in the average) if T denotes the chopper attenuating factor and M_1, M_2 represent semitransparent mirrors with transmission factor 1/2. The detectors D_2, D_4 are anticorrelated with D_1, D_3 in quantum mechanics.

photon has arrived at D_2, then it must have taken the other route through the interferometer otherwise it would have been blocked by the obstacle. Therefore it didn't *touch* the obstacle. This means that we can deduce the presence of an obstacle without touching it. The obstacle could, for example, be a layer of atoms located on the tooths of a fast chopper which, when it is in the state $|A\rangle$, absorbs the photon with probability 1. If the atoms are in any other state, they will not absorb the photon. Thus a detection at D_2 lets us know that the atoms are in the state $|A\rangle$ without interacting with them.

(b) An idea by Ghose, Home, and Agarwal[22] to test the existence of the wave aspect of single-photon states along a given path (such as path II) by obliging them to pass through a combination of two prisms[8] arranged as in Fig. 10. To justify this, one can reproduce the argument developed by Ghose *et al.*[22] which shows that the photons transmitted through the prisms behave like waves if they arrive in D_2. If we analyze in quantum mechanical terms the situation described in Fig. 11, we know that the field amplitudes a, c, d obey (in classical electrodynamics) the relations

$$d = \gamma a, \qquad c = \alpha a \tag{14}$$

where α and γ are, respectively, the reflection and transmission amplitudes. For certain angles of incidence, the total internal reflection occurs and the waves in the region I are evanescent. If the thickness (h/λ) is large enough, then by the time the fields reach the surface of the second prism, the amplitude has decayed to almost zero and no transmission or tunneling takes place. In quantum theory, the quantities d, c, and a are to be treated as annihilation operators. Moreover, in order to maintain the commutation relations, we have to add the vacuum field b at the open port. Thus Eq. (14) has to be modified to

$$c = \alpha a + \beta b, \qquad d = \gamma a + \delta b \tag{15}$$

[8] The classical analogue of this set-up was performed by Bose[23] as reported in Sommerfeld's *Optics*[24]. Bose took two asphalt prisms and placed them opposite each other with a large air gap between them, as in Fig. 10. When microwaves with $\lambda = 20$ cm were incident on the first prism, they were found to be totally internally reflected by it. As he decreased the air gap and made it of the order of several cm, Bose found that the waves could tunnel through the gap. This was a striking confirmation of the wave nature of microwaves. Similar experiments can be done with visible light. Feynman[25] has given a detailed explanation of this effect based on the wave theory of classical electrodynamics. When the gap between the prisms is larger than the wave length the incident photons suffer total reflection into the first prism (registered by the counter D_3). When the gap is shorter than the wavelength the wave tunnels through the gap and photons can be registered by the counter D_2.

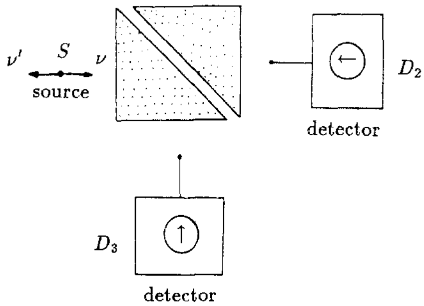

Fig. 10. If we know that a single photon has started from S and appears in D_2, it must have been tunneled through the gap between the two prisms, a property which implies the existence of a wave. The quantum mechanical formalism tells us that any photon which appears (clicks) in D_2 must have tunneled through the gap since the detectors D_2, D_3 must click in absolute anticoincidence.

and one has the commutation relations

$$\begin{aligned} &[a, a^\dagger] = [b, b^\dagger] = 1 \qquad [a, b^\dagger] = 0 \\ &[c, c^\dagger] = [d, d^\dagger] = 1 \end{aligned} \tag{16}$$

Note that $|\alpha|^2 + |\gamma|^2 = 1$, since the prisms are supposed to be loseless. Moreover, β is related to γ through, at most, a phase

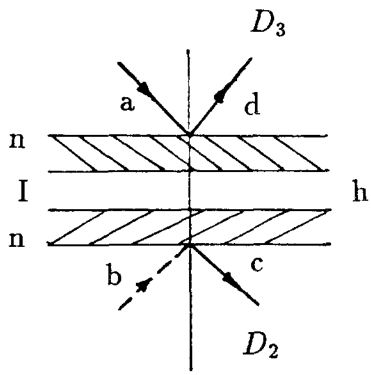

Fig. 11. Schematic representation of the tunneling setup.

factor. The probability $P_d(1)(P_c(1))$ of detecting a photon at the detector D_1 (D_2) is given by

$$P_d(1) = \mathrm{Tr}(p\,|1\rangle_{dd}\langle 1|) \tag{17}$$

where $|1\rangle_d$ is the single-photon state associated with the mode *d*. Assuming the input states as $|1\rangle_a|0\rangle_b$, this probability can be calculated as

$$P_d(1) = |\gamma|^2 \qquad \text{and} \qquad P_c(1) = |\alpha|^2 \tag{18}$$

Note that the results (18) are the same as one would get on the basis of classical electrodynamics. To find out if the detectors D_2 and D_3 click in coincidence or anticoincidence, we need to know the joint probability $P_{c,d}(1, 1)$ of detecting one photon at D_3 and one photon at D_2. This is given by

$$P_{c,d}(1, 1) = \mathrm{Tr}(p\,|1\rangle_c\,|1\rangle_{dd}\langle 1|_c\,\langle 1|) \tag{19}$$

Using (15), (19) reduces to

$$P_{c,d}(1, 1) = 0 \tag{20}$$

which implies that according to quantum mechanics the two detectors D_2 and D_3 must click in anticoincidence.

This complete our demonstration. Indeed the appearance of a photon in D_2 combined with the presence of the completely absorbing material $|A\rangle$ implies that this photon has travelled as a particle along path II only (i.e., a *Welcherweg* knowledge) and has a wave aspect (because of the tunnel effect) simultaneously. Based on the utilization of the usual formalism of quantum mechanics this *Welcherweg* argument strenghtens the point already made by one of the authors (J. P. V.)[8] that in certain specific experimental situations the quantum formalism itself favors the Einstein–de Broglie views against the usual Copenhagen views. He is conscious that the problem of the connection between a statistical formalism, strongly supported by experiment, and possible conflicting interpretations is a difficult, still unsolved, question. A similar difficulty has existed for 45 years (before the discovery of atoms) between the conflicting interpretations of thermodynamics in terms of fluids (Mach and the Vienna school) and the atomic interpretation of Maxwell and Boltzmann. However, the existence of specific *Einweg* and *Welcherweg* feasible *gedanken* experiment shows (if the quantum mechanical predictions are confirmed by the experiment as believed by both the authors) that one has too lightly and for too long discarded the realistic causal views of Einstein and de Broglie and their followers (John Bell included) which should/can now be tested by experiment.

5. TRAJECTORIES IN THE STOCHASTIC MECHANICS

As remarked at the end of the Sec. 3, the orthodox interpretation of the quantum mechanics explains the results of the particle interference experiments in a very peculiar way containing statements that are hard to believe. If other, less upsetting, interpretations are really possible, why did so many people pick up exactly the most incredible one? Why are we obliged to choose the so-called Copenhagen interpretation? The fact is that the very formal structure of the quantum mechanics seems to rule out the possibility of speaking of particle trajectories when we observe interference phenomena, and this idea is based on an analysis of experimental situations very similar to that exposed in the first two sections. If this is true, we should accept the orthodox point of view as the only available and coherent one (even if one could possibly not like it), since the other (making reference to trajectories) is untenable. In order to clarify this point, let us briefly summarize the theoretical origin of the quantum mechanical statements about the non-existence of particle trajectories in interference experiments.

The crucial point here is the fact that apparently in quantum mechanics only the amplitudes can (always) be superposed and not the probability densities. To discuss this statement of far reaching consequences, let us consider a quantum system obeying the Schrödinger equation

$$i\hbar\partial_t\psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}, t) \right] \psi(\mathbf{r}, t) \tag{21}$$

If $K(\mathbf{r}, t; \mathbf{r}', t')$, $t > t'$, is the corresponding Green function, we know that the wave function

$$\psi(\mathbf{r}, t) = \int_{R^3} K(\mathbf{r}, t; \mathbf{r}', t_i)\, \psi_i(\mathbf{r}')\, d^3\mathbf{r}' \tag{22}$$

represents the solution of (21) obeying the initial condition $\psi(\mathbf{r}, t_i) = \psi_i(\mathbf{r})$. From another point of view, $K(\mathbf{r}, t; \mathbf{r}_i, t_i)$ itself is the solution of (21) corresponding to the initial condition

$$\lim_{t \downarrow t_i} \psi(\mathbf{r}, t) = \delta^3(\mathbf{r} - \mathbf{r}_i) \tag{23}$$

and $|K(\mathbf{r}, t; \mathbf{r}_i, t_i)|^2$ is the position probability density at the time t, under the condition that the particle was in $\mathbf{r}_i$ at the time t_i, in the sense of (23). The functions K satisfy also the relations $(t_f > t > t_i)$

$$K(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i) = \int_{R^3} K(\mathbf{r}_f, t_f; \mathbf{r}, t)\, K(\mathbf{r}, t; \mathbf{r}_i, t_i)\, d^3\mathbf{r} \tag{24}$$

Despite an evident analogy with the classical probabilistic relations, we should immediately point out that the differences between the two situations are deep. In fact, if ρ and p are respectively the probability density and the transition probability density of a classical Markov process describing the position of a particle, namely if they are the solutions of a Fokker–Planck equation satisfying respectively the initial conditions

$$\rho(\mathbf{r}, t_i) = \rho_i(\mathbf{r}) \tag{25}$$

$$\lim_{t \downarrow t_i} p(\mathbf{r}, t; \mathbf{r}_i, t_i) = \delta^3(\mathbf{r} - \mathbf{r}_i) \tag{26}$$

we know that they satisfy the Chapman–Kolmogorov relations

$$\rho(\mathbf{r}, t) = \int_{R^3} p(\mathbf{r}, t; \mathbf{r}', t_i)\, \rho_i(\mathbf{r}')\, d^3\mathbf{r}' \tag{27}$$

$$p(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i) = \int_{R^3} p(\mathbf{r}_f, t_f; \mathbf{r}, t)\, p(\mathbf{r}, t; \mathbf{r}_i, t_i)\, d^3\mathbf{r} \tag{28}$$

Evidently the relations (27) and (28) are not the analog of (22) and (24) since ψ and K are not probability densities but only probability amplitudes. In fact, if we calculate the probability densities by means of a square modulus, we obtain immediately that in general

$$|\psi(\mathbf{r}, t)|^2 \neq \int_{R^3} |K(\mathbf{r}, t; \mathbf{r}', t_i)|^2\, |\psi_i(\mathbf{r}')|^2\, d^3\mathbf{r}' \tag{29}$$

$$|K(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i)|^2 \neq \int_{R^3} |K(\mathbf{r}_f, t_f; \mathbf{r}, t)|^2\, |K(\mathbf{r}, t; \mathbf{r}_i, t_i)|^2\, d^3\mathbf{r} \tag{30}$$

namely that the quantum probability densities (square modulus of some wave function) do not satisfy the Chapman–Kolmogorov relations (27) and (28). Since these classical equations are seen as an expression of the fact that a Markov process, going from $\mathbf{r}_i$ to $\mathbf{r}_f$, must also go through some of the possible intermediate positions at intermediate times, the inequalities (29) and (30) are generally considered as a form of the opposite statement: *in quantum mechanics only the complex amplitudes can always be added, as* *in* (22) *and* (24), *but not in general the probability densities.* More precisely, if a particle travels from $\mathbf{r}_i$ to $\mathbf{r}_f$, we cannot think of it as being somewhere, with a given probability, at the intermediate times, unless we verify our statement by directly measuring the intermediate position. However, we know also that such a measurements would change the state of our system in such a particular way that afterwards the relations (27) and (28) will be verified, but a number of quantum effects, like interferences, will inevitably be lost. It is clear that in this situation it is apparently very difficult to speak of a trajectory between $\mathbf{r}_i$ and $\mathbf{r}_f$.

Let us illustrate this discussion by means of a simplified Young two-slit experiment (or, equivalently, a Rauch neutron interferometry experiment). Let us suppose that we have a screen, with two holes located in $\mathbf{r}_1$ and $\mathbf{r}_2$, between a source of particles and a detector located in $\mathbf{r}$, and let us ask for the probability densities of the detected particles in the following three situations:

Situation	Hole 1	Hole 2
"1"	open	closed
"2"	closed	open
"1, 2"	open	open.

From the Green function of the free Schrödinger equation[26]

$$K_0(\mathbf{r}, t; \mathbf{r}', t') = \left[\frac{m}{2\pi i\hbar(t-t')}\right]^{3/2} \exp\left[\frac{i}{\hbar}\frac{m(\mathbf{r}-\mathbf{r}')^2}{2(t-t')}\right]$$

and from the initial conditions $(t = t_i)$

$$\psi_i^{(1)}(\mathbf{r}) = \delta^3(\mathbf{r}-\mathbf{r}_1)$$
$$\psi_i^{(2)}(\mathbf{r}) = \delta^3(\mathbf{r}-\mathbf{r}_2)$$
$$\psi_i^{(1,2)}(\mathbf{r}) = c_1\delta^3(\mathbf{r}-\mathbf{r}_1) + c_2\delta^3(\mathbf{r}-\mathbf{r}_2)$$

(where $|c_1|^2 + |c_2|^2 = 1$) we can calculate the wave functions respectively in the three proposed situations:

$$\begin{aligned} \psi^{(1)}(\mathbf{r}, t) &= K_0(\mathbf{r}, t; \mathbf{r}_1, t_i) \\ \psi^{(2)}(\mathbf{r}, t) &= K_0(\mathbf{r}, t; \mathbf{r}_2, t_i) \\ \psi^{(1,2)}(\mathbf{r}, t) &= c_1 K_0(\mathbf{r}, t; \mathbf{r}_1, t_i) + c_2 K_0(\mathbf{r}, t; \mathbf{r}_2, t_i) \end{aligned} \tag{31}$$

and we can verify directly that the probability densities do not add:

$$|\psi^{(1,2)}(\mathbf{r}, t)|^2 \neq |c_1|^2\, |\psi^{(1)}(\mathbf{r}, t)|^2 + |c_2|^2\, |\psi^{(2)}(\mathbf{r}, t)|^2 \tag{32}$$

the difference between the two sides being, of course, in the interference terms. We should now contrast this situation with the classical description where, from (22) and the initial conditions

$$\rho_i^{(1)}(\mathbf{r}) = \delta^3(\mathbf{r} - \mathbf{r}_1)$$
$$\rho_i^{(2)}(\mathbf{r}) = \delta^3(\mathbf{r} - r_2)$$
$$\rho_i^{(1,2)}(\mathbf{r}) = a_1\delta^3(\mathbf{r} - \mathbf{r}_1) + a_2\delta^3(\mathbf{r} - \mathbf{r}_2)$$

(where $a_1 + a_2 = 1$; $a_1 \geqslant 0$, $a_2 \geqslant 0$), we get

$$\rho^{(1)}(\mathbf{r}, t) = p(\mathbf{r}, t; \mathbf{r}_1, t_i)$$
$$\rho^{(2)}(\mathbf{r}, t) = p(\mathbf{r}, t; \mathbf{r}_2, t_i)$$
$$\begin{aligned}\rho^{(1,2)}(\mathbf{r}, t) &= a_1\, p(\mathbf{r}, t; \mathbf{r}_1, t_i) + a_2\, p(\mathbf{r}, t; \mathbf{r}_2, t_i)\\ &= a_1\rho^{(1)}(\mathbf{r}, t) + a_2\rho^{(2)}(\mathbf{r}, t)\end{aligned}$$

and hence the probability densities would add. This striking contrast between the two descriptions of the case "1, 2" is the basis of the quantum mechanical rejection of the idea that particles come either from a hole or from the other (or, in the case of neutron interferometry, pass through a coil or the other) and, by extension, of the idea that the particles follow a *nondirectly observed* path when they travel from $\mathbf{r}_i$ to $\mathbf{r}_f$. As a consequence, the Feynman formula giving the Green function as a path integral,[(26)] namely

$$K(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i) = \int_{i \to f} \exp\left[\frac{i}{\hbar}\int_{t_i}^{t_f} \mathscr{L}\left(\frac{d\mathbf{r}}{dt}(t), \mathbf{r}(t), t\right) dt\right] \mathscr{D}[\mathbf{r}(\cdot)] \tag{33}$$

with the usual meaning of the symbols, cannot be assimilated (if not very formally) to a sum of probabilistic weights for real paths, but it represents only a formal addition of complex amplitudes.

However, today there are theories, inherently based on the idea that particles follow random trajectories in space and time, that extensively simulate the quantum results.[(27)] We will adhere in the following to a more recent formulation of this *stochastic mechanics*[9] derived from stochastic variational principles in the framework of a control theory.[(29)] This formulation has several advantages with respect to the previous one, beyond elegance and symmetry: first of all, a number of *assumptions* of the theory

[9] We should mention here that the stochastic mechanics is in a strict relation with the stochastic interpretation of the quantum mechanics (even if they do not coincide) started in the fifties with the work of Bohm and one of the authors (J.P.V.).[(28)]

can be deduced here from a unique variational principle; and secondly, and perhaps principally, it is a formulation which is not tied to a representation in the configuration space since a true transformation theory can be given. Of course we will limit ourselves here only to a few remarks on this theory that are essential to pursue our discussion and we will refer to the quoted bibliography for more details.

If, for the sake of simplicity, we consider a scalar nonrelativistic particle, the quantum behavior can be simulated by means of a stochastic process $\xi(t)$, describing the evolution of the particle position, which is a solution of a stochastic differential equation of the form

$$d\xi(t) = \mathbf{v}_{(+)}(\xi(t), t)\, dt + d\boldsymbol{\beta}(t) \tag{34}$$

where $\boldsymbol{\beta}(t)$ is a Wiener process with diffusion constant $\nu = \hbar/2m$. It is fundamental now to make the following remark: there are two possible different interpretational schemes of (34) leading to two completely different results.[30]

First of all we can think of $\mathbf{v}_{(+)}$ as a given field, fixed once and for all by the physics of our problem, and $\xi(t)$ as a complete description of the state of our system. In this case it can be shown that our theory is a stochastic generalization of the classical mechanics, leading to Langevin-type equations and to a dissipative dynamics that cannot reproduce the quantum effects.

However, there is a second way of looking at (34): we can suppose that $\xi(t)$ does not contain all the information about the state of our system, but is only a sort of configurational variable, the rest of the information being stored elsewhere. The analogy with a classical phase space description, where we need both the $q(t)$'s and the $p(t)$'s to define the state of a system, is somehow illuminating. But it must be recalled also that this analogy is only half correct, because here we do not have a direct analog for the conjugate momenta. In fact, it is well known that the process $\xi(t)$ is almost nowhere derivable, so that nothing like a *velocity* is definible here. This suggests the idea that somehow the conjugate variables must have a character formally (and maybe substantially) different from that of the positional variable. To do this, the stochastic mechanics introduces the so called *forward and backward derivatives* of $F(\xi(t), t)$, where $F(\mathbf{r}(t), t)$ is an arbitrary regular function, as

$$(D_{(\pm)}F)(\mathbf{r}, t) = \pm \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} E(\Delta^{(\pm)}F \mid \xi(t) = \mathbf{r}) \tag{35}$$

where

$$\Delta^{(\pm)}F = F(\xi(t \pm \Delta t), t \pm \Delta t) - F(\xi(t), t)$$

and hence the *forward and backward velocities*[27]

$$\mathbf{v}_{(\pm)}(\mathbf{r}, t) = (D_{(\pm)}\xi)(\mathbf{r}, t) \tag{36}$$

Here we utilized the fact that the operation of conditional expectation produces a smoothing of the fluctuations of the stochastic process that allows the definition of a derivative. A central feature of this model is that we will take both $\xi(t)$ and $\mathbf{v}_{(+)}(\mathbf{r}, t)$ as the dynamical variables of our problem so that $\mathbf{v}_{(\pm)}$ will not be given *a priori* but will be determined, as a part of the problem, by means of a *stochastic variational principle* which will select the physically meaningful *(measurable)* processes among all the possible *(virtual)* processes described by (34). By doing so we will have a model which is a stochastic control theory.[29–31] and which, by means of a suitable choice of the Lagrangian which we cannot discuss here, leads directly to a perfect reproduction of the quantum results.

It is also interesting to look at this procedure from the opposite side: starting from the Schrödinger equation (21) for a given quantum system, we can reconstruct the stochastic processes that simulate the quantum behavior. In fact, we should remember that, for a given Schrödinger equation (namely, for a given quantum system) there is not only one process associated, but rather an entire family of processes. Namely, it is possible to show that we can associate a different process to every quantum state, i.e., to every solution $\psi(\mathbf{r}, t)$ of (21), in the following way: from the decomposition

$$\psi(\mathbf{r}, t) = R(\mathbf{r}, t)\, e^{iS(\mathbf{r}, t)/\hbar} \tag{37}$$

we can calculate the forward velocity as

$$\mathbf{v}_{(+)}(\mathbf{r}, t) = 2\nu\nabla W_{(+)}(\mathbf{r}, t) \tag{38}$$

where

$$W_{(+)}(\mathbf{r}, t) = \ln[R(\mathbf{r}, t)\, e^{S(\mathbf{r}, t)/\hbar}]$$

and hence we fix (for this particular, given wave function) the form of (34). The uniqueness of its solution is guaranteed by an initial condition[32]

$$\xi(t_i) = \xi_i \tag{39}$$

which can also be deduced by determining the random variable ξ_i from the given initial probability density $|\psi(\mathbf{r}, t_i)|^2$. The unique solution of (34) under the initial conditions (39) will coincide with the process chosen by our stochastic variational principle. Of course to a different ψ will be associated a different process.

The manner of dependence of the process ξ on the wave function is also apparent from the remark that, if through (37) we separate (21) into its real and imaginary parts

$$\partial_t R^2 + \nabla\left(R^2 \frac{\nabla S}{m}\right) = 0 \tag{40}$$

$$\partial_t S + \frac{m}{2}\left(\frac{\nabla S}{m}\right)^2 - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V = 0 \tag{41}$$

we can always cast the continuity equation (40) in the form of a forward Fokker–Planck equation for a density $\rho = R^2$ with $\mathbf{v}_{(+)}$ given in (36)

$$\partial_t \rho = -\nabla(\rho \mathbf{v}_{(+)}) + \nu \nabla^2 \rho \tag{42}$$

but we must also remember that now (42) is not a Fokker–Planck equation in the usual sense since $\mathbf{v}_{(+)}$, as remarked before, is not an *a priori* given function and in fact it depends in its turn on the solution ρ of (42) through (39). We can see that even from another point of view: if we fix a solution ψ of (21), the form of (42) (namely $\mathbf{v}_{(+)}$) will also be fixed; however, (42) will have an infinity of solutions ρ. Among these solutions only one satisfies the stochastic variational principle. This solution verifies $\rho = R^2 = |\psi|^2$, so that it corresponds to the actual probability density and can be considered as selected through the initial condition (25) with $\rho_i(\mathbf{r}) = R(\mathbf{r}, t_i)$. Of course other solutions corresponding to different initial conditions are formally available: for example, there are transition probabilities $p(\mathbf{r}, t; \mathbf{r}_i, t_i)$ solutions of (42) for initial condition like (26). But in general they are in some sense virtual since they do not satisfy the stochastic variational principle, so that we cannot associate to them a direct meaning of physical observability.[30] Of course, these solutions do not correspond to the square modulus of a wave function solution of (21). Summarizing: to a given Schrödinger equation (21) we can associate, by calculating $\mathbf{v}_{(+)}$ from (38), an entire family of Fokker–Planck equations, one for every solution ψ; every particular Fokker–Planck equation describes an entire family of stochastic processes, one for every solution determined by a initial condition; however, only one among these stochastic processes satisfies the stochastic variational principle, corresponds to the initial condition deducible from ψ, and has a probability density coincident with $|\psi|^2$; all the other (virtual) processes, which nevertheless are formally perfectly meaningful from the point of view of the stochastic differential equation (34), cannot be put in correspondence with a wave function solution of (21).[10]

Let us now rediscuss the two-slit experiment in the light of these remarks. From the wave functions (31) we determine the Fokker–Planck equations corresponding to every situation by calculating the suitable form of $\mathbf{v}_{(+)}$ from (38). Of course in the situations "1" and "2" the transition probabilities (from the slits to the screen) are

$$\begin{aligned} p^{(1)}(\mathbf{r}, t; \mathbf{r}_1, t_i) &= |\psi^{(1)}(\mathbf{r}, t)|^2 \\ p^{(2)}(\mathbf{r}, t; \mathbf{r}_1, t_i) &= |\psi^{(1)}(\mathbf{r}, t)|^2 \end{aligned} \tag{43}$$

As for the situation "1, 2" the solution selected by the quantum mechanics (or equivalently by the stochastic variational principle) for the corresponding Fokker–Planck equation is $|\psi^{(1,2)}|^2$, giving the probability density on the screen (interference) when initially the probability is concentrated around both the hole "1" and the hole "2." Of course in our scheme $|\psi^{(1,2)}|^2$ is the probability distribution function of a stochastic process and hence it follows trajectories in the spacetime. However, if we try to consider this probability distribution function as the sum of suitable transition probabilities, we get in trouble. In fact, for a fixed $\mathbf{v}_{(+)}$, the Fokker–Planck equation is a perfectly classical one, so that we can calculate the solutions $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_1, t_i)$ and $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_2, t_i)$, namely the transition probability densities, respectively from "1" and from "2" to the screen *when two holes are open*, but we should bear in mind that

$$\begin{aligned} p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_1, t_i) &\neq p^{(1)}(\mathbf{r}, t; r_1, t_i) \\ p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_2, t_i) &\neq p^{(2)}(\mathbf{r}, t; \mathbf{r}_2, t_i) \end{aligned} \tag{44}$$

and that, differently from $p^{(1)}(\mathbf{r}, t; \mathbf{r}_1, t_i)$ and $p^{(2)}(\mathbf{r}, t; \mathbf{r}_2, t_i)$, the transition probabilities $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_1, t_i)$ and $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_2, t_i)$ cannot be calculated as the square modulus of a wave function; namely that they are the probability densities of *virtual* processes. Of course, since $|\psi^{(1,2)}(\mathbf{r}, t)|^2$, $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_1, t_i)$, and $p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_2, t_i)$ are solutions of the same classical Fokker–Planck equation, classical probabilities superpose as usual. We will thus have

$$|\psi^{(1,2)}(\mathbf{r}, t)|^2 = |c_1|^2\, p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_1, t_i) + |c_2|^2\, p^{(1,2)}(\mathbf{r}, t; \mathbf{r}_2, t_i)$$

[10] In that sense the Schrödinger equation can be assimilated to a constraint describing an average moving equilibrium situation. The *virtual* processes described in this paragraph can be considered physically real in the usual sense of the word but do not satisfy the least action principle associated to the Schrödinger equation, so that their properties cannot be directly tested experimentally.

and this result is not in contradiction with (32) because we have to take into account (43) and (44). As a consequence, the interference pattern can always be considered as the sum of suitable transition probability densities for particles coming either from the hole "1" or from the hole "2," but only in the above described sense of the *virtual* processes. Of course here the interference effects could be seen as incorporated in the form of the $\mathbf{v}_{(+)}$ deduced from $\psi^{(1,2)}$. In other words, in the framework of stochastic mechanics we get the following situation: on the one hand, we can always calculate suitable classical conditional probabilities and use them in a classical way, even in the interference experiments, but we should remember that in this case some of these processes are *virtual*. On the other hand, we must also remember that the function $|\psi^{(1,2)}|^2$ giving the interference pattern is the probability distribution function of a classical stochastic process with all its spacetime trajectories.

In order to stress once more the differences between the two points of view, we will show now that in a stochastic mechanical context we can calculate all the transition probability densities by means of path integrals if all the processes (*measurable* and *virtual*) solutions of the stochastic differential equation (34) are taken into account, and hence every transition from $\mathbf{r}_i$ to $\mathbf{r}_f$ can always be considered as built (in a probabilistic way) by means of all the (*observable* or *virtual*) paths between $\mathbf{r}_i$ and $\mathbf{r}_f$. Here, however, differently from (33), the statistical weight of each trajectory is real and positive and not complex.[33] We will briefly remember that an arbitrary stochastic process $\boldsymbol{\eta}(t)$, $t \in T \equiv [t_i, t_f]$, defined on a probability space $(\Omega, \mathscr{F}, P)$ takes its values on the probabilizable space $(R^{3T}, \mathscr{B}(R^{3T}))$ and induces on it a probability P_η. For example, the Wiener process $\boldsymbol{\beta}$ characterized by a transition probability density

$$p_w(\mathbf{r}+\Delta\mathbf{r}, t+\Delta t; \mathbf{r}, t) = (4\pi\nu\Delta t)^{-3/2} \exp\left[-\frac{(\Delta\mathbf{r})^2}{4\nu\Delta t}\right], \qquad \Delta t \geqslant 0 \quad (45)$$

defines on the trajectory space the well-known Wiener measure P_w by extension of the probability measure defined on the cylinders of $\mathscr{B}(R^{3T})$. The other measures that we will consider here are defined by the processes solutions of (34). In particular, the process $\boldsymbol{\xi}_0(t)$ corresponding to $\mathbf{v}_{(+)} \equiv 0$ will define a measure coincident with P_w. A theorem due to Girsanov states that all these measures are connected in the sense that, if P_1 and P_2 are defined by two solutions $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ corresponding to two different $\mathbf{v}_{(+)}^{(1)}$ and $\mathbf{v}_{(+)}^{(2)}$, but to the same initial conditions, then P_2 is absolutely continuous with respect to P_1 and the Radon derivative takes the form

$$\frac{dP_2}{dP_1}[\boldsymbol{\xi}_1(\cdot)] = e^{U[\boldsymbol{\xi}_1(\cdot)]} \qquad (46)$$

where U is defined by means of an Itô stochastic integral

$$U[\xi_1(\cdot)] = \frac{1}{2\nu}\int_{t_i}^{t_f} \delta\mathbf{v}_{(+)}(\xi_1(t), t)\, d\boldsymbol{\beta}(t) - \frac{1}{4\nu}\int_{t_i}^{t_f} [\delta\mathbf{v}_{(+)}(\xi_1(t), t)]^2\, dt \tag{47}$$

where $\delta\mathbf{v}_{(+)} = \mathbf{v}_{(+)}^{(2)} - \mathbf{v}_{(+)}^{(1)}$, which, in particular, when $\xi_1 = \xi_0$, becomes

$$U[\xi_0(\cdot)] = \frac{1}{2\nu}\int_{t_i}^{t_f} \mathbf{v}_{(+)}(\xi_0(t), t)\, d\boldsymbol{\beta}(t) - \frac{1}{4\nu}\int_{t_i}^{t_f} [\mathbf{v}_{(+)}(\xi_0(t), t)]^2\, dt \tag{48}$$

Hence the conditional expectation values of an arbitrary functional $F[\xi(\cdot)]$ can always be calculated by means of the Wiener measure induced by ξ_0:

$$E(F[\xi(\cdot)] \mid \xi_i) = E(F[\xi_0(\cdot)]\, e^{U[\xi_0(\cdot)]} \mid \xi_i) \tag{49}$$

If now $J_A[\mathbf{r}(\cdot); t]$ is the indicator functional of the one-dimensional cylinder

$$C_t(A) = \{\mathbf{r}(\cdot) : \mathbf{r}(t) \in A\};\ t \in R;\ A \in \mathscr{B}(R^3)$$

we have

$$\int_A p(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i)\, d^3\mathbf{r}_f = E(J_A[\xi_0(\cdot); t_f]\, e^{U[\xi_0(\cdot)]} \mid \xi_i = \mathbf{r}_i)$$

and calculating it as a sequence of approximations by means of partitions of $[t_i, t_f]$ in n subintervals, we have that the nth iteration

$$\int_{R^3} d^3\mathbf{r}_1 \cdots \int_{R^3} d^3\mathbf{r}_{n-1} (4\pi\nu\Delta t)^{-3n/2}$$
$$\times \exp\left[-\frac{1}{4\nu\Delta t}\sum_{k=0}^{n-1} (\Delta\mathbf{r}_k - \mathbf{v}_{(+)}(\mathbf{r}_k, t_k)\,\Delta t)^2\right] \tag{50}$$

tends to p when eventually $n \to \infty$, and the path integral

$$p(\mathbf{r}_f, t_f; \mathbf{r}_i, t_i)$$
$$= \int_{i \to f} \exp\left[-\frac{1}{4\nu}\int_{t_i}^{t_f}\left(\frac{d\mathbf{r}}{dt}(t) - \mathbf{v}_{(+)}(\mathbf{r}(t), t)\right)^2 dt\right] \mathscr{D}[\mathbf{r}(\cdot)] \tag{51}$$

is the formal statement of the fact that p is the limit of (50) when $n \to \infty$. This expression, despite the formal analogy with (33), is now a true probabilistic statement, as the outlined derivation shows, and is based on the idea that the particles travel along a path when going from $\mathbf{r}_i$ to $\mathbf{r}_f$. It represents also, in some sense, a partial answer to the problems posed by Feynman and discussed in the Introduction: only partial, since we must also remember that in (51) both *observable* and *virtual* processes are taken into account.

6. CONNECTIONS AND POSSIBLE DEVELOPMENTS

We want now connect the previous analysis with other fields of research by making three remarks.[11] The first deals with evident consequences from the existence of real spacetime (timelike) single-particle paths in the spacetime of G.R.T. They can be listed as follows:

— The particles can be individually described by extended structures and canonically conjugated pairs of variables (P_μ, Q_μ) with respect to a Lagrangian and Hamiltonian formalism which explicitly contains terms representing the effects of the quantum potential introduced into the theory by de Broglie, Bohm, and others.[(34)] This quantum potential which acts on the particle velocity also contains terms (torques) which modify its associated spin vector, i.e., its internal angular momentum.

— The real existence of real P, Q particle variables does not invalidate the Heisenberg uncertainty relations $\Delta P \Delta Q \geqslant \hbar/2$ since the corresponding ΔP and ΔQ in these relations reflect, in this model, uncertainties introduced by the real physical interaction between real macroscopic measuring devices with individual microobjects, i.e., particles and their surrounding physical ψ field, as suggested by Popper *et al.*[(35)] This interpretation of the Heisenberg uncertainties can be tested by experiment.

— The existence of particle trajectories under the action of the quantum potential implies that their energies vary (with varying boundary conditions) even in free space in the absence of all known physical external interactions. This implies that the Einstein–de Broglie wave-plus-particle model should be completed and cannot survive in its point-particle pilot-wave form. Since the total energy of the waves is conserved by the quantum equations, the particle energy variation cannot be drawn from the wave itself. This also implies that quantum particle motions do not satisfy energy conservation unless one finds another energy

[11] These remarks represent, at the present, only the opinions of one of the authors (J.P.V.).

source associated with the quantum potential. The only known reasonable answer is to be found in de Broglie's double-solution model where the wave-plus-particle system corresponds to a nonlinear wave solution (which contains a particle-like soliton-like singularity) of a nonlinear wave equation. In this situation it has been shown, at least in the case of scalar waves, that everything goes as if the singularity is effectively piloted by a linear solution of the linear part of the wave equation.[36]

The second remark deals with the consequences and the implications of the existence of correlated quantum particle trajectories in many-body systems described by *entangled* quantum states. As we shall see, this is where quantum mechanics introduces, thanks to Bohm, Bell, Aspect *et al.* (but contrarily to Einstein's and de Broglie's hopes) a new concept in G.R.T., i.e., the concept of causal action at a distance described by the many-body quantum potential. To clarify this point, one must first discuss the physical signification of nonlocality and causality in configuration and phase space in relativistic quantum mechanics and its consequences. We limit this discussion to two correlated relativistic particles (since its extension to the N particle case is evident) described by their paths $x^1_\mu(\tau_1)$, $x^2_\mu(\tau_2)$. Following Einstein's ideas, if one wants to justify the Bohr–Sommerfeld quantization laws (i.e., $\oint P\, dQ = nh$), one must work with a particular set of canonical variables P_i, Q_i associated with an action function S by the relations

$$P^{1,2}_\mu = \partial^{1,2}_\mu S(Q_1, Q_2, \tau) \tag{52}$$

and work with the special Hamiltonian $H(\partial^1 S, \partial^2 S, Q^1, Q^2, \tau)$. Relations (52) evidently show that the motion remains on an eight-dimensional surface Ω in phase space (defined by $H = \text{const}$) which corresponds to configuration space. Any allowed path on this Ω represents a possible correlated motion in Σ_4 of our two particles 1 and 2. If quantum particles move in Σ_4, this raises a crucial theoretical point: i.e., the causal (or not) nature of such correlated motions tied to *entangled* quantum states. As one now knows the theoretical analysis by E.P.R., Bohm and Bell has shown that the present quantum mechanical formalism is essentially nonlocal, a fact now strongly confirmed by the experiments of Aspect, Mandel, and others. One can thus only save Einstein's causality by showing the causal nature of the particular nonlocal interactions tied to the quantum potential, as has been done by the authors and Droz-Vincent[37] in the following way. We first define what we mean with the word *causality*:

(a) the system of our two particles can be solved in the forward (or backward) time direction in the sense of the Cauchy problem;

(b) the paths of all material particles must be timelike;

(c) the formalism must be invariant under the Poincaré group $P = T \otimes \mathscr{L} \uparrow$.

It is possible to show[38] that in this case one can have action at a distance and preserve causality. If we start with the Hamiltonians

$$H_i = H_{0i} + V_i, \qquad i = 1, 2 \tag{53}$$

where $H_{0i} = p_i^2/2 = m_i^2/2$ and V_i are nonlocal potentials, the existence of world lines requires, for identical particles, the vanishing of the Poisson brackets $\{H_1, H_2\}$. With the separation of internal and external variables

$$\begin{aligned} P^\mu &= p_1^\mu + p_2^\mu, \qquad & y^\mu &= \frac{p_1^\mu - p_2^\mu}{2} \\ Q^\mu &= \frac{q_1^\mu + q_2^\mu}{2}, \qquad & z^\mu &= q_1^\mu - q_2^\mu \end{aligned} \tag{54}$$

the condition for the existence of causal timelike world lines becomes $\{y \cdot P, V\} = 0$, which implies that V depends on $\tilde{z}^2$, P^2, $\tilde{y}^2$, $\tilde{z} \cdot \tilde{y}$, $y \cdot P$, where $\tilde{z}^\mu = z^\mu - (z_\nu P^\nu) P^\mu / P^2$, $\tilde{y}^\mu = y^\mu - (y_\nu P^\nu) P^\mu / P^2$, but it must not depend on $z \cdot P$.

In the framework of the covariant canonical transformation theory the Hamilton Jacobi system for the characteristic function W is

$$H_i\left(q_1^\mu, q_2^\mu; \frac{\partial W}{\partial q_1^\mu}, \frac{\partial W}{\partial q_1^\mu}\right) = \frac{m^2}{2}, \qquad i = 1, 2 \tag{55}$$

One remarks here[39] that the canonical variables q_i^μ are not coincident with the positions x_i^μ except when the interaction vanishes.

By quantizing this system of two free particles, we obtain two Klein–Gordon equations which are equivalent to the system

$$(\Box_1 + \Box_2)\, \psi(x_1, x_2) = 2m^2 \psi(x_1, x_2) \tag{56}$$

$$(\Box_1 - \Box_2)\, \psi(x_1, x_2) = 0 \tag{57}$$

If now we separate the real and imaginary parts of (56) with $\psi = \exp(R + iW)$, we obtain the system

$$\frac{1}{2}(\partial_{i_\mu} W \partial_i^\mu W) + U_i = \frac{m^2}{2}, \qquad i = 1, 2 \tag{58}$$

with quantum potentials[34]

$$U_i = -\tfrac{1}{2}(\Box_i R + \partial_{1_\mu} R \partial_2^\mu R), \qquad i = 1, 2 \tag{59}$$

Despite an obvious analogy the system (58) cannot be immediately identified with (55). To be more specific, let us consider

$$\psi = e^{iK_\mu(x_1^\mu + x_2^\mu)/2}\psi(z_\mu)$$

where K_μ is a constant timelike vector. In this case R depends only on $z_\perp^\mu = z^\mu - (z_\nu K^\nu)\, K^\mu/K^2$ so that $U_1 = U_2 = U = f(z_\perp^\mu)$ and hence U cannot satisfy the condition $\{y \cdot P, V\} = 0$ unless we make the substitution $z_\perp^\mu \to \tilde{z}^\mu$. Only in this case can we get the correct dependence $V = f(\tilde{z})$ and interpret V as a relativistic potential.

At this point we have exhibited a mathematical analogy between a system of two quantum free particles and a system of *fictitious*, but causally interacting, particles. We are going to recall now the physical interpretation in four points:

(A) We can give a physical basis to our quantum potential only if we consider the ψ-field of a quantum particle not as a pure mathematical tool but as a real wave field on a subquantal medium.[40] Indeed, it is well known, since Dirac's pioneer work,[41] that Einstein relativity theory is compatible with a relativistic stochastic ether model, so that quantum statistics will reflect the real random fluctuations of a particle embedded in this ether.[42] More precisely, the quantum potential is now interpreted as a real interaction among the particles and the subquantal fluid polarized by the presence of the particles. The quantum potential now represents a true stochastic potential.

(B) One has shown that the existence of a quantum ether allows one to deduce the relativistic quantum equations for single free particles[42] and for systems of two particles[43] as describing the stochastic motion of classical particles in interaction with the ether, if the random jumps are made at the velocity of light.

(C) The causality implied in our model is absolute in the sense that the measuring processes themselves satisfy the same causal law and are real physical processes with antecedents in time. The measuring process (observer plus apparatus plus observed particles) is a set of particles which are part of an overall causal process. In this scheme the intervention of a measuring process contains no supranatural *free will* or *observer consciousness*. The causal character is related to the constraint $\{H_1, H_2\} = 0$, which implies that the canonically conjugated proper times τ_1 and τ_2 are independent variables.

(D) The nonlocal interactions associated with quantum mechanics cannot propagate signals since the corresponding particles cannot exchange energy in their rest frames. As a consequence, the order of events along the timelike paths followed by all known particles is identical (i.e., invariant) for all inertial observers. Of course such nonlocal interaction cannot be instantaneous in all frames (a property evidently incompatible with relativity theory) but only in the particle center-of-mass rest frame. In that case no energy is exchanged, i.e., there are no possible signals carried by the quantum potential.

The last remark is that the existence of particle trajectories opens a path to unify G.R.T. and quantum mechanics in a completely different way which contradicts the still unsuccessful attempts to quantize G.R.T. and reduce it to a part of the world of quantum theory. Indeed if it could be shown in general that

— the real linear ψ-field modifies (as in the case of massive particles in G.R.T.) the $g_{\mu\nu}$-field in such a way that the corresponding de Broglie–Bohm particle path corresponds to geodetics of the metric perturbed by the real ψ-field (as has been done in particular cases[44]),

— that the waves associated to N particles of the set $\{n_i\}$, $i = 1,..., N$, yield N paths which correspond to ψ_i-states and N-states of their sourrounding metric $g_{\mu\nu}$, these paths thus representing real possible motions between N pairs of fixed points, i.e., being weighted with positive probabilities in such a way that the Fokker–Planck equation derived from the average statistical ψ-waves represents an average geodetic between these points,

we would be well on our way (1) to consider the stochastic interpretation of quantum mechanics as a result of particle motions in G.R.T. under the influence of a fluctuation surrounding $g_{\mu\nu}$ background; (2) to construct a nonlocal realist version of quantum mechanics within the frame of Einstein's world model.

7. CONCLUSIONS

As is clear from the previous discussion, no conclusive, theoretical or experimental, argument is at hand today on the problem of the trajectory description of quantum interferences. In the theoretical field we can say that in the interfering case "1, 2" we can always reconstruct the interference pattern $|\psi^{(1,2)}|^2$ by means of the transition probabilities of going from "1" to the screen or from "2" to the screen, and hence we can now *think* that particles follow trajectories. However, the processes that realize these configurations contain some virtual paths that do not satisfy the equilibrium stochastic variational principle. In other words, all the transition probabilities of our problem can be calculated as superpositions of positive real probabilities (not only amplitudes), but we should take into account the virtual processes discussed above. Hence, even if we can say that it is now thinkable to describe the quantum interference phenomena in terms of trajectories of stochastic processes, we must also admit that the question still involves some ambiguity.

From the experimental point of view, the last proposals on neutron interferometry now seem to make it possible to gather strong indirect evidence for the fact that the neutron pass through one coil only (even if nobody pretends to say which one), namely for so-called *Einweg* statements. However, the discussion of the previous sections also shows that, in the absence of a direct measurement showing which way the neutron has passed (namely, in the absence of a true *Welcherweg* experiment), it is still possible to argue that nothing has been conclusively decided. In the words of Feynman himself: "...the statement 'B has some value' may be meaningless whenever we make no attempt to measure B."

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