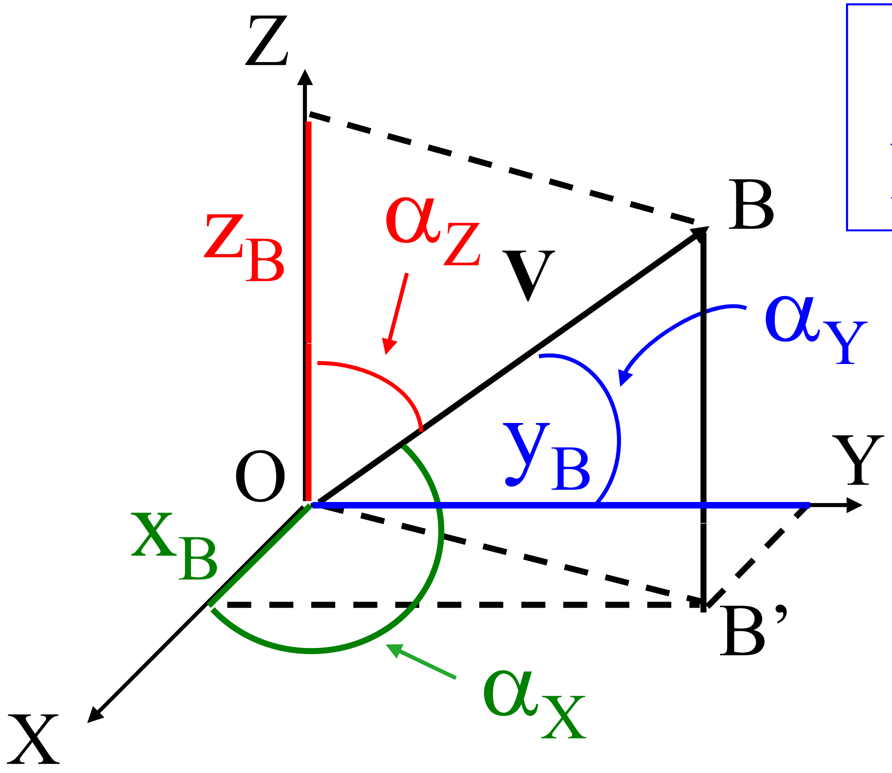
I VETTORI

grandezze scalari:
vengono definite dal loro valore numerico

grandezze vettoriali:
oltre al loro valore numerico,
vengono assegnati una direzione e un verso



modulo di **V** =
lunghezza del segmento OB

direzione di **V** determinata da

$\cos \alpha_X$

$\cos \alpha_Y$

$\cos \alpha_Z$

$\mathbf{V} = \mathbf{OB}$

O (0,0,0) B (x_B, y_B, z_B)

$V_X = x_B$
$V_y = y_B$
$V_Z = z_B$

proiezioni di **OB** sui tre assi = componenti di **V** lungo i tre assi cartesiani

V (V_X , V_Y , V_Z)

Se il primo estremo di **V** non coincide con O

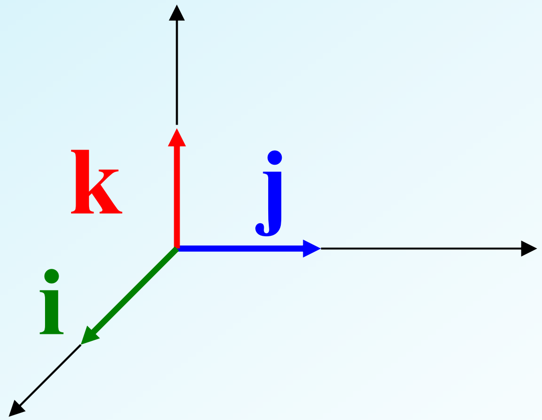
$\mathbf{V} = \mathbf{AB}$

A (x_A, y_A, z_A) B (x_B, y_B, z_B)

$V_X = x_B - x_A$
$V_y = y_B - y_A$
$V_Z = z_B - z_A$

Versore = vettore di lunghezza unitaria

i **j** **k** versori degli assi coordinati



i (1,0,0)

j (0,1,0)

k (0,0,1)

ALGEBRA VETTORIALE

PRODOTTO DI UN VETTORE A PER UNO SCALARE m

$\mathbf{B} = m\mathbf{A}$ vettore parallelo ad **A**

$|\mathbf{B}| = |m|\,|\mathbf{A}|$

verso di **B**:
- concorde col verso di **A** se $m > 0$
- opposto al verso di **A** se $m < 0$

SOMMA DI DUE VETTORI A E B

$\mathbf{A}\ (A_X, A_Y, A_Z)$ $\mathbf{B}\ (B_X, B_Y, B_Z)$

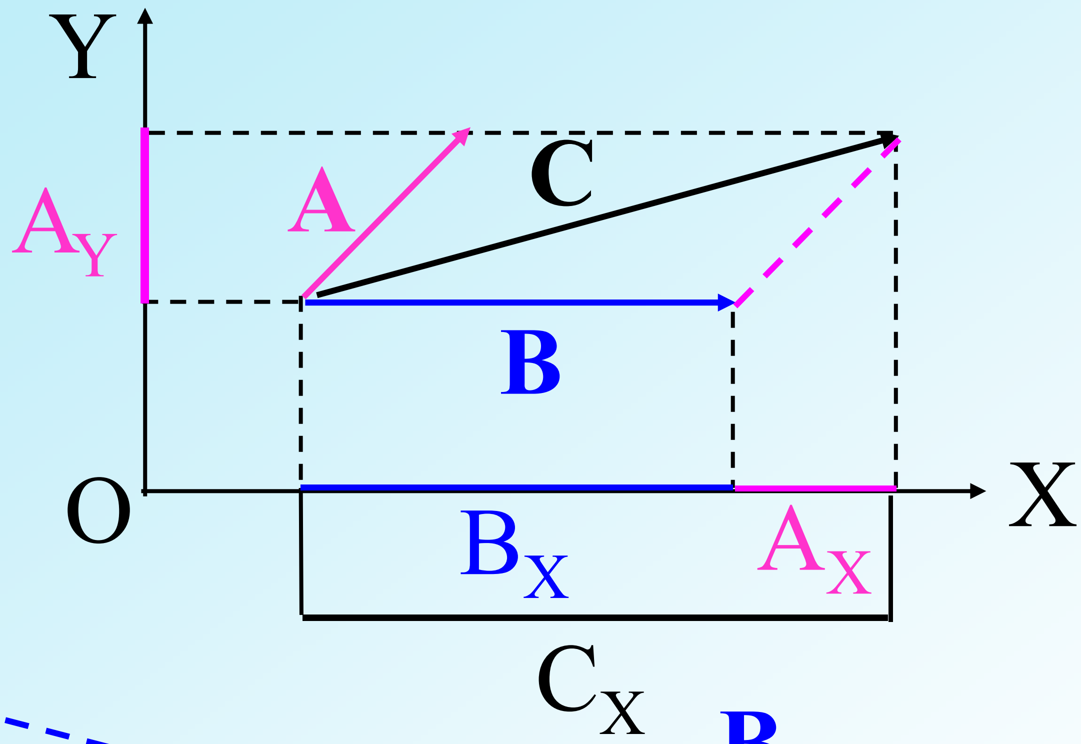
$\mathbf{C} = \mathbf{A} + \mathbf{B}$ $\mathbf{C}\ (C_X, C_Y, C_Z)$

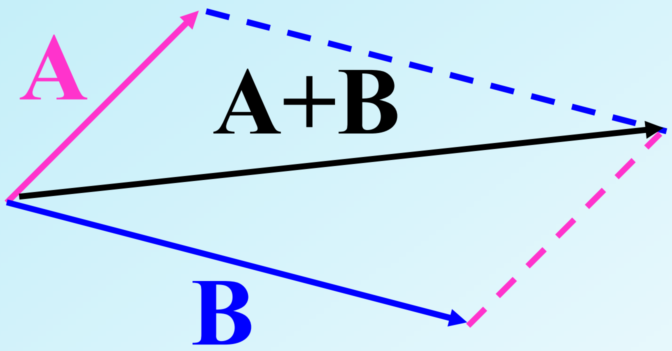
$$C_X = A_X + B_X$$

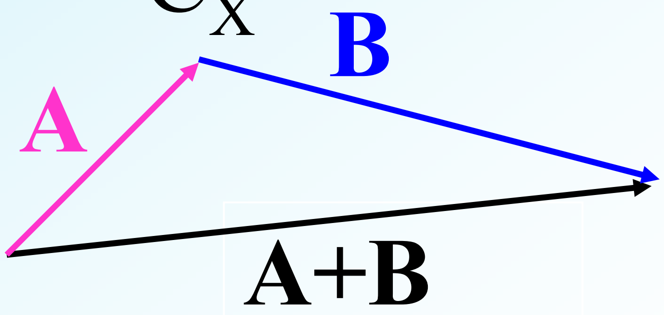
$$C_Y = A_Y + B_Y$$

$$C_Z = A_Z + B_Z$$

In un piano XY



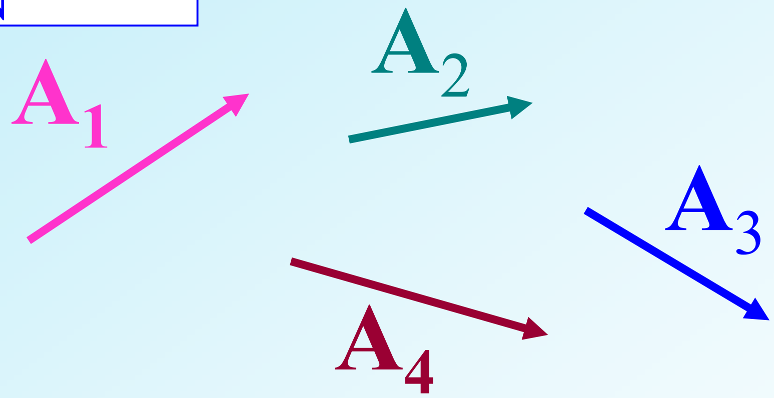




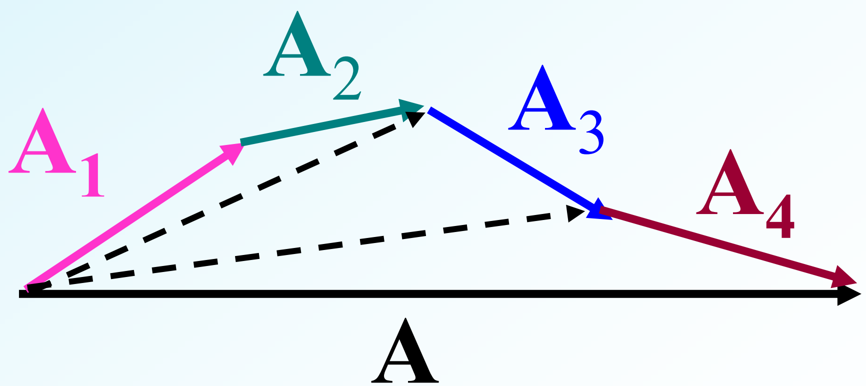
C vettore somma =
diagonale del parallelogramma
avente per lati i vettori **A** e **B**

SOMMA DI N VETTORI

$\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots \mathbf{A}_N$



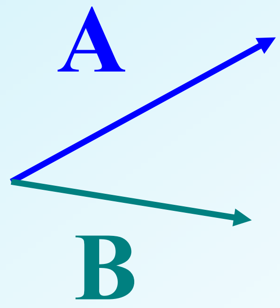
$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \ldots \mathbf{A}_N$ vettore che congiunge il primo estremo di $\mathbf{A}_1$ con il secondo estremo di $\mathbf{A}_N$

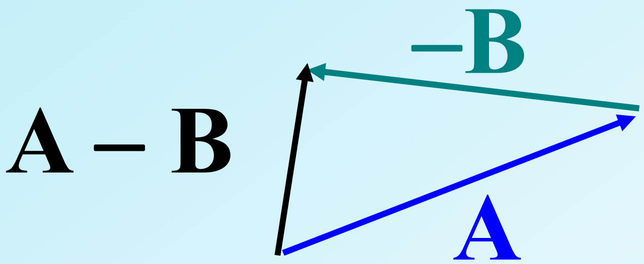


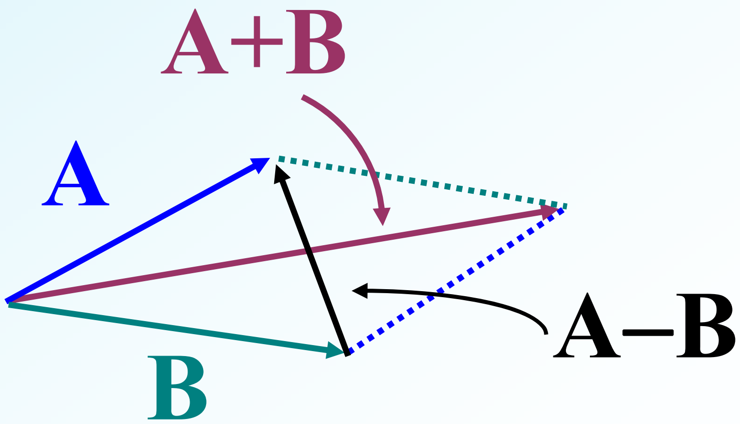
DIFFERENZA DI DUE VETTORI A e B :

⇕

somma dei vettori **A** e **–B**



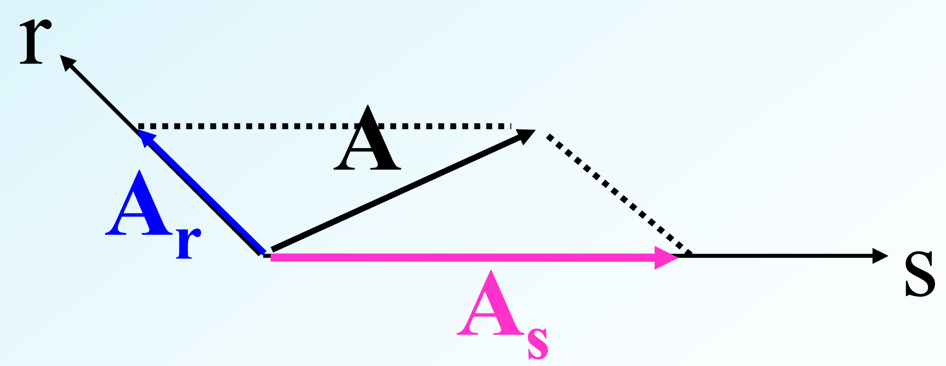




SCOMPOSIZIONE DI UN VETTORE **A** LUNGO DUE DIREZIONI ORIENTATE r ed s:

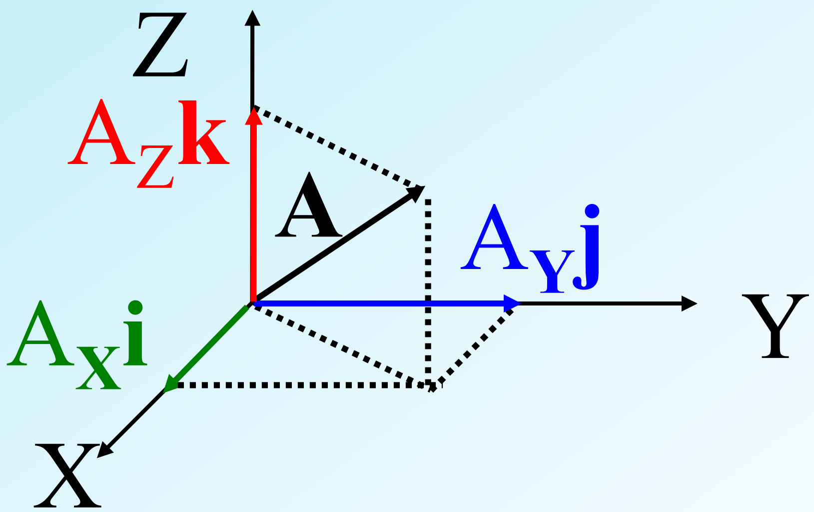
determinazione di due vettori paralleli a r ed s la cui somma è **A**

$$\mathbf{A} = \mathbf{A_r} + \mathbf{A_s}$$



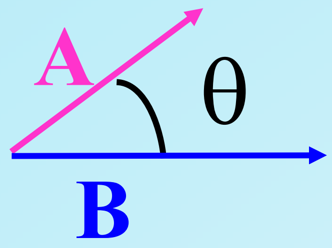
SCOMPOSIZIONE LUNGO GLI ASSI CARTESIANI

$$\mathbf{A} = A_X\,\mathbf{i} + A_Y\,\mathbf{j} + A_Z\,\mathbf{k}$$



PRODOTTO SCALARE TRA DUE VETTORI **A** e **B**

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}| \cdot |\mathbf{B}| \cos\theta$$



Si ottiene una grandezza scalare

$$\mathbf{A} \bullet \mathbf{B} = 0 \rightarrow \begin{cases} \mathbf{A} = 0 \\ \mathbf{B} = 0 \\ \mathbf{A} \perp \mathbf{B} \end{cases}$$

$$\mathbf{A} \bullet \mathbf{A} = A\,A \cos 0 = A^2$$

$$\mathbf{i} \bullet \mathbf{i} = 1 \qquad \mathbf{j} \bullet \mathbf{j} = 1 \qquad \mathbf{k} \bullet \mathbf{k} = 1$$

$$\mathbf{i} \bullet \mathbf{j} = 0 \qquad \mathbf{j} \bullet \mathbf{k} = 0 \qquad \mathbf{i} \bullet \mathbf{k} = 0$$

Proprietà del prodotto scalare:

proprietà commutativa

$$\mathbf{A} \bullet \mathbf{B} = \mathbf{B} \bullet \mathbf{A}$$

proprietà distributiva

$$\mathbf{A} \bullet (\mathbf{B} + \mathbf{C}) = \mathbf{A} \bullet \mathbf{B} + \mathbf{A} \bullet \mathbf{C}$$

In termini di componenti cartesiane

$$\mathbf{A} \bullet \mathbf{B} = (A_X\,\mathbf{i} + A_Y\,\mathbf{j} + A_Z\,\mathbf{k}) \bullet (B_X\,\mathbf{i} + B_Y\,\mathbf{j} + B_Z\,\mathbf{k}) = A_X B_X + A_Y B_Y + A_Z B_Z$$

$$\mathbf{A} \bullet \mathbf{A} = A^2 = A_X^2 + A_Y^2 + A_Z^2$$

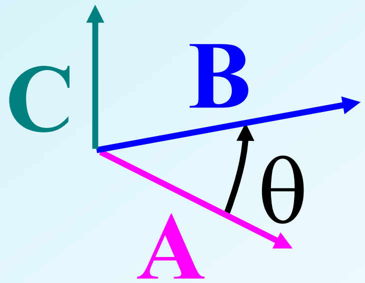
PRODOTTO VETTORIALE DI DUE VETTORI **A** e **B**

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

oppure

$$\mathbf{C} = \mathbf{A} \wedge \mathbf{B}$$

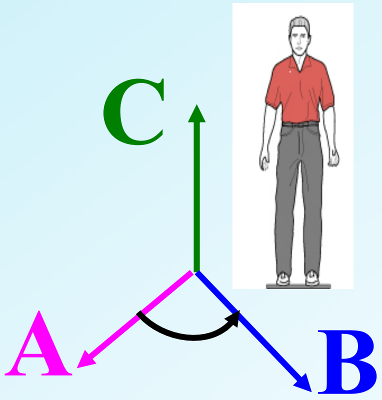
C vettore

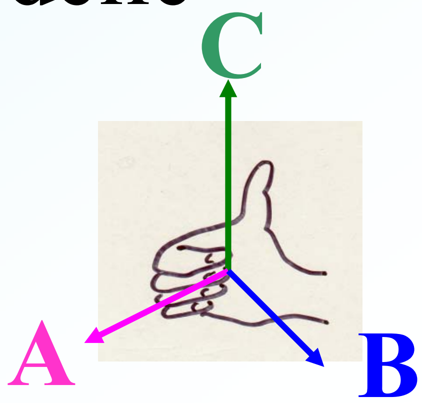


modulo di **C** : $C = A\,B\,\mathrm{sen}\,\theta$

direzione di **C** : perpendicolare al piano definito da **A** e **B**

verso di **C** : definito da una delle seguenti regole





Proprietà del prodotto vettoriale:

proprietà anticommutativa

$$\mathbf{A} \wedge \mathbf{B} = -\,\mathbf{B} \wedge \mathbf{A}$$

proprietà distributiva

$$\mathbf{A} \wedge (\mathbf{B} + \mathbf{C}) = \mathbf{A} \wedge \mathbf{B} + \mathbf{A} \wedge \mathbf{C}$$

$$\mathbf{i} \times \mathbf{i} = 0 \qquad \mathbf{j} \times \mathbf{j} = 0 \qquad \mathbf{k} \times \mathbf{k} = 0$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} = -\,\mathbf{j} \times \mathbf{i}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i} = -\,\mathbf{k} \times \mathbf{j}$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j} = -\,\mathbf{i} \times \mathbf{k}$$

In termini di componenti cartesiane

$$
\begin{aligned}
\mathbf{A} \times \mathbf{B} &= (A_X \, \mathbf{i} + A_Y \, \mathbf{j} + A_Z \, \mathbf{k}) \times \\
&\quad (B_X \, \mathbf{i} + B_Y \, \mathbf{j} + B_Z \, \mathbf{k}) = \\
&= A_X B_Y \, \mathbf{k} - A_X B_Z \, \mathbf{j} + \\
&\quad - A_y B_X \, \mathbf{k} + A_Y B_Z \, \mathbf{i} + \\
&\quad + A_Z B_X \, \mathbf{j} - A_Z B_Y \, \mathbf{i} = \\
&= (A_Y B_Z - A_Z B_Y) \, \mathbf{i} + (A_Z B_X - A_X B_Z) \, \mathbf{j} + \\
&\quad + (A_X B_Y - A_Y B_X) \, \mathbf{k}
\end{aligned}
$$

Regola mnemonica

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

Prodotto Triplo Misto Di Tre Vettori

$$(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = (A_Y B_Z - A_Z B_Y)\, C_X +$$

$$+ (A_Z B_X - A_X B_Z)\, C_Y +$$

$$+ (A_X B_Y - A_Y B_X)\, C_Z$$

$$(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = \begin{vmatrix} C_X & C_Y & C_Z \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

$$(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = \mathbf{A} \bullet (\mathbf{B} \times \mathbf{C})$$

DERIVATA DI UN VETTORE

A vettore

$$\frac{d\mathbf{A}}{dt} \quad \text{vettore}$$

$$\frac{d\mathbf{A}}{dt} = \lim_{\Delta t \to 0} \frac{\mathbf{A}(t+\Delta t) - \mathbf{A}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{A}}{\Delta t}$$

$$\frac{d(\mathbf{A}+\mathbf{B})}{dt} = \frac{d\mathbf{A}}{dt} + \frac{d\mathbf{B}}{dt}$$

$$\frac{d(m\mathbf{A})}{dt} = m\frac{d\mathbf{A}}{dt} + \mathbf{A}\frac{dm}{dt}$$

Se $\quad m = \text{costante}$

$$\frac{d(m\mathbf{A})}{dt} = m\frac{d\mathbf{A}}{dt}$$

$$\boxed{\frac{d}{dt}(\mathbf{A}\bullet\mathbf{B}) = \frac{d\mathbf{A}}{dt}\bullet\mathbf{B} + \mathbf{A}\bullet\frac{d\mathbf{B}}{dt}}$$

$$\boxed{\frac{d}{dt}(\mathbf{A}\times\mathbf{B}) = \frac{d\mathbf{A}}{dt}\times\mathbf{B} + \mathbf{A}\times\frac{d\mathbf{B}}{dt}}$$

In termini di componenti cartesiane

$$\mathbf{A} = A_X\,\mathbf{i} + A_Y\,\mathbf{j} + A_Z\,\mathbf{k}$$

Se **i** **j** **k** costanti

$$\frac{d\mathbf{A}}{dt} = \frac{dA_X}{dt}\mathbf{i} + \frac{dA_Y}{dt}\mathbf{j} + \frac{dA_Z}{dt}\mathbf{k}$$

In generale

$$\frac{d\mathbf{A}}{dt} = \frac{dA_X}{dt}\mathbf{i} + \frac{dA_Y}{dt}\mathbf{j} + \frac{dA_Z}{dt}\mathbf{k} + A_X\frac{d\mathbf{i}}{dt} + A_Y\frac{d\mathbf{j}}{dt} + A_Z\frac{d\mathbf{k}}{dt}$$