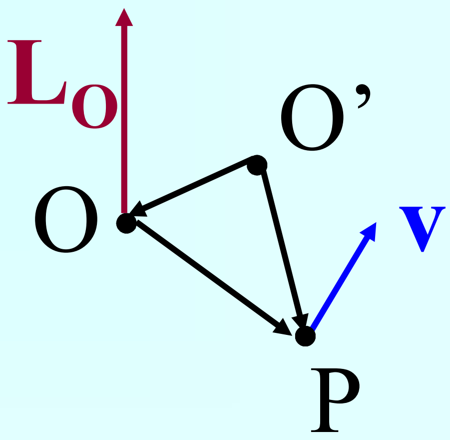
MOMENTO ANGOLARE

P punto materiale in moto con velocità **v**

Si definisce
momento angolare rispetto ad un polo O

$$\mathbf{L}_O = \mathbf{OP} \times m\mathbf{v} = \mathbf{r} \times m\mathbf{v}$$



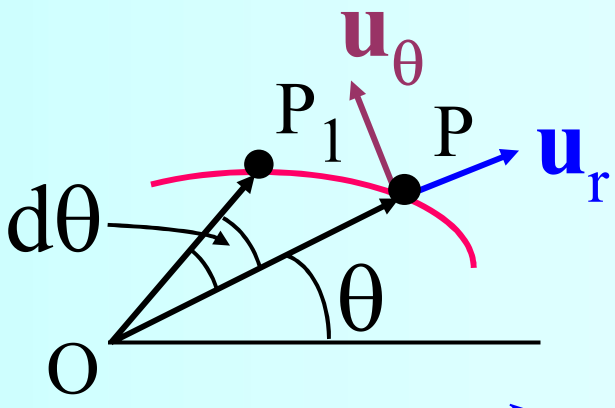
$$\mathbf{L}_{O'} = \mathbf{O'P} \times m\mathbf{v} =$$

$$= (\mathbf{O'O} + \mathbf{OP}) \times m\mathbf{v} = \mathbf{L}_O + \mathbf{O'O} \times m\mathbf{v}$$

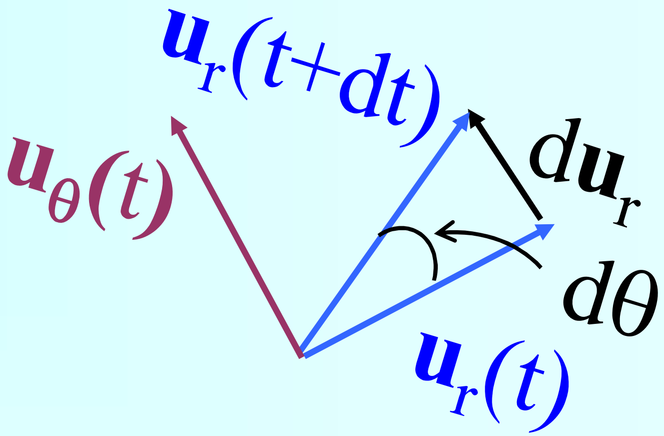
MOTO PIANO IN COORDINATE POLARI

$$\mathbf{OP} = r\,\mathbf{u}_r$$

$$\mathbf{u}_\theta \perp \mathbf{u}_r$$



$$\frac{d\,\mathbf{u}_r}{dt} = \frac{d\theta}{dt}\mathbf{u}_\theta$$



$$\mathbf{v} = \frac{d}{dt} r\,\mathbf{u}_r = \frac{dr}{dt}\mathbf{u}_r + r\frac{d\,\mathbf{u}_r}{dt} =$$

$$= \frac{dr}{dt}\mathbf{u}_r + r\frac{d\theta}{dt}\mathbf{u}_\theta = \mathbf{v}_r + \mathbf{v}_\theta$$

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2}$$

Moto piano in coordinate polari

$$\mathbf{L_O} = \mathbf{r} \times m\,(\mathbf{v_r} + \mathbf{v_\theta}) =$$

$$= \mathbf{r} \times m\mathbf{v_r} + \mathbf{r} \times m\mathbf{v_\theta} = \mathbf{r} \times m\mathbf{v_\theta}$$

con $\quad v_\theta = r\dfrac{d\theta}{dt}$

Se O appartiene al piano del moto

$$L_O = rmv_\theta = mr^2\frac{d\theta}{dt}$$

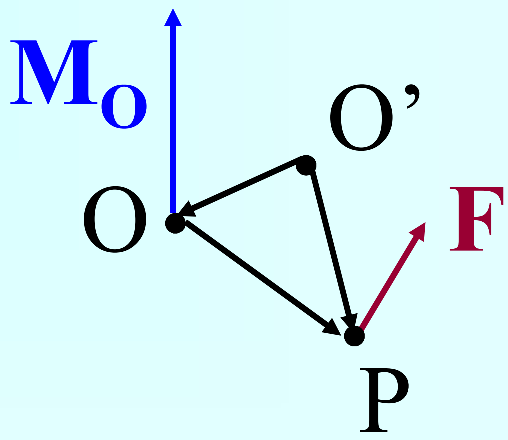
$\mathbf{L_O}$ costante in direzione

MOMENTO DELLA FORZA

F forza applicata in un punto P:

Si definisce
momento della forza rispetto ad un polo O

$$\mathbf{M_O} = \mathbf{OP} \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$



$$\mathbf{M_{O'}} = \mathbf{O'P} \times \mathbf{F} = (\mathbf{O'O} + \mathbf{OP}) \times \mathbf{F} =$$
$$\mathbf{M_O} + \mathbf{O'O} \times \mathbf{F}$$

$\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots\ldots \mathbf{F}_N$ applicate in P

$\mathbf{R} = \sum_i \mathbf{F_i}$ risultante delle forze

$\mathbf{M}_{Oi} = \mathbf{r} \times \mathbf{F}_i$ momento di $\mathbf{F}_i$

Momento risultante $\mathbf{M}_O = \sum_i \mathbf{M}_{Oi} = \sum_i \mathbf{r} \times \mathbf{F}_i =$

$$= \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \ldots \mathbf{r} \times \mathbf{F}_N =$$

$$= \mathbf{r} \times \left(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots \mathbf{F}_N\right) = \mathbf{r} \times \sum_i \mathbf{F}_i =$$

$= \mathbf{r} \times \mathbf{R}$ momento della forza risultante

Valutiamo $\dfrac{d\mathbf{L}_O}{dt} = \dfrac{d}{dt}\left(\mathbf{r}\times m\,\mathbf{v}\right) =$

$$= \frac{d\mathbf{r}}{dt}\times m\,\mathbf{v} + \mathbf{r}\times m\frac{d\mathbf{v}}{dt} =$$

$$= \mathbf{v}\times m\,\mathbf{v} + \mathbf{r}\times m\,\mathbf{a} = \mathbf{r}\times\mathbf{F} = \mathbf{M}_O$$

$$\mathbf{M}_O = \frac{d\mathbf{L}_O}{dt}$$

Teorema del momento angolare

$$\mathbf{M_O} = 0$$

$$\mathbf{L_O} = \text{costante}$$

conservazione del momento angolare

$$\int_{L_{oi}}^{L_{of}} d\mathbf{L}_o = \int_0^t \mathbf{M}_o \, dt \quad \Rightarrow \quad \mathbf{L}_{Of} - \mathbf{L}_{Oi} = \int_0^t \mathbf{M}_o \, dt$$