Entropy-driven phase transitions of entanglement

Giuseppe Florio

Museo Storico della Fisica e Centro Studi e Ricerche "Enrico Fermi" & Dipartimento Interateneo di Fisica "M. Merlin" & Istituto Italiano di Fisica Nucleare

In collaboration with:

- P. Facchi, S. Pascazio (Bari)
- G. Parisi (Rome)
- K. Yuasa (Waseda University, Tokyo)

Based on:

- Physical Review A 87, 052324 (2013)

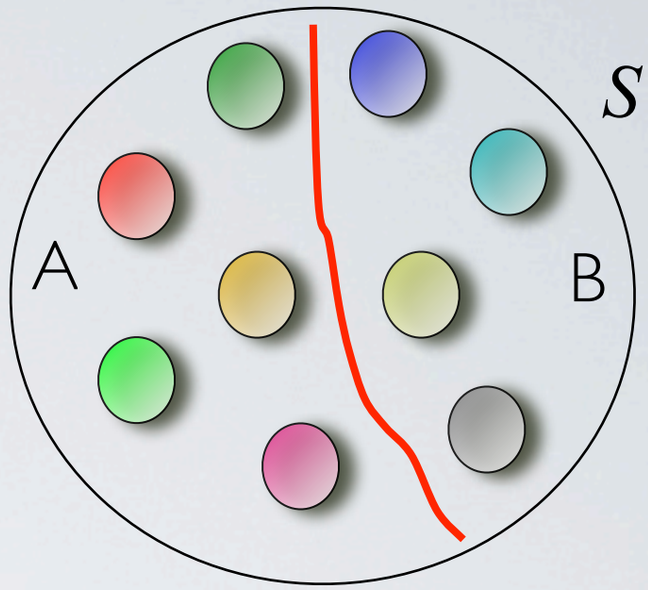
Bari Xmas Workshop 2013

Entanglement

Consider a quantum system in a pure state $|\eta\rangle$

Consider two subsystems A and B

If one can write the state in a factorized form



$$|\eta\rangle = |\psi\rangle_A |\phi\rangle_B$$

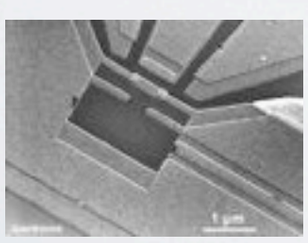
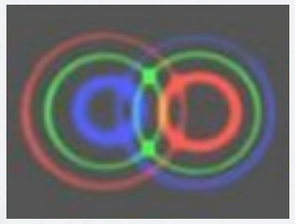
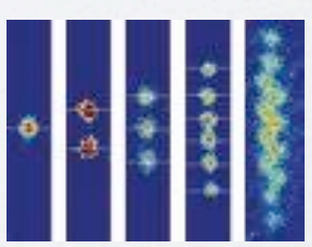
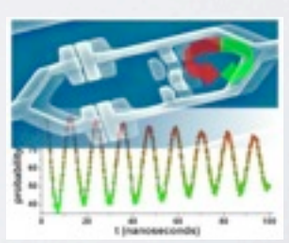
$$|\psi\rangle_A \in \mathcal{H}_A \quad |\phi\rangle_B \in \mathcal{H}_B$$

then the state is SEPARABLE

Otherwise it is ENTANGLED!

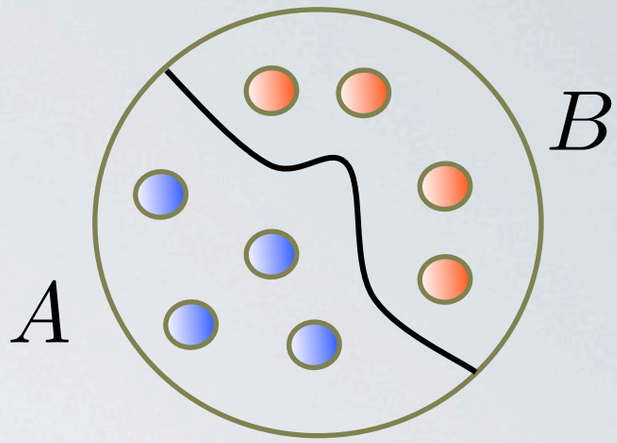
Entanglement for: communication, cryptography, quantum many-body systems...

Structure of the quantum state at fixed value of bipartite entanglement?

A measure of the amount of bipartite entanglement between A and B

$$S_{\mathrm{vN}}(\vec{\lambda}) = -\mathrm{tr}_A(\varrho_A \ln \varrho_A) = -\sum_{k=1}^{N} \lambda_k \ln \lambda_k$$



von Neumann Entropy of ϱ_A

Reduced density matrix of subsystem A: $\varrho_A = \mathrm{tr}_B |\psi\rangle\langle\psi|$

$\vec{\lambda} = (\lambda_1, \ldots, \lambda_N)$ $\quad \lambda_k \geq 0, \quad \sum_k \lambda_k = 1$ Eigenvalues of ϱ_A

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \qquad \dim \mathcal{H}_A = \dim \mathcal{H}_B = N \qquad \dim \mathcal{H} = N^2$$

$$\varrho_A = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \longleftarrow \quad 0 \leq S_{\mathrm{vN}} \leq \ln N \quad \longrightarrow \varrho_A = \frac{1}{N} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Global system in a product state: no entanglement

Global system in a maximally bipartite entangled state

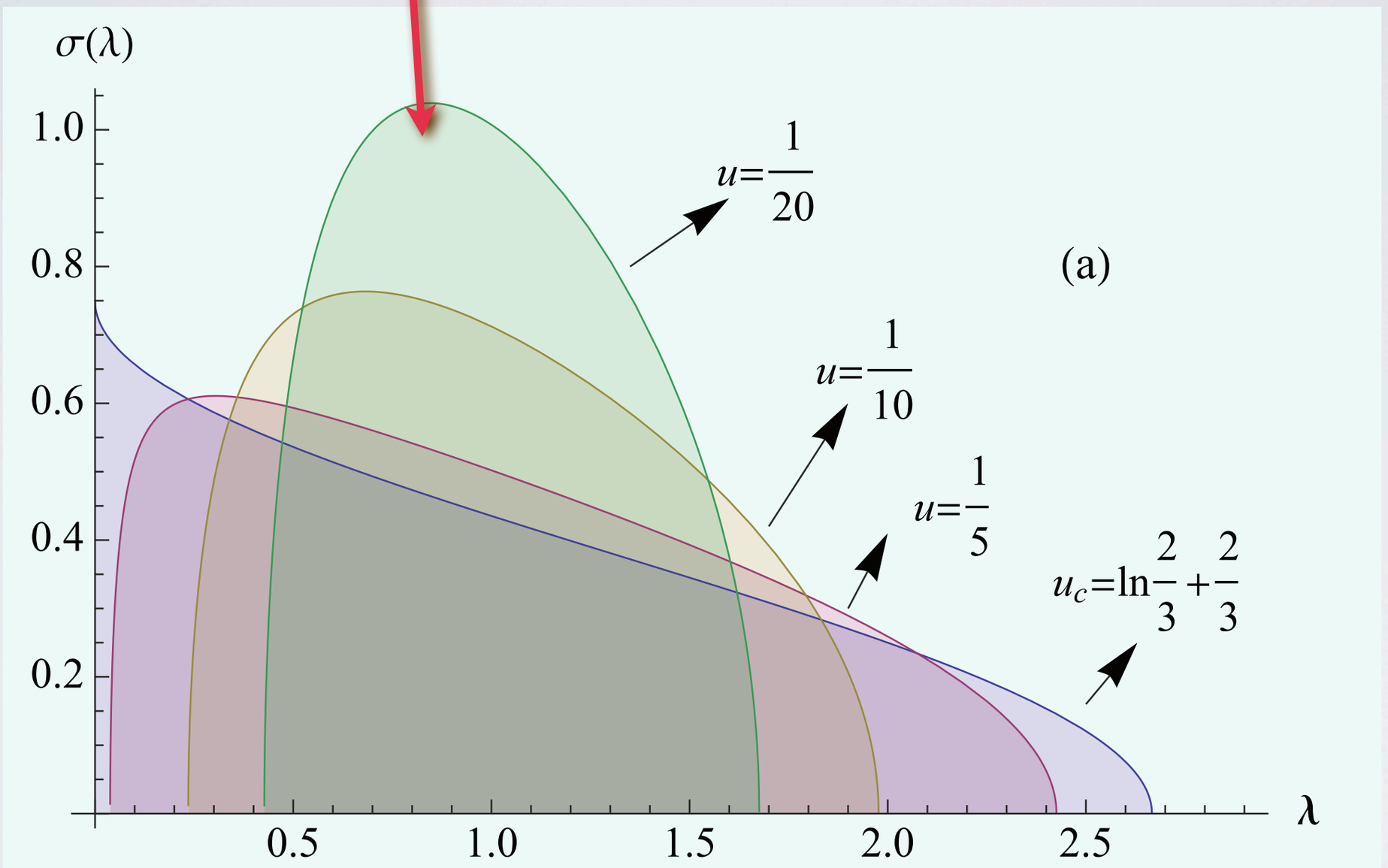
$$S_{\mathrm{vN}} = \ln N - u$$

$$u < u_c \simeq 0.26$$

Deformed Wigner's semicircle law

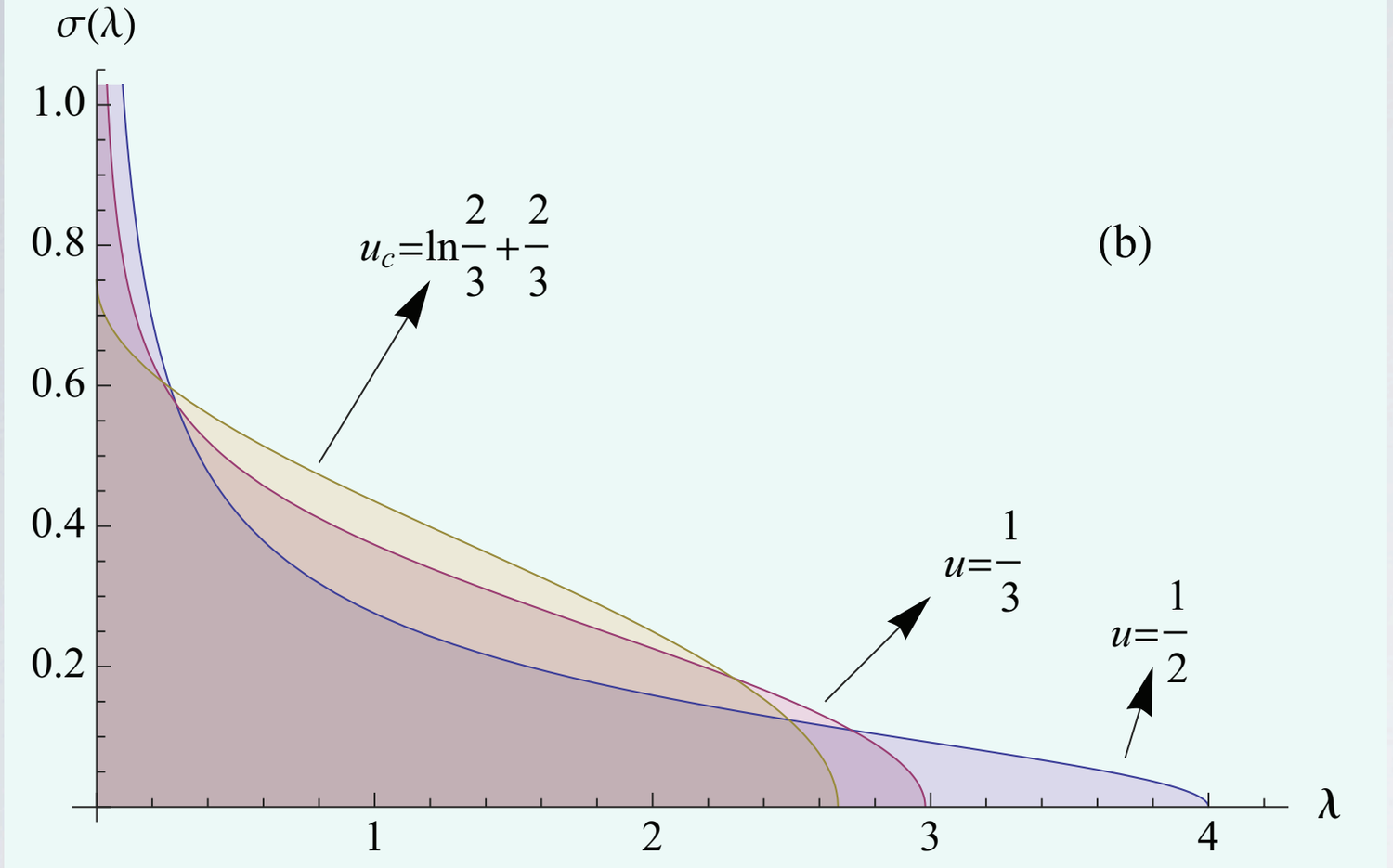
$$\propto \sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}$$

towards max entanglement



Previous work on "Purity" and "Rényi entropy"

Facchi et al. PRL2008, DePasquale et al. PRA2010, Nadal et al. PRL2010

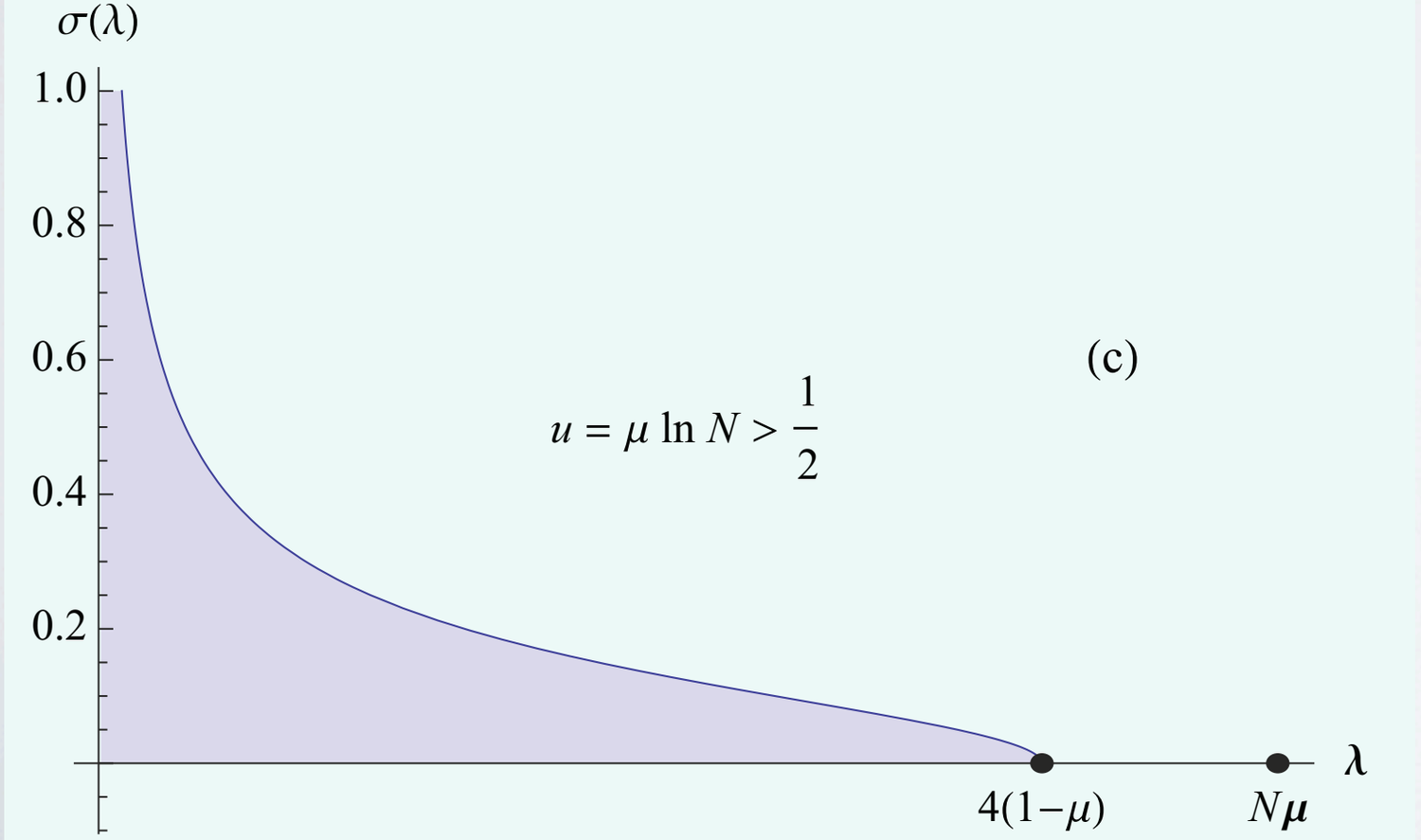


$$S_{\rm vN} = \ln N - u$$

$$u > u_c \simeq 0.26$$

Deformed Marchenko-Pastur law

$$|\psi\rangle \neq |u_A\rangle \otimes |v_B\rangle$$



$$u > 1/2$$

towards separable states

$$\vec{\lambda} \simeq (1, 0, \ldots, 0)$$

$$\lambda_1 = \mu = O(1)$$

$$|\psi\rangle = |u_A\rangle \otimes |v_B\rangle$$

Merry Christmas!!!!