SELF-INDUCED FLAVOR EVOLUTION OF SUPERNOVA NEUTRINOS WITHOUT AXIAL SYMMETRY

(Based on 1308.1402; 1308.5255)

(Alessandro MIRIZZI, Hamburg U.)
3ν FRAMEWORK

- **Mixing parameters:** \( U = U (\theta_{12}, \theta_{13}, \theta_{23}, \delta) \) as for CKM matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
1 & & \\
\frac{c_{13}}{s_{13}} & c_{13} & e^{-i\delta} s_{13} \\
-s_{23} & c_{23} & 1
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} \\
-s_{12} & c_{12}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

- \( c_{12} = \cos \theta_{12}, \) etc., \( \delta \) CP phase

- **Mass-gap parameters:** \( M^2 = \left\{ \frac{-\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm \Delta m^2 \right\} \)

\[
\nu_3 \quad +\Delta m^2 \quad \text{inverted hierarchy}
\]

\[
\nu_1 \quad +\delta m^2/2 \quad \nu_1 \quad +\delta m^2/2
\]

\[
\nu_2 \quad -\delta m^2/2 \quad \nu_2 \quad -\delta m^2/2
\]

- **Normal hierarchy**

- **2ν subsector:** \( (\Delta m^2, \theta_{13}) \)
SNAP-SHOTS OF SN DENSITY PROFILES

- Matter bkg potential
  \[ \lambda = \sqrt{2G_F N_e} \sim R^{-3} \]
- \(\nu-\nu\) interaction
  \[ \mu = \sqrt{2G_F n_\nu} \sim R^{-2} \]
- Vacuum oscillation frequencies
  \[ \omega = \frac{\Delta m^2}{2E} \]

When \(\mu \gg \lambda\), SN \(\nu\) oscillations dominated by \(\nu-\nu\) interactions

Collective flavor transitions at low-radii \([O (10^2 - 10^3 \text{ km})]\]

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

[see Duan et al, arXiv:1001.2799 for a review]
**DENSITY MATRIX FOR THE NEUTRINO ENSEMBLE**

Diagonal elements related to flavor content

\[
\rho_{\alpha\alpha} = \frac{F_{\nu\alpha}(E, r)}{F(E, r)}
\]

Off-diagonal elements responsible for flavor conversions

\[
\rho = \begin{pmatrix}
\rho_{ee} & \rho_{e\mu}^* & \rho_{e\tau} \\
\rho_{e\mu} & \rho_{\mu\mu} & \rho_{\mu\tau}^* \\
\rho_{e\tau} & \rho_{\mu\tau} & \rho_{\tau\tau}
\end{pmatrix}
\]

- In 2ν scenario. Decompose density matrix over Pauli matrices to get the "polarization" (Bloch) vector \( \mathbf{P} \). Survival probability \( \text{Pee} = 1/2(1 + P_z) \).
  
  \( P_z = -1 \to \text{Pee} = 0 \); \( P_z = 0 \to \text{Pee} = 1/2 \) (flavor decoherence)

- The EOMs for the time evolution in a homogeneous medium are the Liouville equations (e.g. Early Universe)

\[
i\partial_t \rho_p = [H_p, \rho_p]
\]

Alessandro Mirizzi

Xmas Theory Workshop

23 December 2013
Evolution in space for $\nu$'s streaming from a SN core in quasi-stationary situation

$$i \vec{\nu}_p \cdot \vec{\nabla}_x \rho_{p,x} = \left[H(\omega, \lambda, \rho_{p',x}), \rho_{p,x}\right]$$

Liouville operator for free streaming $\nu$

**MULTI-ANGLE $\nu-\nu$ HAMILTONIAN**

$$H_{\nu\nu} = \sqrt{2} G_F \int d\vec{q} \left(1 - \vec{\nu}_p \cdot \vec{\nu}_q\right) \left(\rho_{q,x} - \bar{\rho}_{q,x}\right)$$
Neutrinos are emitted uniformly and (half)-isotropically from the surface of a sphere (ν-sphere), like in a blackbody.

Physical conditions depend only on the distance $r$ from the center of the star (azimuthal symmetry).

Only multi-zenith-angle (MZA) effects in terms of $u = \sin^2 \theta_R$.

Project evolution along radial direction (ODE problem) $\vec{v}_p \cdot \vec{\nabla}_x \rightarrow v_r \, \! dr$.
SINGLE-ZENITH-ANGLE (SZA) APPROXIMATION

Introduce an average of the MZA factor \((1 - \cos \theta_{pq})\).

**SZA ν–ν Hamiltonian**

\[
H_{\nu\nu} = \mu(r)(\rho - \bar{\rho})
\]

\[
\mu(r) = \sqrt{2} G_F n_\nu \langle 1 - \cos \theta \rangle
\]

[Hannestad et al., astro-ph/0608695]

Toy model:
Monochromatic ν beam.

Only \(\nu_e\) and \(\overline{\nu_e}\), with an excess of \(\nu_e\)

- No effect in NH
- Complete self-induced conversions in IH. Bimodal instability
MULTI-ZENITH-ANGLE DECOHERENCE

\[ H_{\nu\nu} = \sqrt{2} G_F \int d\mathbf{q} \left( 1 - \cos \theta_{pq} \right) \left( \rho_{q,x} - \bar{\rho}_{q,x} \right) \]

Is the MZA decoherence relevant for SN neutrinos?

Symmetric $\nu_e, \bar{\nu}_e$ systems decoheres in both hierarchies [Raffelt & Sigl, hep-ph/0701182]

Flux term does not vanish in a non-isotropic medium, like $\nu$ streaming off a SN

Alessandro Mirizzi  Xmas Theory Workshop  23 December  2013
MZA EFFECTS FOR SN NEUTRINOS

Flavor asymmetry

\[ \varepsilon = \frac{F(\nu_e) - F(\bar{\nu}_e)}{F(\nu_e) - F(\bar{\nu}_x)} \]

- Flavor equilibration in both NH & IH
- Nothing occurs in NH (stable configuration)
- Complete conversions in IH (bimodal instability)

[Esteban-Pretel et al., 0706.2498]

- Flavor asymmetry
- Quasi single-angle behaviour

Large $\nu_e\bar{\nu}_e$ asymmetry required to suppress multi-angle decoherence
Self-induced flavor conversions are associated to an instability in the
flavor space [Sawyer, 0803.4319; Banerjee, Dighe & Raffelt, 1107.2308]

Instability required to get started (exponential growth of the off-
diagonal density matrix part)

The onset of the conversions can be found through a stability analysis
of the linearized EOMs.

In [Raffelt, Sarikas, Seixas, 1305.7140] a stability analysis of the EOMs
has been performed including the azimuthal angle $\phi$ of the $\nu$
propagation and without enforcing axial symmetry also starting with
an intial axial symmetric $\nu$ emission.

A new multi-azimuthal-angle (MAA) instability has been found!!

In the unstable case, numerical simulations are mandatory.
MAA EFFECTS FOR SN NEUTRINOS

Monochromatic $\nu$ beam

System of only $\nu_e$ and $\bar{\nu}_e$ with an excess of $\nu_e$

NH becomes unstable under MAA effects!

SZA vs MZA marginal impact

New effects in IH only for small $\epsilon$

Alessandro Mirizzi

Xmas Theory Workshop

23 December 2013
System of only $\nu_e$ and $\bar{\nu}_e$ with an excess of $\nu_e$.

Fermi-Dirac energy distributions with $\langle E \rangle = 15$ MeV.

$$g_\omega \equiv \frac{|\Delta m^2|}{2\omega^2} \times \begin{cases} f_{\nu_e}(E_\omega) - f_{\nu_x}(E_\omega) & \text{for } \omega > 0 \\ f_{\nu_x}(E_\omega) - f_{\bar{\nu}_e}(E_\omega) & \text{for } \omega < 0 \end{cases}$$

$$E_\omega = \left| \frac{\Delta m^2}{2\omega} \right|$$
SPECTRAL SPLITS ($\varepsilon=1.5$)

Oscillated spectra

Initial

Final SZA & MAA

Final MZA & MAA

Swap function

\[ s(\omega) = \frac{g(\omega)_{\text{final}}}{g(\omega)_{\text{initial}}} \]

SZA & MAA

MZA & MAA

Spectral swaps and splits in both NH (MAA) & IH (Bimodal)

Antinu split smeared by MZA effects
SPECTRAL SPLITS FOR SN NEUTRINO FLUXES

\[ F_{\nu_e} : F_{\bar{\nu}_e} : F_{\nu_x} = 2.4:1.6:1.0 \]

\[ \langle E_e \rangle = 12 \text{ MeV} ; \]
\[ \langle E_x \rangle = 18 \text{ MeV} \]
\[ \epsilon = 1.34 \]

- Spectral swaps and splits in both NH & IH!!
- In the axial symmetric case, only IH instable

[A.M., 1308.5255]

23 December 2013
SPECTRAL SPLITS FOR SN NEUTRINO FLUXES

Nu fluxes present a spectral split at $E = 12$ MeV (fixed by lepton number conservation)

Antineu fluxes are swapped

Alessandro Mirizzi
Xmas Theory Workshop
23 December 2013
CONCLUSIONS

- A new multi-azimuthal-angle (MAA) instability emerges in the dense SN neutrino gas, removing the axial symmetry in the nu-nu interaction term.

- For simple systems of only $\nu_e$ and $\bar{\nu}_e$, with an excess of $\nu_e$, MAA effects can lead to spectral splits (for large flavor asymmetry) or flavor decoherence (for small flavor asymmetry) in NH.

- For SN $\nu$ spectra as in the accretion phase, MAA effects would induce swaps and splits in NH.

- A change of paradigm in the SN neutrino self-induced effects?

Still many open points (e.g. role of dense ordinary matter, initial nu angular distributions...)

LOT OF WORK TO BE DONE ... WAITING THE NEXT SN EXPLOSION!