Anderson localization of interacting particles
(a.k.a. Many-Body-Localization)

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Anderson Localization
without interactions

Conduction of electron in metals with impurities

\[-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = i\hbar \partial_t \psi\]

Interference enhances the probability that a particle starting from A goes back to A even in the presence of a random background

But is it enough to suppress conductivity in the metal?

Metal-Insulator transition?
Anderson Localization without interactions

Anderson (1958) answered in the affirmative
A sufficient amount of disorder suppresses completely the conductance

In one and two dimensions the effect is particularly strong:
All the states are localized for arbitrary weak disorder

\[ \sigma \sim 0 \quad \text{independent of temperature} \]

though the coupling to phonons could actually give

\[ \sigma \sim e^{-\left(\frac{A}{T}\right)^{\frac{1}{d+1}}} \]
Anderson Localization without interactions

In 3 dimensions (and above) only the low-energy states are localized and a mobility edge exists

\[ E_F > E_c, \quad \sigma \sim \sigma_0 \]

While

\[ E_F < E_c, \quad \sigma \sim e^{-(E_c - E_F)/T} \]

although in the presence of phonons again

\[ \sigma \sim e^{-\left(\frac{A}{T}\right)^\frac{1}{d+1}} \]
The role of interactions

Is it possible that the role played by the phonons is played by the quasiparticles generated by the interaction?

Localized states support hopping (variable range hopping)

Energy mismatch is matched by phonons (continuous spectrum)

Fleishman and Anderson: NO
Basko, Aleiner and Altshuler (2006) have proposed a new analysis of the problem with interactions.

By analyzing the perturbation theory in the interaction $\lambda$ they have identified a region of convergence of the series as a function of $T$.

One obtains the following picture:

\[ Z_{\text{T}_{c}} = \int_{0}^{1} \frac{E_{c}}{T} \, dE \]

where the critical temperature is determined by Eq. (18).

The schematic temperature dependence of the conductivity is summarized on Fig. 1.

Therefore, the temperature dependence of the dissipative coefficient in the system shows the singularity typical for a phase transition.

To prove Eqs. (22) we use the Gibbs distribution and find

\[ r(T) = \sum_{k} P_{k} r(E_{k}) = \int_{0}^{1} dE \frac{e^{-S(E)/T}}{C_{0}} E = r(E) \int_{0}^{1} dE \frac{e^{-S(E)/T}}{C_{1}} E \]

where the entropy $S(E)$ is proportional to volume, and $E$ is counted from the ground state.

The integral is calculated in the saddle point or in the steepest decent approximations, except for $V_{\lambda,1}$. The saddle point $E_{c}(T)$ is given by

\[ \frac{dS}{dE} = \frac{E_{c}}{T} \]

Taking into account $r(E) = 0$ for $E < E_{c}$ we find:

\[ r(T) = \frac{r(E_{c}(T))}{\exp \left( \frac{E_{c}}{T} \right)} \]

As both energies entering the exponential are extensive, $E_{c}(T), E_{c}/V$, we obtain Eqs. (22).

As we already mentioned, vanishing of the dissipative conductivity at $T < T_{c}$ means freezing of all relaxation processes. In particular the microcanonical distribution could never be established for the closed system. In this respect, the dynamics of the system resembles the glassy state [19].

To establish the thermal equilibrium in such insulating state requires finite coupling of the system with the external reservoir (i.e., phonons). The presence of the finite electron–phonon interaction (as phonons are usually delocalized), smears out the transition, and

Fig. 1. Schematic temperature dependence of the dc conductivity $\sigma(T)$. Below the point of the many-body metal–insulator transition, $T < T_{c}$, $\sigma(T) = 0$, as shown in Section 6. Temperature interval $T > T_{c}$ corresponds to the developed metallic phase, where Eq. (17d) is valid. In this regime for the model described in Section 3 $\sigma(T)$ is given analytically by Eqs. (93)–(99) and plotted on Fig. 10. At $T > T_{c}^{(el)}$ the high-temperature metallic perturbation theory of [15] is valid.
A simplified model

Based on these ideas we can review the 1 and 2 dimensional models with interactions: they might have delocalized phases!

\[ H = -g \sum_i (s_i^x s_{i+1}^x + s_i^y s_{i+1}^y + \Delta s_i^z s_{i+1}^z) - \sum_i h_i s_i^z \]

**XXZ chain with disorder** \( h_i \in [-h, h] \)

After fermionization, 1-d fermions hopping with on-site disorder and 4-body interactions

\[ H = -g \sum_i c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i + \Delta (c_i^\dagger c_i)(c_{i+1}^\dagger c_{i+1}) - \sum_i h_i (c_i^\dagger c_i) \]

**Number of particles** number of spins up: conserved
A simplified model

For $\Delta = 0$

the model is single particle hopping in a random potential:

*whole spectrum is localized*

What happens for $\Delta > 0$, $\Delta = 1$, say?  


A transition is observed in the spectrum: from WD to Poisson  

$h_c = 3.5 \pm 1.0$

also signals in the correlation functions
A simplified model

Many other indications of the transition

We are studying the following problem:

Let us prepare the system in a particular configuration of the computational basis spins

\[ |\psi_0\rangle = |\uparrow\downarrow\downarrow \ldots \uparrow\rangle \]

and let us calculate the survival probability

\[ P(t) = |\langle \psi_0 | e^{-iHt} \psi_0 \rangle|^2 \]

\[ \frac{1}{T} \int_0^T dt P(t) \simeq \sum_n |\langle \psi_0 | E_n \rangle|^4 \equiv R \]

defines participation ratio
A simplified model

localized: \[ R = O(1) \]
delocalized: \[ R = O(1/2^N) \]

Preliminary results:
A fully-connected simplified model

\[
H = -g \sum_{i,j} \vec{s}_i \cdot \vec{s}_j - \sum_i h_i s_i^z
\]

\( h_i \in [-1, 1] \)

The model is integrable: can be solved by Bethe Ansatz

\[
\lim_{t \to \infty} \frac{\langle s_z(t) \rangle}{\langle s_z(0) \rangle} = q
\]

\[
q \approx \frac{1}{1 + g}
\]
A fully-connected simplified model

$q > 0$ is a signal that the system is always localized. Integrability prevents the delocalization.

\[
\bar{q} \geq 1 - \sqrt{3\pi g} \int_{-\infty}^{\infty} dx P(x)^2 e^{3\pi P(x)^2 g^2} \Phi(\sqrt{3\pi P(x)} g)
\]

Mazur’s inequality
Conclusions

Anderson (de)localization can occur in 1d as well but it requires the presence of interactions: Many-Body-Localization

It is the quantum analog of spin-glass transition, with the freezing of the initial many-body state

Speculations relate it to the infinite-randomness critical point

It is still a vastly unexplored topic