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Quantum interference with expanding BEC in microgravity

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Xmas Workshop 20/12/12

Why matter interferometers?

1. Gravimeters
2. Gravity gradiometers
3. Magnetometers
4. Atomic clocks
5. Gyroscopes

Main motivations

1. Study of coherence evolution of quantum object delocalized in space-time

2. Test of the universality of free falls with quantum objects

3. Detection of gravitational waves

4. Test of the gravitational red-shift

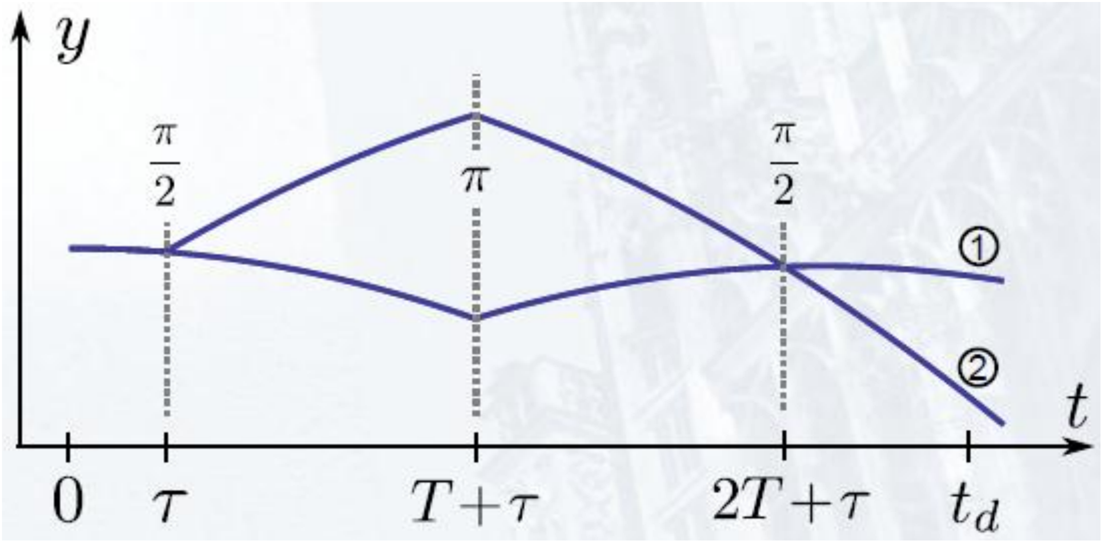
Test of the universality of free falls

$$V(y) = m\gamma y$$

$$\varphi_\gamma = k\gamma T^2$$

M. Kasevich and S. Chu, *PRL* **67**, 181 (1991)

BEC center-of-mass trajectories in a SMZI



BEC AMZI in microgravity

Why BEC interferometry?

BEC →
- Slow spreading
- Excellent mode properties

→ **High resolution Interferometers**

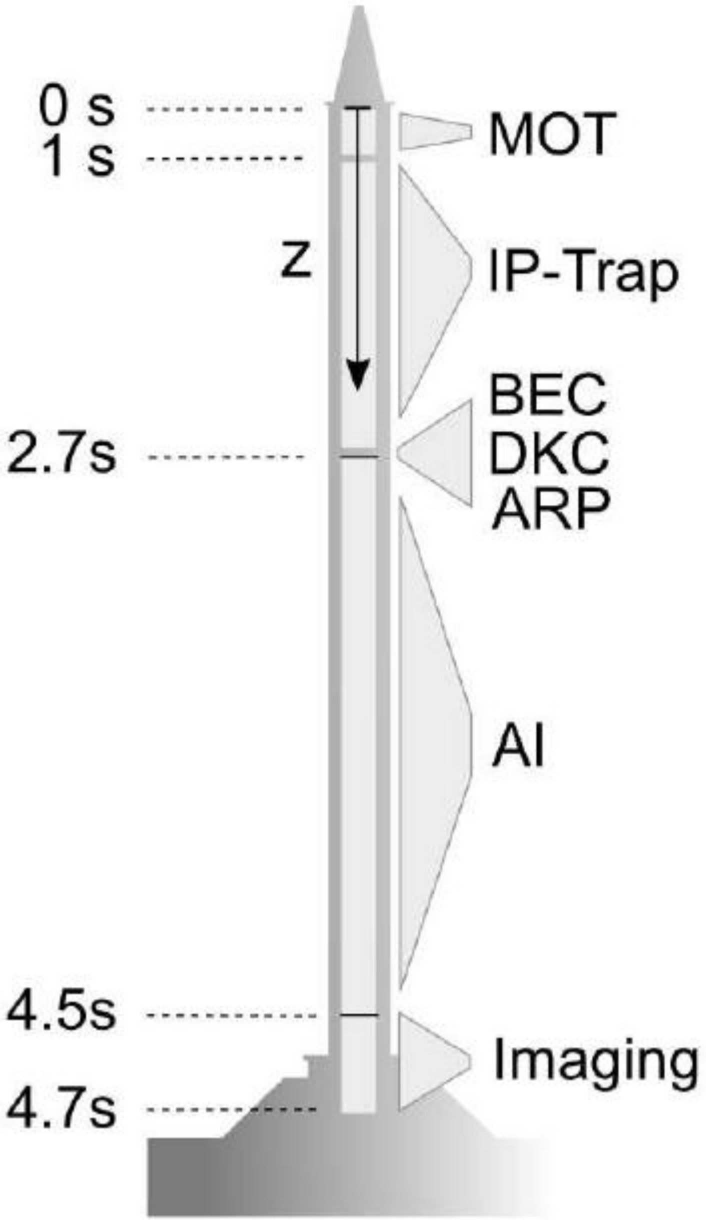
Why microgravity?

Much longer expansion times

Why asymmetric?

Fringe detection in a single experiment

Outline



- BEC evolution in microgravity
- Beam splitters and mirrors for atoms: Bragg pulses
- Experiment with a BEC Asymmetric Mach Zehnder Interferometer (AMZI)

BEC evolution in microgravity

Dynamics of N~10^4 interacting bosons in a BEC:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla_{\boldsymbol{x}}^2 + V(t,\boldsymbol{x}) + g\,|\psi(t,\boldsymbol{x})|^2\right]\psi(t,\boldsymbol{x})$$

Repulsive interaction between bosons
described by the coupling constant g

Trap approximated locally by a quadratic potential

$$V(0,\boldsymbol{x}) = \frac{m}{2}(\boldsymbol{x}-\boldsymbol{\rho}(0))^{\mathrm{T}}\Omega^2(0)\,(\boldsymbol{x}-\boldsymbol{\rho}(0))$$

$\Omega(t) = 0$ for $t > 0$

How to find a solution for the GP equation?

Time-dependent TF approximation

$$\varepsilon = \varepsilon(\boldsymbol{r}, t) \equiv \frac{\hbar^2}{2m} \frac{\Delta \psi^{(\mathrm{TF})}}{\psi^{(\mathrm{TF})}}$$ is neglected!

Approximated solution of the GP equation:

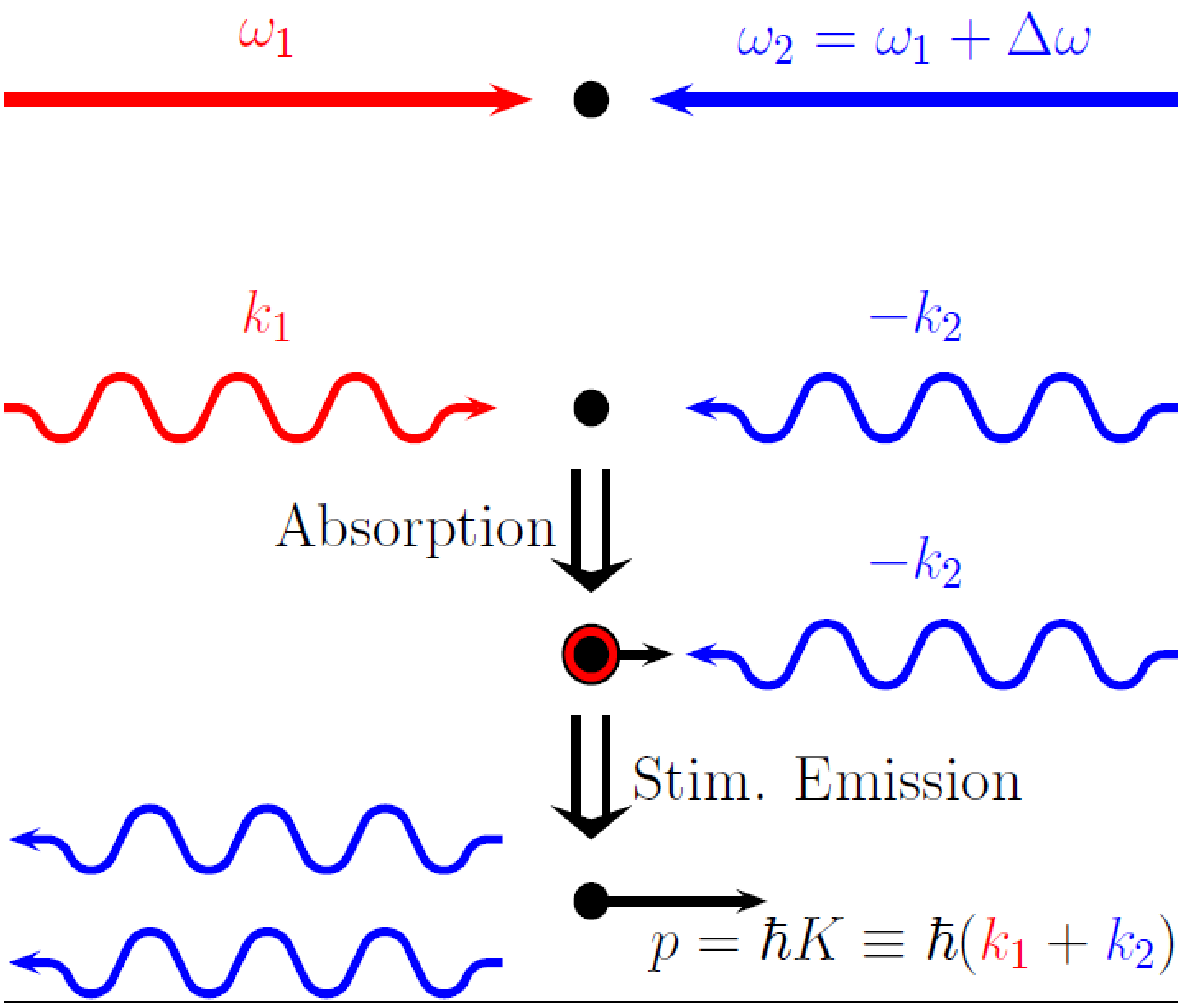
$$\psi(\boldsymbol{r}, t) = \exp\left[\mathrm{i}\Phi\left(\boldsymbol{r} - \boldsymbol{\mathcal{R}}(t), t; \boldsymbol{\mathcal{R}}(t), \dot{\boldsymbol{\mathcal{R}}}(t)\right)\right] \times \psi^{(\mathrm{TF})}\left(\boldsymbol{r} - \boldsymbol{\mathcal{R}}(t); t\right)$$

Y. Castin and R. Dum, *PRL* **77**, 5315 (1996)
T. van Zoest et al., *Science* **328**, 1540 (2010)
Y. Kagan et al., *PRA* **54**, R1753 (1996)
P. Storey and M. Olshanii, *PRA* **62**, 033604 (2000)

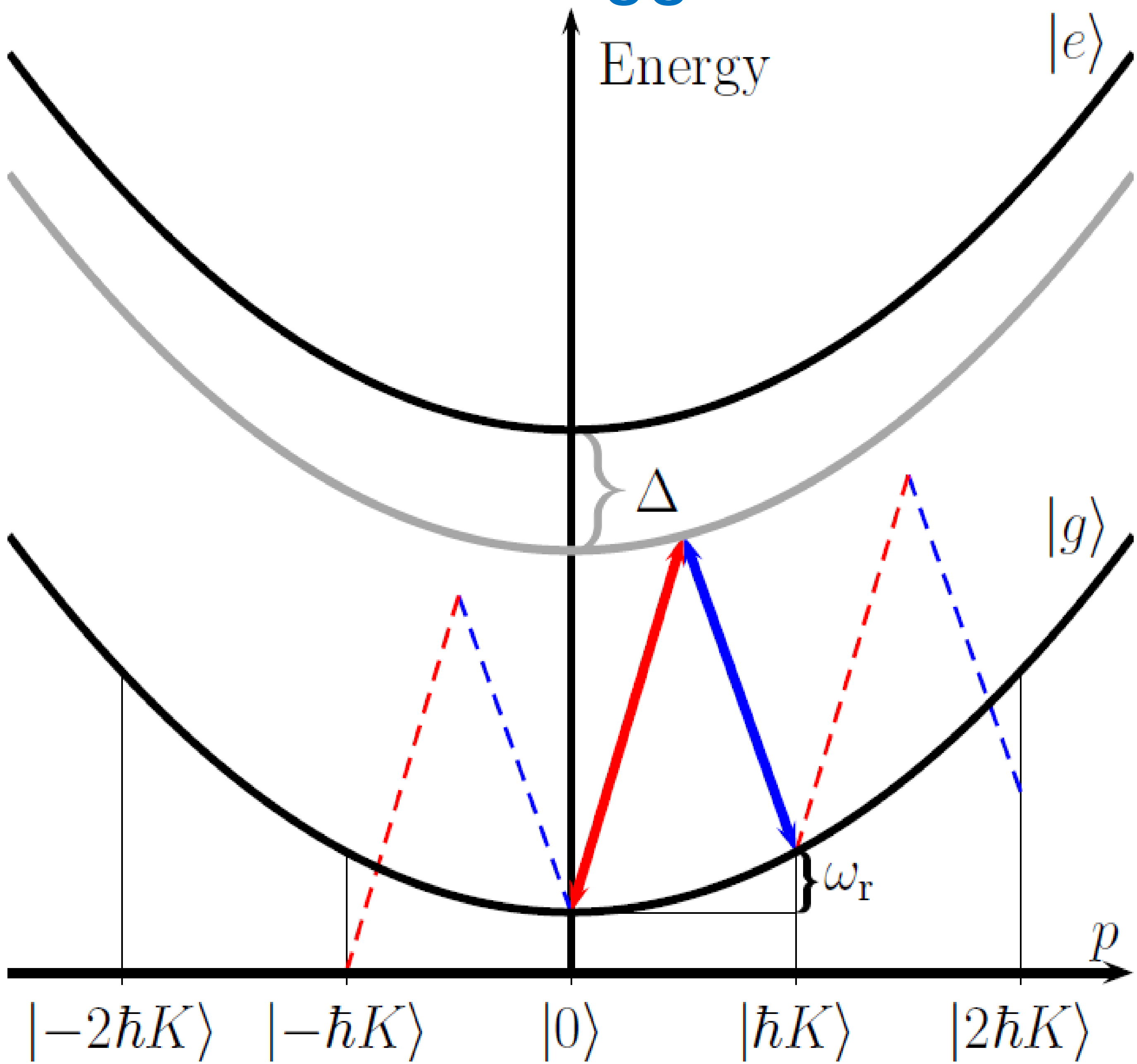
Ellipsoidal condensate with a non trivial space-time dependent phase

BEC interferometery:
Interfering BEC wave amplitudes
with different phases

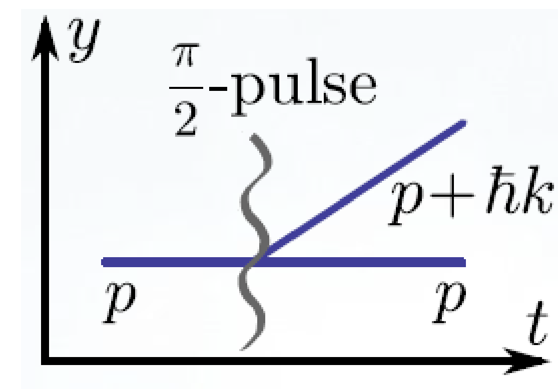
Beam splitters and mirrors for atoms: Bragg diffraction



Beam splitters and mirrors for atoms: Bragg diffraction



Beam splitter



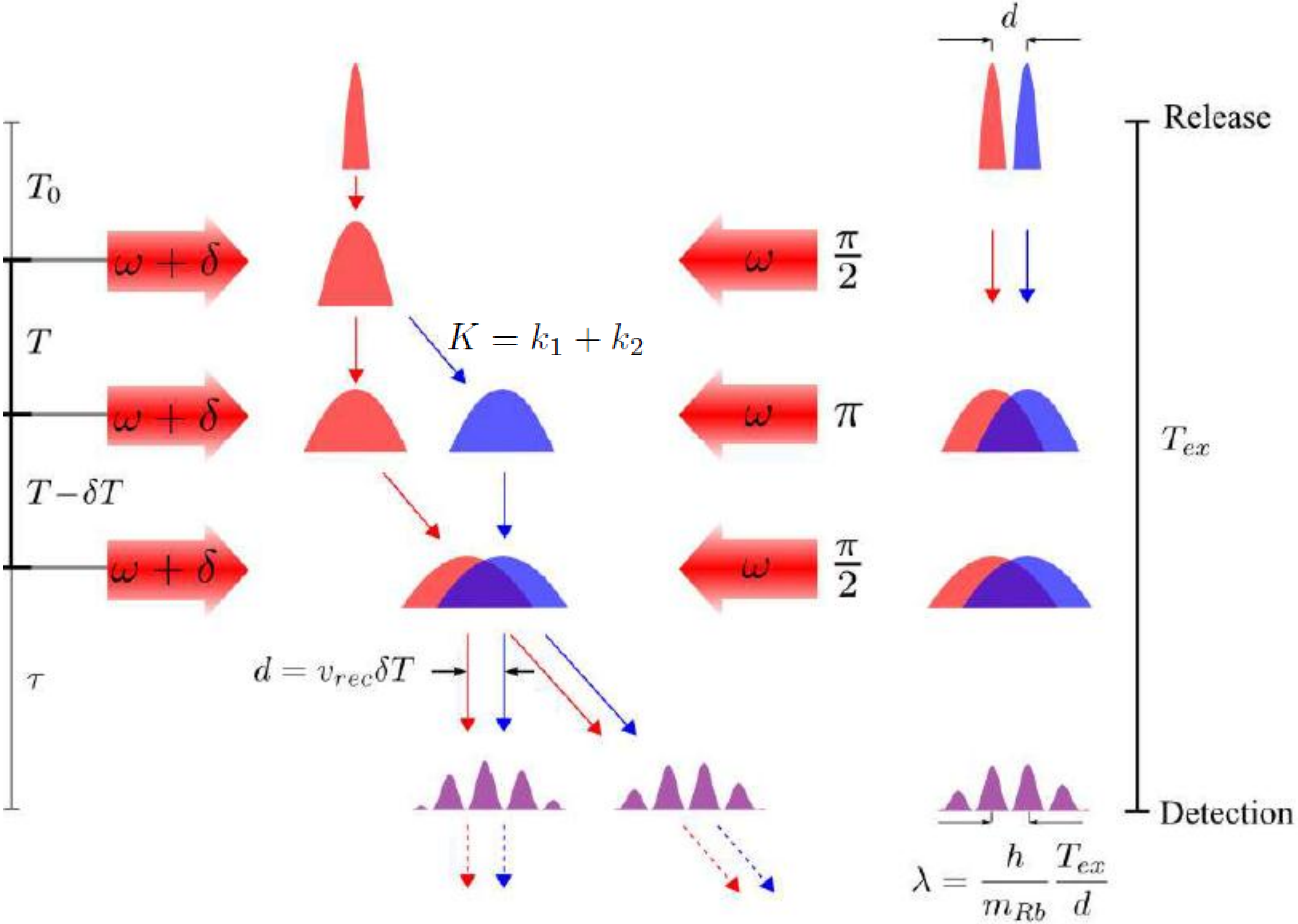
Resonant condition for first order Bragg

$$\omega_1 - \omega_2 = \omega_r$$

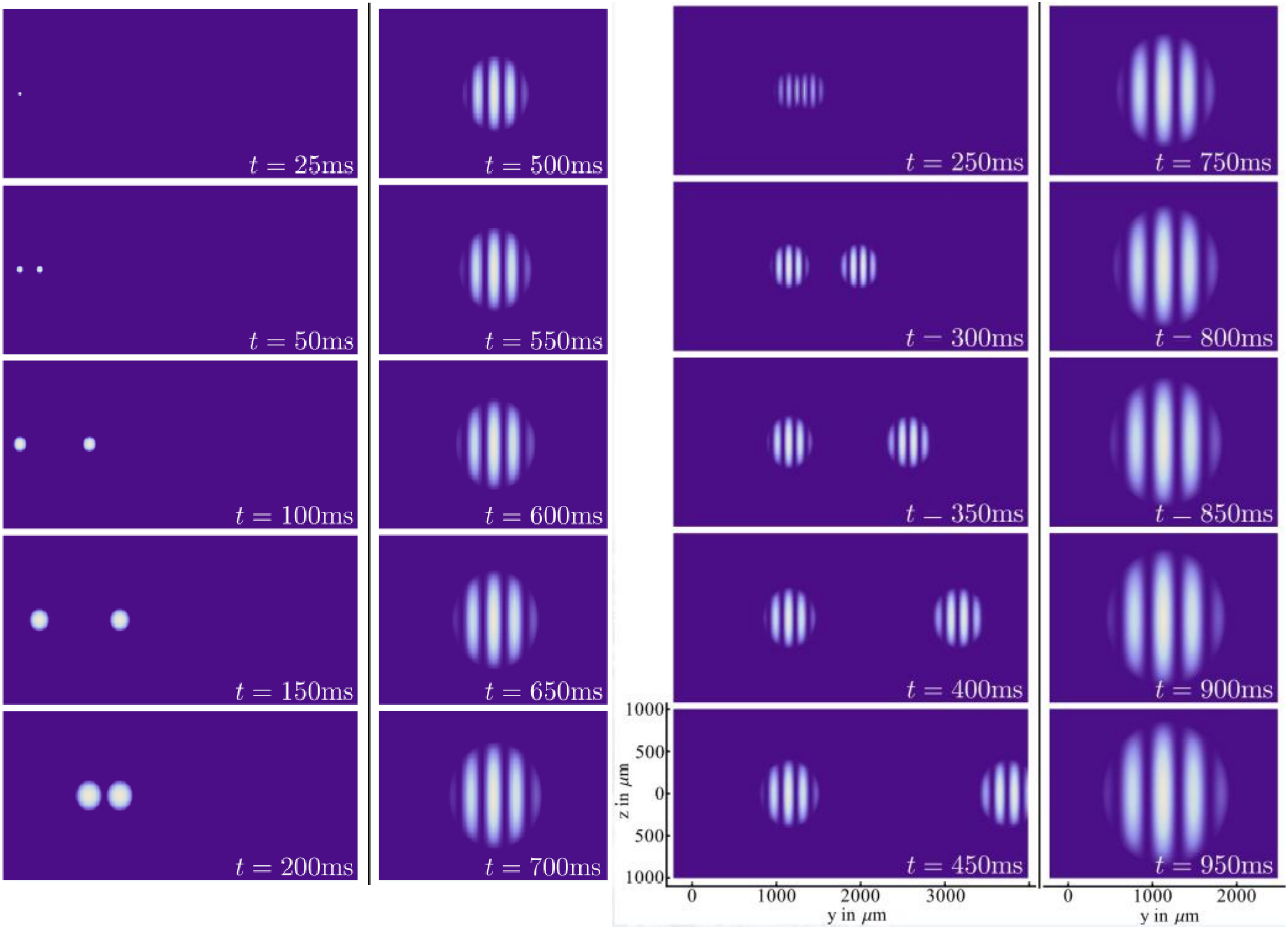
$$\omega_r = \frac{\hbar K^2}{2m}$$

$$K = k_1 + k_2$$

BEC AMZI in microgravity



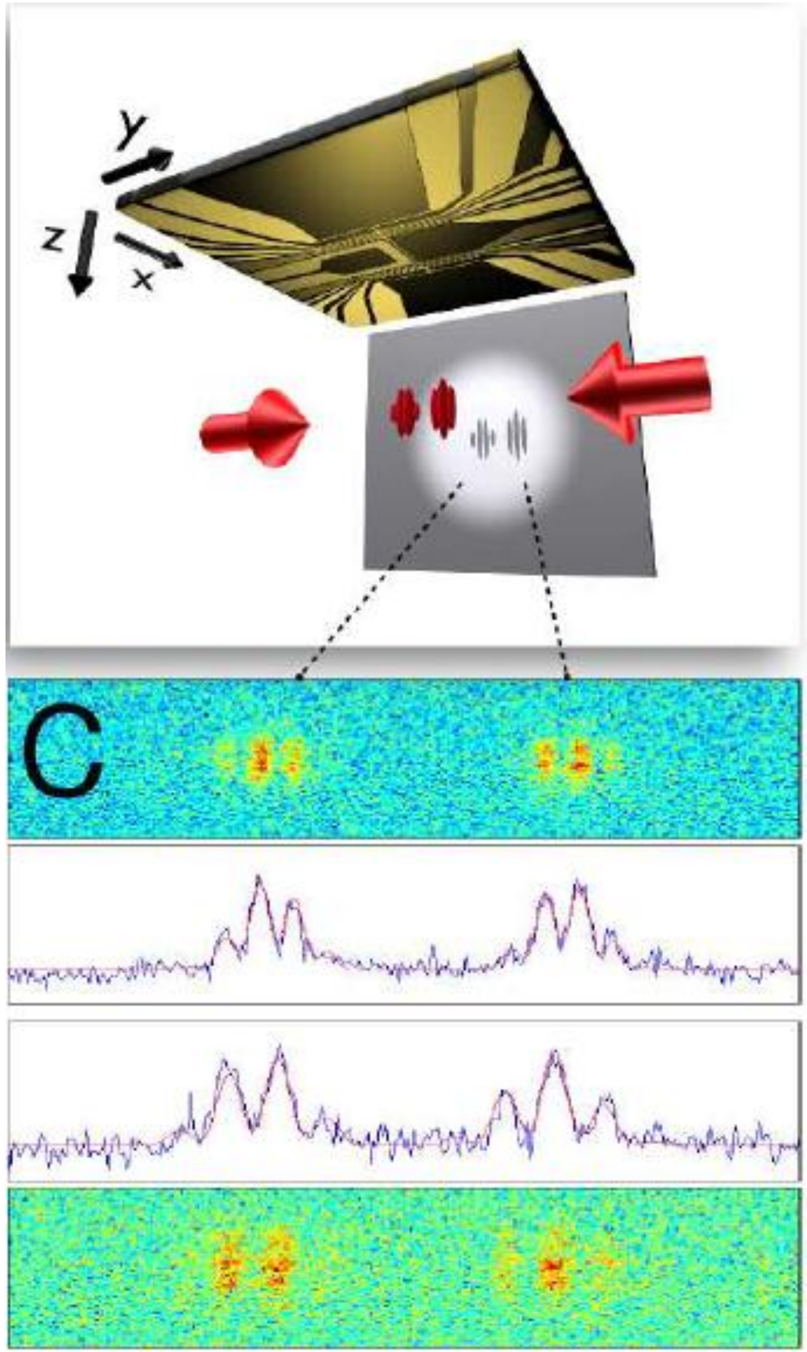
3D density plot for an asymmetric MZ interferometer in the x=0 plane as a function of time



Observed interference fringes

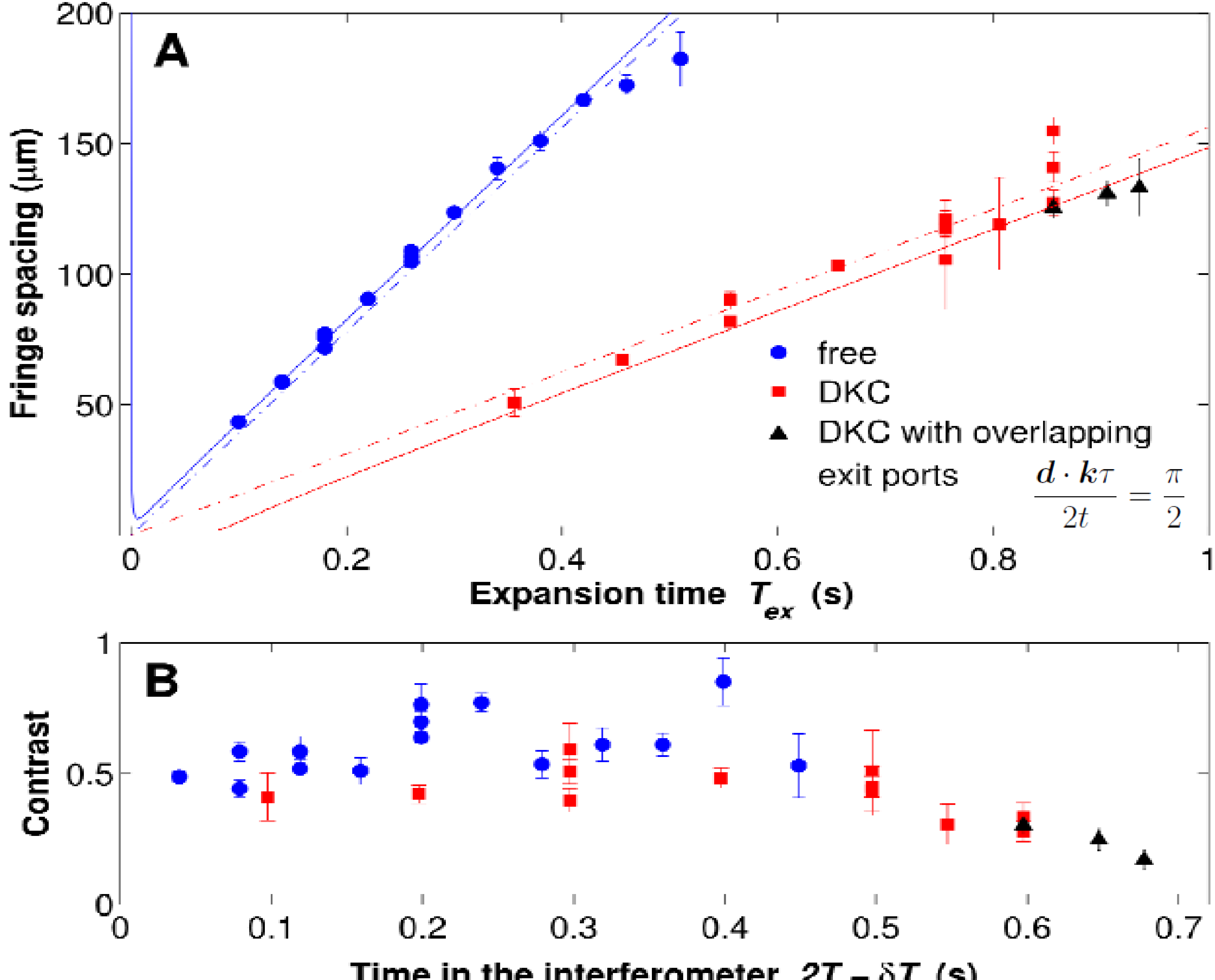
$\lambda = 75\mu m$

$T_{ex} = 180ms$



$\lambda = 107\mu m$

$T_{ex} = 260ms$


A
Fringe spacing (μm)
Expansion time T_{ex} (s)
free
DKC
DKC with overlapping exit ports $\frac{d \cdot k\tau}{2t} = \frac{\pi}{2}$
B
Contrast
Time in the interferometer $2T - \delta T$ (s)

Outline and future developments

- Experimental measurement of BEC quantum interference in microgravity

- Theoretical model for the study of quantum interference in microgravity

- Test of the universality of free falls by measuring at the same time quantum interference for two different quantum species with different mass

- Study of the fundamental physics at the border between quantum mechanics and general relativity

QUANTUS Team in Ulm

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Wolfgang Zeller

Stephan Kleinert

Enno Giese

Thank for your attention!

Time-dependent TF approximation

$$\psi(\boldsymbol{r},t) = \exp\left[\mathrm{i}\Phi\left(\boldsymbol{r}-\boldsymbol{\mathcal{R}}(t),t;\boldsymbol{\mathcal{R}}(t),\dot{\boldsymbol{\mathcal{R}}}(t)\right)\right] \times \psi^{(\mathrm{TF})}(\boldsymbol{r}-\boldsymbol{\mathcal{R}}(t);t)$$

$$\Phi\left(\boldsymbol{r},t;\boldsymbol{\mathcal{R}}(t),\dot{R}(t),\underline{\Lambda}(t)\right) \equiv \Phi_L + \Phi_{\boldsymbol{\mathcal{P}}} + \Phi_{\Lambda}$$

$$\Phi_{\Lambda}(\boldsymbol{r},t) \equiv \frac{m}{2\hbar}\boldsymbol{r}^T\,\underline{\dot{\Lambda}}(t)\underline{\Lambda}^{-1}(t)\,\boldsymbol{r}$$

$$\Lambda^{\mathrm{T}}(\tau)\left(\frac{\mathrm{d}^2\Lambda}{\mathrm{d}\tau^2}+\Omega^2(\tau)\,\Lambda(\tau)\right)=\frac{\Omega^2(0)}{\det\Lambda(\tau)}$$

$$\Lambda(0)=\mathbb{1} \quad \text{and} \quad \left.\frac{\mathrm{d}\Lambda}{\mathrm{d}\tau}\right|_{\tau=0}=0$$

Gross–Pitaevskii equation

Dynamics of N~10^4 interacting bosons in a BEC:

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla_{\boldsymbol{x}}^2 + V(t, \boldsymbol{x}) + g \left| \psi(t, \boldsymbol{x}) \right|^2 \right] \psi(t, \boldsymbol{x})$$

Repulsive interaction between bosons described by the coupling constant g

Trap approximated locally by a quadratic potential

$$V(0, \boldsymbol{x}) = \frac{m}{2} (\boldsymbol{x} - \boldsymbol{\rho}(0))^{\mathrm{T}} \Omega^2(0) \left(\boldsymbol{x} - \boldsymbol{\rho}(0) \right)$$

Normalization of the macroscopic wave function:

N atoms $\Rightarrow$ $$\int_{\mathbb{R}^d} \left| \psi(t, \boldsymbol{x}) \right|^2 \mathrm{d}^d \boldsymbol{x} = N$$

How to extract the internal dynamic of the BEC from the GP equation?

BEC output state in the “far field” regime

$$\Psi(\mathbf{x},t) = e^{i\Phi_1(\mathbf{x},t)}\psi_1(\mathbf{x},t) + e^{i\Phi_2(\mathbf{x},t)}\psi_2(\mathbf{x},t) + e^{i\Phi_3(\mathbf{x},t)}\psi_3(\mathbf{x},t) + e^{i\Phi_4(\mathbf{x},t)}\psi_4(\mathbf{x},t)$$

Two BEC amplitudes, for each momenta, separated by a distance $\boldsymbol{d}^T = (0,d,0)$ $\quad d = \frac{\hbar k}{m}\delta$

$$\psi_1(\mathbf{x},t) = \psi_2(\mathbf{x}+\mathbf{d},t) = \psi_3(\mathbf{x}+\frac{\mathbf{P}t}{m},t) = \psi_4(\mathbf{x}+\mathbf{d}+\frac{\mathbf{P}t}{m},t) \qquad \mathbf{P} = \hbar(0,K,0)$$

$$K = k_1 + k_2$$

Overlapping! Overlapping!

“Far field” condition
for long expansion time

BEC 1D width $\Delta y^{(\psi)}(t) < \frac{\hbar K t}{m}$

$$|\Psi(\mathbf{x},t)|^2 = |e^{i\Delta\Phi_{1-2}(\mathbf{x},t)}\psi_1(\mathbf{x},t) + \psi_2(\mathbf{x},t)|^2 + |e^{i\Delta\Phi_{3-4}}\psi_3(\mathbf{x},t) + \psi_4(\mathbf{x},t)|^2$$

Two set of interference fringes

“Far field” interference fringes

$$|\Psi(\mathbf{x},t)|^2 = |e^{i\Delta\Phi_{1-2}(\mathbf{x},t)}\psi_1(\mathbf{x},t) + \psi_2(\mathbf{x},t)|^2 + |e^{i\Delta\Phi_{3-4}}\psi_3(\mathbf{x},t) + \psi_4(\mathbf{x},t)|^2$$

Linear approximation
for long time BEC expansion

$$\Lambda_{ii} \approx v_i t$$

BEC internal dynamics

Bragg pulses
interaction

$$\Delta\Phi_{1-2}(y,t) \approx \frac{mdy}{\hbar t} + \Delta C^{(B)}_{1-2} + f(t)$$

$$\Delta\Phi_{3-4}(y,t) \approx \frac{mdy}{\hbar t} + \Delta C^{(B)}_{3-4} + f(t)$$

Fringe spacing

$$\lambda_{1-2}(t) = \lambda_{3-4}(t) = \frac{ht}{md}$$

$$\lambda_{dB}(t) = \frac{h}{mv(t)} \qquad v = d/t$$

Phase delay between
the two set of fringes

$$\Delta C^{(B)}_{1-2} - \Delta C^{(B)}_{3-4} = -\pi$$

Conservation of particles!