Can anisotropic expansion of the spacetime generate quantum interference in light propagation?

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Let's consider the Bianchi type I metric

$$ds^2 = dt^2 - b_1^2(t)\left[dx^2 + dy^2\right] - b_2^2(t)dz^2$$

Maxwell equations in a curved spacetime without electromagnetic sources are

$$\nabla_\mu F^{\mu\nu} = 0$$

where

$$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$$

After imposing the generalized Coulomb gauge $A_0 = 0$ and $\frac{\partial_1 A_1}{b_1(t)} + \frac{\partial_2 A_2}{b_1(t)} + \frac{\partial_3 A_3}{b_2(t)} = 0$ let us expand fileds in normal modes:

$$A_i(t,\vec{r}) = \int \frac{d^3k}{(2\pi)^3\sqrt{2k}} \sum_{\alpha=1,2} \epsilon_{i\vec{k}\alpha} a_{i\vec{k}\alpha} A_{ik}(t) e^{i\vec{k}\cdot\vec{r}} + h.c.$$

and require the usual bosonic commutation rule in order to quantize fields:

$$\left[a_{\perp\vec{k},\alpha}, a^{+}_{\perp\vec{k'}\beta}\right] = \delta_{\vec{k}\vec{k'}}\delta_{\alpha\beta}$$

$$\left[a_{\parallel\vec{k}\alpha}, a^{+}_{\parallel\vec{k'}\beta}\right] = \delta_{\vec{k}\vec{k'}}\delta_{\alpha\beta}$$

$$\text{All others} = 0$$

In such a way, we obtain two different evolutions

$$\ddot{A}_{\perp k}(t) + \left(2\,\frac{\dot{b}_1}{b_1} - \frac{\dot{b}_2}{b_2}\right)\dot{A}_{\perp k}(t) + k^2 A_{\perp k}(t) = 0$$

$$\ddot{A}_{\parallel k}(t) + \frac{\dot{b}_2}{b_2}\,\dot{A}_{\parallel k}(t) + k^2 A_{\parallel k}(t) = 0$$

respectively referring to the evolution along b_2 direction and the perpendicular one.

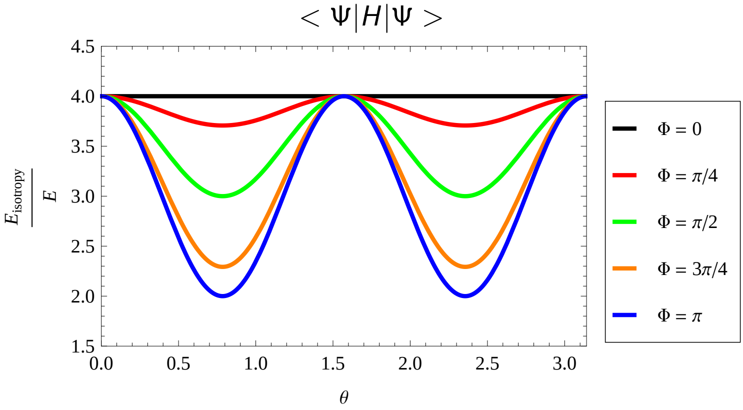
Let us consider the generalized hamiltonian

$$H \equiv \sum_{\vec{k}} \left(E_{\parallel \vec{k}}\, a^{+}_{\parallel \vec{k}}\, a_{\parallel \vec{k}} + E_{\perp \vec{k}}\, a^{+}_{\perp \vec{k}}\, a_{\perp \vec{k}} \right)$$

that collapse to the usual normal ordered one in of the Minkowski spacetime $(b_1 = b_2 = 1)$.
In such a way, the one particle state and the two particle one are respectively:

$$|\psi >= \cos\theta\, a^{+}_{\perp \vec{k}} |0> + e^{i\alpha} \sin\theta\, a^{+}_{\parallel \vec{k}} |0>$$

$$|\Psi >= \left(\cos\theta\, a^{+}_{1\perp} + e^{i\alpha} \sin\theta\, a^{+}_{1\parallel} \right) \left(\cos\phi\, a^{+}_{2\perp} + e^{i\beta} \sin\phi\, a^{+}_{2\parallel} \right) |0>$$



where $\Phi = \alpha - \beta$ and $\theta = \phi$.