Random States and typicality in Hilbert spaces

Theory Xmas Workshop 2012

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When something is too complicated. . . you can use a random object to mimic it.

Random States $\leftrightarrow$

Random Matrix Theory

Founding fathers: Wishart, Wigner, Dyson

RMT in Physics:

Quantum Mechanics
Statistical Mechanics

H random hamiltonian
U random evolution
ρ random density matrix
$\partial^2 V$ random landscape

RMT in Math:

Probability and Statistics
Geometry
Linear Algebra and Numerical Analysis

Σ random covariances
$Ax = b$ random linear system

This is a rich and attractive area for physicists. Especially for a student!

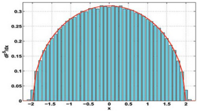
Random Matrix Theory, universality and concentration phenomena

Universality of the spectral statistics of random matrices

For random matrices a kind of Central Limit Theorem holds. In fact, many CLT's!
The behaviour of a function $f(X_1, \ldots, X_N)$ of iid random variables $X_1, \ldots, X_N$ for N large does not depend very much on the actual distribution of the X_i.
An idea inherited from Statistical Mechanics.

Example: Wigner's semicircle law

For a large class of hermitian random matrices, the spectral distribution $p(\lambda)$ tends to the semicircle distribution (analogue of the Gaussian distribution in a noncommutative setting).



Concentration of measure phenomenon

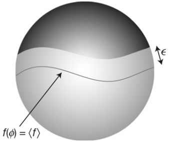
Some properties are shared by an overwhelming majority of the states.
A high dimensional effect, or a property of a large number of variables, for which many observables are almost constant and close to their mean value.
A basic feature in Statistical Mechanics.

Example: Levy's lemma

Nice functions f on a high dim sphere $\mathbb{S}^n$ are almost constant:

$f = \langle f \rangle$ with overwhelming probability, in the sense that

$\mathbf{Pr}\left(|f(X) - \langle f \rangle| \geq \epsilon\right) \leq e^{-Cn\epsilon^2}$

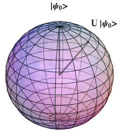


Random Quantum States: an overview

Examples of random states: $|\psi\rangle \sim p(\psi)\mathrm{d}\psi$ (random unit vectors)

Unbiased ensemble A quantum system with a cutoff Λ on the energy H.
A crude approach: the probability measure is uniform: $\mathbf{Pr}(|\psi\rangle) = \mathbf{Pr}(U|\psi\rangle)$.

Maximal symmetry = minimal prior knowledge
Many results have been achieved. Most of the technology works well!

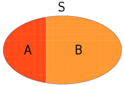


Any physical system obeys some constraint: conservation laws.
The accessible portion of the Hilbert space is a submanifold.

Fixed "energy" ensemble equal prob. to states with the same energy: $\langle\psi| H |\psi\rangle = E$.
Hard problem. In the large dimensional limit one want to recover the Gibbs measure.
The typical local states ρ must be canonical $e^{-\beta H}/Z$(according to Statistical Mechanics)

Fixed "entanglement" ensemble: states of composite systems $|\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$.

Explore the isoentangled surfaces $S(|\psi\rangle_{AB}) = s$.
Hard because the geometry of quantum states is not fully understood.



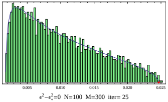
Random Quantum States: my work.

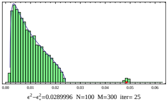
Typical local properties of bipartite systems via Coulomb gas method and orthogonal poly.

(*preprint*: FDC, P.Facchi, G.Florio, S.Pascazio)

Random states with fixed entanglement: unitarily invariant measure interacts nicely with the superposition principle.

New polarized ensembles: isoentangled ensemble and phase transitions.





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Locally invariant ensembles: the Hilbert space of a bipartite quantum system is foliated in manifolds of equal Schmidt rank.

On each leave, by projecting the unbiased measure we obtain a family of pdfs.

(*work in progress*)

Typicality in Hilbert spaces: random states and the **foundations of Statistical Mechanics**.

The thermodynamical limit $\frac{\dim \mathrm{system}}{\dim \mathrm{bath}} = \mathrm{O}(1)$.

Quantum Information: random states and ensemble preparation.

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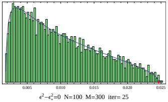

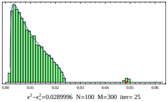

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Merry Xmas!