Multipartite Entanglement and Gaussian states

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- Journal of Mathematical Physics (to appear)
- International Journal of Geometric Methods in Modern Physics (2012)
- Physical Review A (2009)

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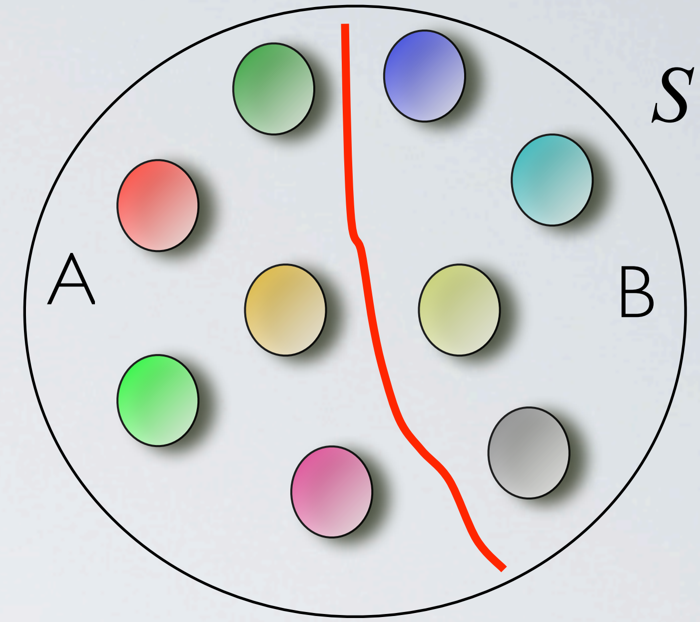
Entanglement

Consider a quantum system in a pure state $|\eta\rangle$

Consider two subsystems A and B

If one can write the state in a factorized form



$|\eta\rangle = |\psi\rangle_A |\phi\rangle_B$

$|\psi\rangle_A \in \mathcal{H}_A \quad |\phi\rangle_B \in \mathcal{H}_B$

then the state is SEPARABLE.

Otherwise it is ENTANGLED.

We consider a system composed of n subsystems described by a Hilbert space

$$\mathcal{H}_S := \bigotimes_{i \in S} \mathfrak{h}_i$$

with

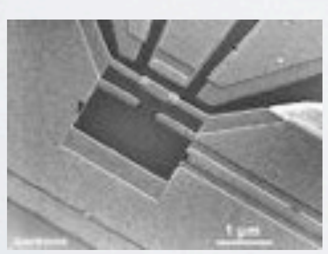
$$\mathfrak{h}_i \simeq \mathfrak{h}$$

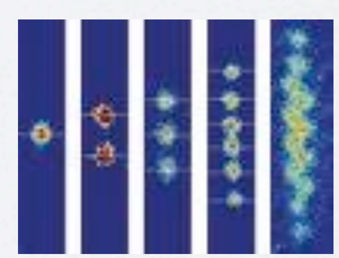
$$S = \{1, 2, \ldots, n\}$$

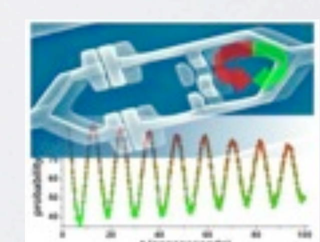
$\mathfrak{h} = \mathbb{C}^2$ qubit

or

$\mathfrak{h} = L^2(\mathbb{R})$ continuous variables







Gaussian states

Let us consider a collection of n bosonic oscillators with canonical variables $\{q_k, p_k\}_{k=1,\dots n}$

The state of the system can be described by the density operator $\rho_{(n)}$ or, in the phase-space, by the so-called **Wigner function**.

By definition, Gaussian states (important for applications in quantum optics) are described by a gaussian Wigner function:

$$W_{(n)}(\mathbf{X}) \quad = \quad \frac{1}{(2\pi)^n\sqrt{\det(\mathbb{V})}} \exp\left[-\frac{1}{2}(\mathbf{X}-\mathbf{X}_0)\mathbb{V}^{-1}(\mathbf{X}-\mathbf{X}_0)^{\mathsf{T}}\right]$$

$$\mathbf{X} = (X_1, \dots X_{2n}) := (q_1, p_1, \dots q_n, p_n)$$

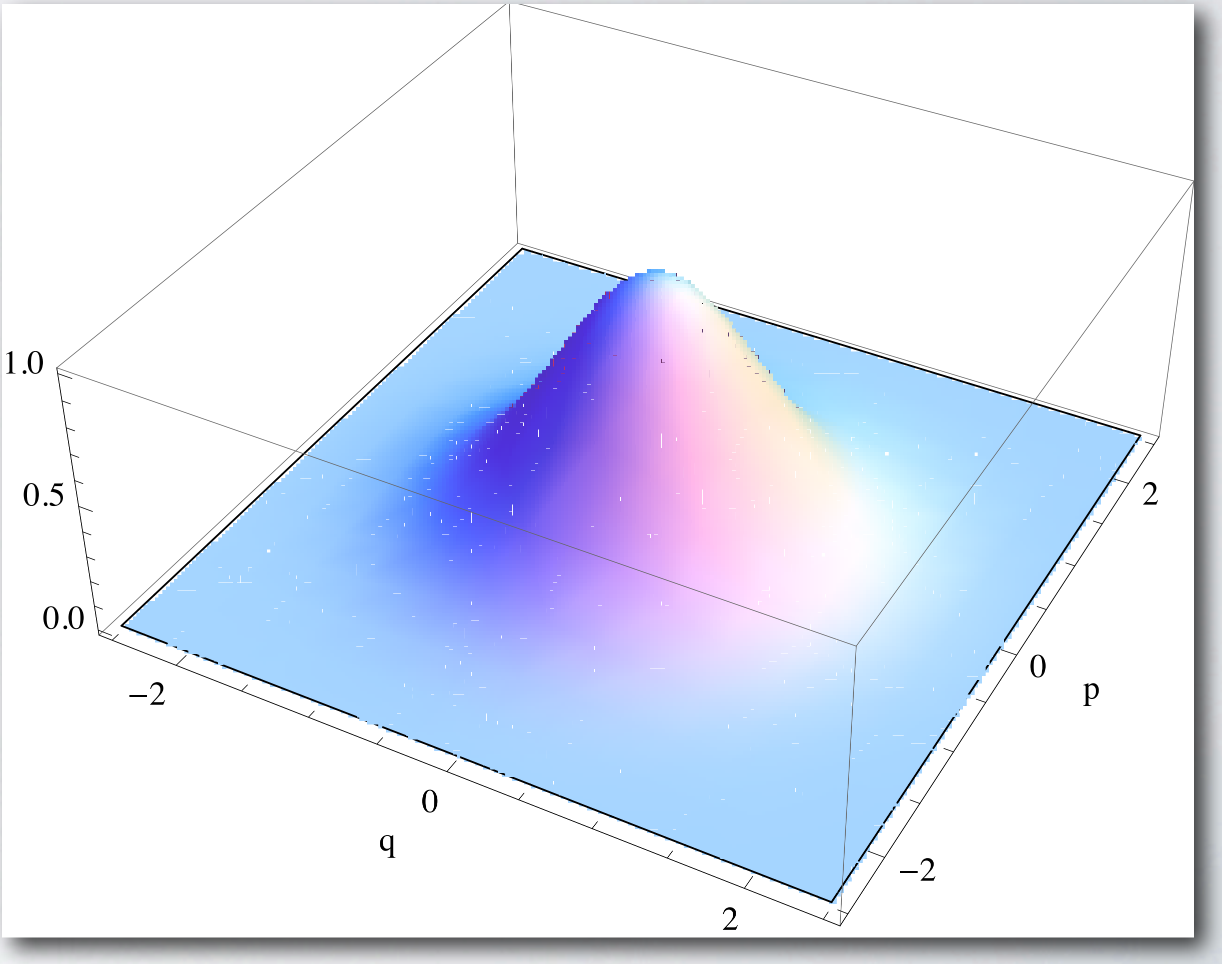
Phase-space coordinate vector

$\mathbf{X}_0 = \langle \mathbf{X} \rangle$ vector of the first moments with

$$\langle f(\mathbf{X})\rangle := \int f(\mathbf{X}) W_{(n)}(\mathbf{X}) d^{2n}\mathbf{X}$$

$$\mathbb{V}_{lm} = \langle (X_l - \langle X_l \rangle)(X_m - \langle X_m \rangle)\rangle$$

Covariance matrix (CM)



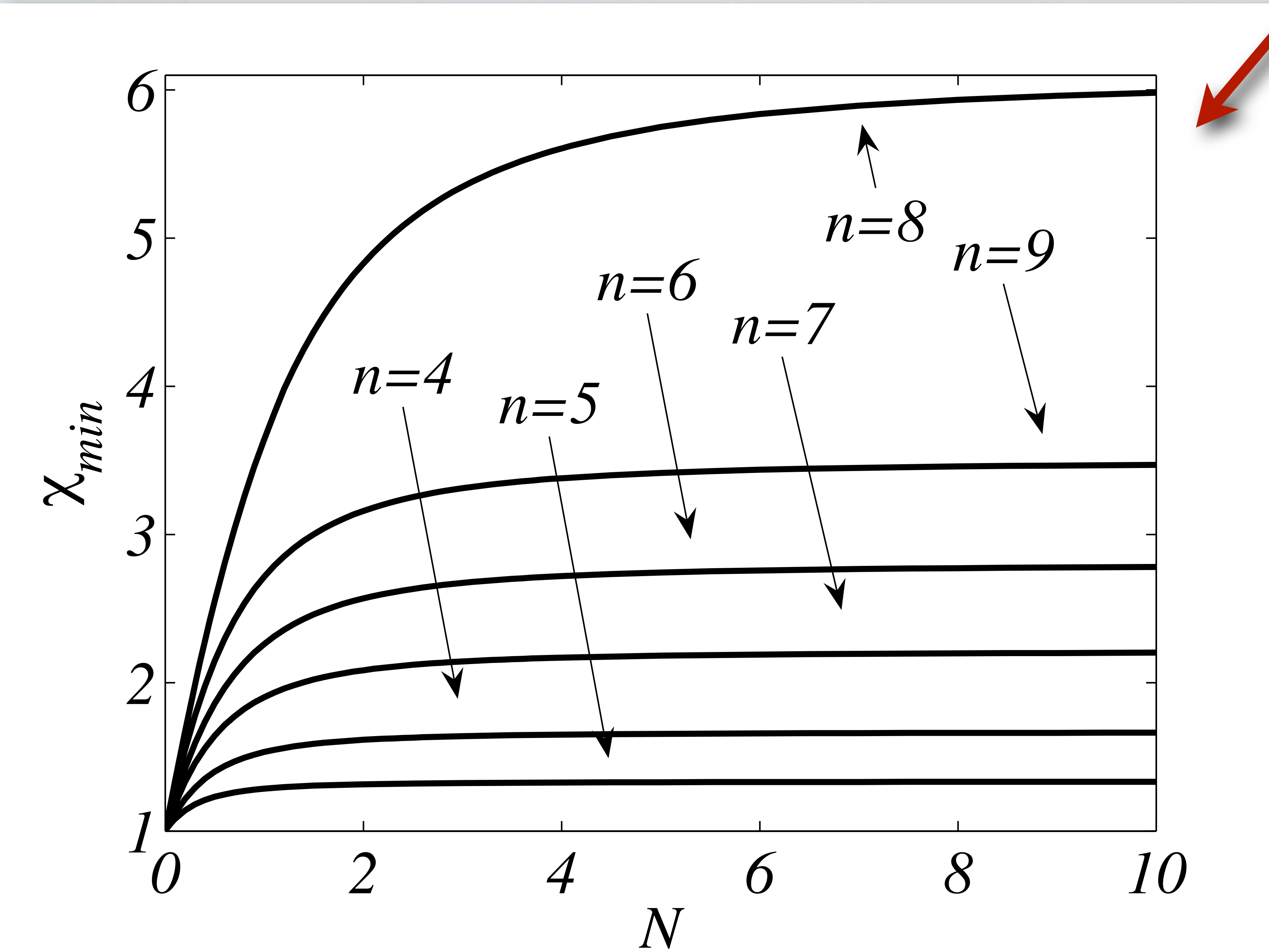
Multipartite entanglement in multimode Gaussian states

Energy constraint: we do not allow more than N mean excitations for each bosonic mode:

$$\frac{\langle q_k^2 + p_k^2 \rangle}{2} \leq N + \frac{1}{2}, \qquad \text{for} \quad k = 1, \dots n$$

Measure of entanglement:
Normalized potential of multipartite entanglement χ

Dependence of the minimum (maximum multipartite entanglement) of number of modes and mean number of excitations per mode



$\chi = 1$ **Ideal minimum**

This is possible only for N=0: the vacuum (separable)

In general we have frustration!

Increasing the value of N we observe a saturation effect

Sampling of Gaussian states

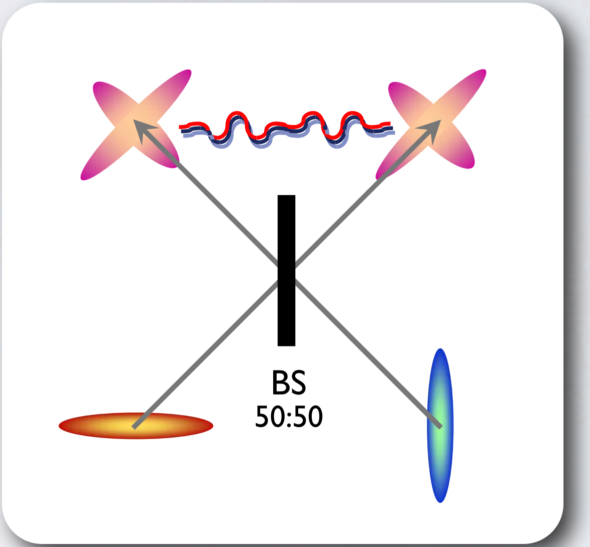
Bipartite system: (A, B) $\quad n_A = |A| \quad n_A + n_B = n \quad 1 \le n_A \le n_B$

$$|\psi_G\rangle = \prod_{k=1}^{n_A} \exp\left[r_k(a_k b_k - a_k^\dagger b_k^\dagger)\right]|0\rangle$$

$$a_k, a_k^\dagger\,, \qquad k = 1, \dots, n_A$$
$$b_k, b_k^\dagger\,, \qquad k = 1, \dots, n_B \qquad r_k \ge 0$$

Two-modes squeezed vacuum states (twin-"beams")

$$|\mathrm{TMSV}\rangle = \exp\left[r(ab - a^\dagger b^\dagger)\right]|0\rangle$$



How do we sample Gaussian states?

Invariant measure of Gaussian states generated in nonlinear optical parametric processes

$$\nu_k = \cosh 2r_k\,, \quad k = 1, \dots, n_A$$

Symplectic eigenvalues

$$d\mu_{\bar{G}} = \bar{K}_{n,n_A} \prod_{h<k=1}^{n_A} (\nu_h - \nu_k)^2 \prod_{j=1}^{n_A} (\nu_j - 1)^{n_B - n_A}\, d\nu\, d\mu(\bar{\alpha}_A)\, d\mu(\bar{\alpha}_B)\, d\theta$$

non-local degrees of freedoom

Merry Christmas!!!!