Phenomenology of Absolute Neutrino Masses

Carlo Giunti

INFN, Sezione di Torino, and

Dipartimento di Fisica Teorica, Università di Torino

giunti@to.infn.it

NOW 2004, 11-17 September 2004, Conca Specchiulla, Otranto, Italy

Open Questions on Neutrino Masses

Value of $\vert U_{e3}\vert$, J_{CP}?	$\Leftarrow$	ν Oscillations
Sterile Neutrinos?	$\Leftarrow$	ν Oscillations
		LSND indication $\Longrightarrow$ wait MiniBooNE
Absolute Scale of Neutrino Masses?	$\Leftarrow$	β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay
Pattern of Neutrino Masses?	$\Leftarrow$	ν Osc., β Dec., Cosmology, $\beta\beta_{0\nu}$ Dec.
Are Neutrinos Majorana?	$\Leftarrow$	$\beta\beta_{0\nu}$ Decay

Oscillations are Insensitive to Absolute Scale of Neutrino Masses

oscillations are due to the interference of different massive components of a flavor ν

$\Downarrow$

oscillations can be sensitive only to mass differences

In Neutrino Oscillations Dirac = Majorana

$$\frac{\mathrm{d}\nu_\alpha}{\mathrm{d}t} = \frac{1}{2E}\left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\nu_\beta \qquad M^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

Dirac: $U \to U_\mathrm{D}$

Majorana: $U \to U_\mathrm{M} = U_\mathrm{D}\,D$

$$U_\mathrm{D} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$D = \mathrm{diag}(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}) \quad \Rightarrow \quad U_\mathrm{M}M^2U_\mathrm{M}^\dagger = U_\mathrm{D}DM^2D^\dagger U_\mathrm{D}^\dagger = U_\mathrm{D}M^2U_\mathrm{D}^\dagger$$

Majorana phases

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495]

[Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Neutrino Oscillation Data $\Rightarrow$ Three-Neutrino Mixing

flavor fields ν_α

$\alpha = e, \mu, \tau$

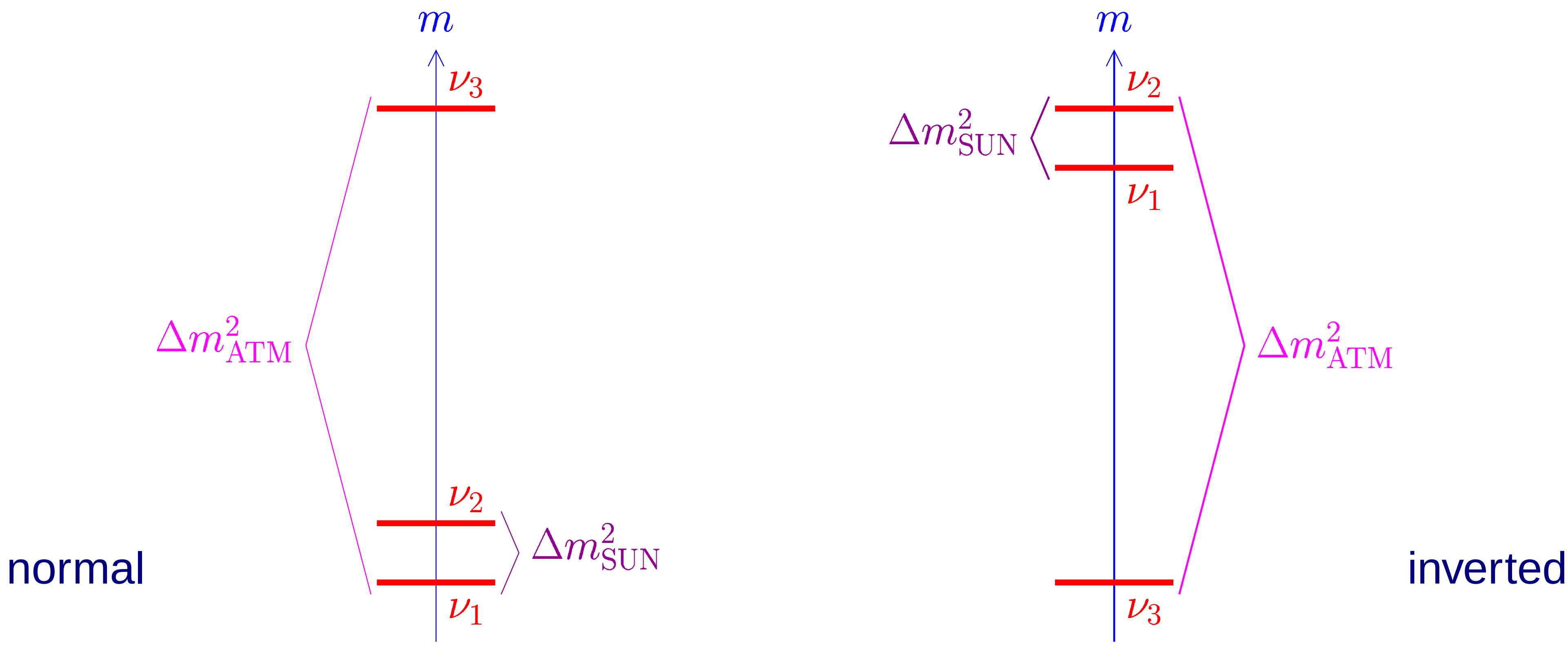
$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$$

massive fields ν_k

$k = 1, 2, 3$

$$\Delta m^2_{\text{SUN}} = \Delta m^2_{21} \simeq 8.3 \times 10^{-5} \, \text{eV}^2$$

$$\Delta m^2_{\text{ATM}} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \times 10^{-3} \, \text{eV}^2$$



Global Fit of Oscillation Data ⇒ Bilarge Mixing

[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

$$\Delta m^{2\,\mathrm{bf}}_{21} \simeq 8.3 \times 10^{-5}\,\mathrm{eV}^2 \qquad 7.4 \times 10^{-5} \lesssim \Delta m^2_{21} \lesssim 9.3 \times 10^{-5} \quad (3\sigma)$$

$$\Delta m^{2\,\mathrm{bf}}_{31} \simeq 2.4 \times 10^{-3}\,\mathrm{eV}^2 \qquad 1.8 \times 10^{-3} \lesssim \Delta m^2_{31} \lesssim 3.2 \times 10^{-3} \quad (3\sigma)$$

$$\sin^2\vartheta^{\mathrm{bf}}_{12} \simeq 0.28 \qquad 0.22 \lesssim \sin^2\vartheta_{12} \lesssim 0.37 \quad (3\sigma)$$

$$\sin^2\vartheta^{\mathrm{bf}}_{13} \simeq 0.01 \qquad \sin^2\vartheta_{13} \lesssim 0.05 \quad (3\sigma)$$

$$\sin^2 2\vartheta^{\mathrm{bf}}_{\mathrm{ATM}} \simeq 1 \qquad \sin^2 2\vartheta_{\mathrm{ATM}} \gtrsim 0.86 \quad (3\sigma)$$

[Fogli, Lisi, Marrone, Montanino, PRD 67 (2003) 093006]

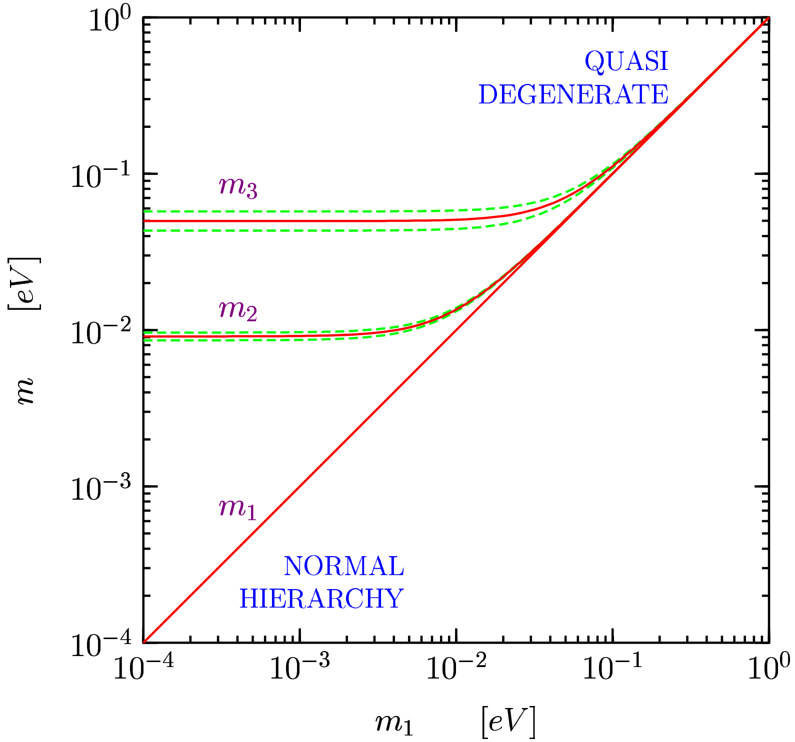
$$\sin^2\vartheta_{\mathrm{ATM}} = |U_{\mu 3}|^2 = \sin^2\vartheta_{23}\,\cos^2\vartheta_{13}$$

$$|U|_{\mathrm{bf}} \simeq \begin{pmatrix} 0.84 & 0.53 & 0.10 \\ 0.31-0.43 & 0.56-0.63 & 0.71 \\ 0.32-0.44 & 0.57-0.64 & 0.70 \end{pmatrix}$$

$$|U| \simeq \begin{pmatrix} 0.77-0.88 & 0.46-0.61 & 0.00-0.22 \\ 0.08-0.60 & 0.30-0.79 & 0.55-0.85 \\ 0.10-0.61 & 0.33-0.81 & 0.51-0.83 \end{pmatrix}$$

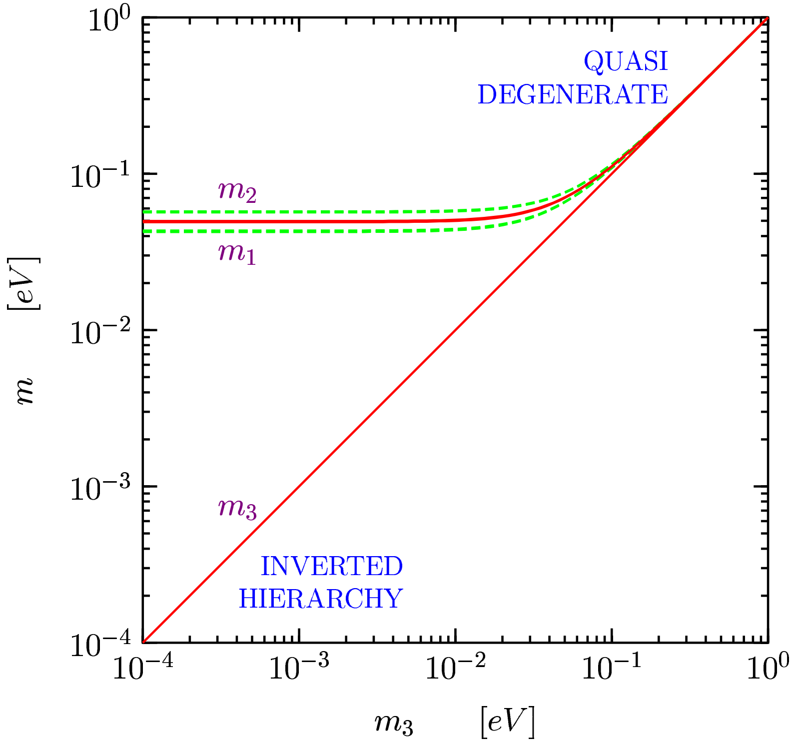
Absolute Scale of Neutrino Masses

normal scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{ATM}}^2$$

inverted scheme



$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{ATM}}^2$$

Quasi Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\mathrm{ATM}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

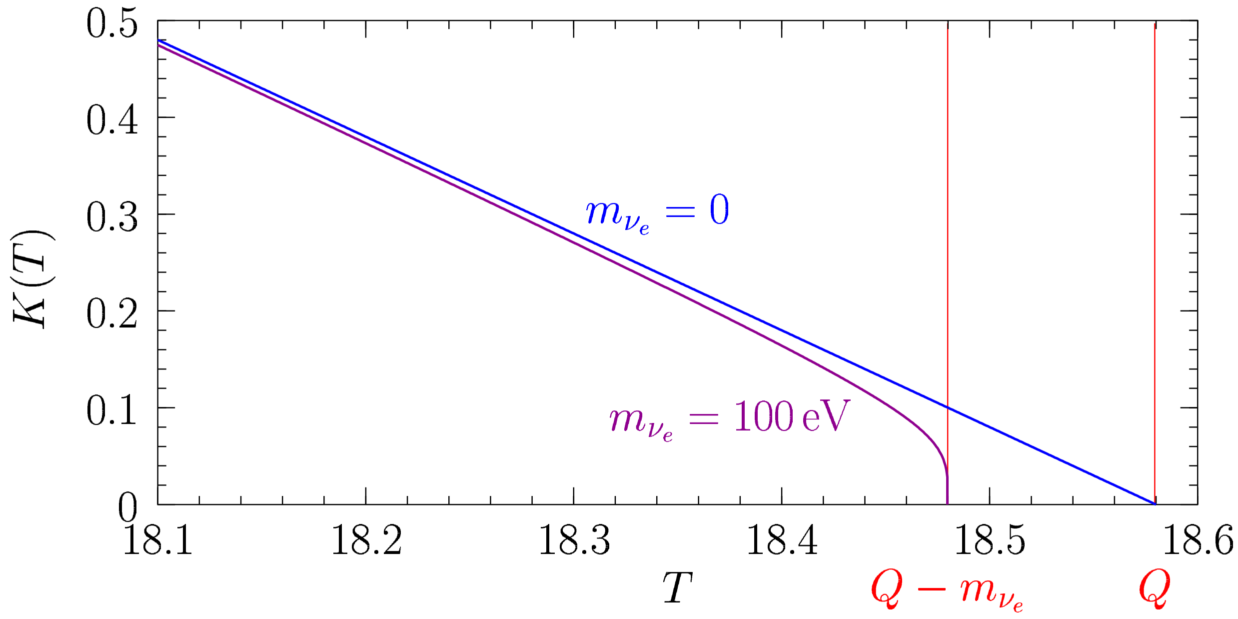
Tritium β Decay

$^{3}\mathrm{H} \to {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot: $$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$m_{\nu_e} < 2.2\,\mathrm{eV}$ (95% C.L.)

Mainz & Troitsk [Weinheimer, hep-ex/0210050]

$m_{\nu_e} < 1.8\,\mathrm{eV}$ (95% C.L.)

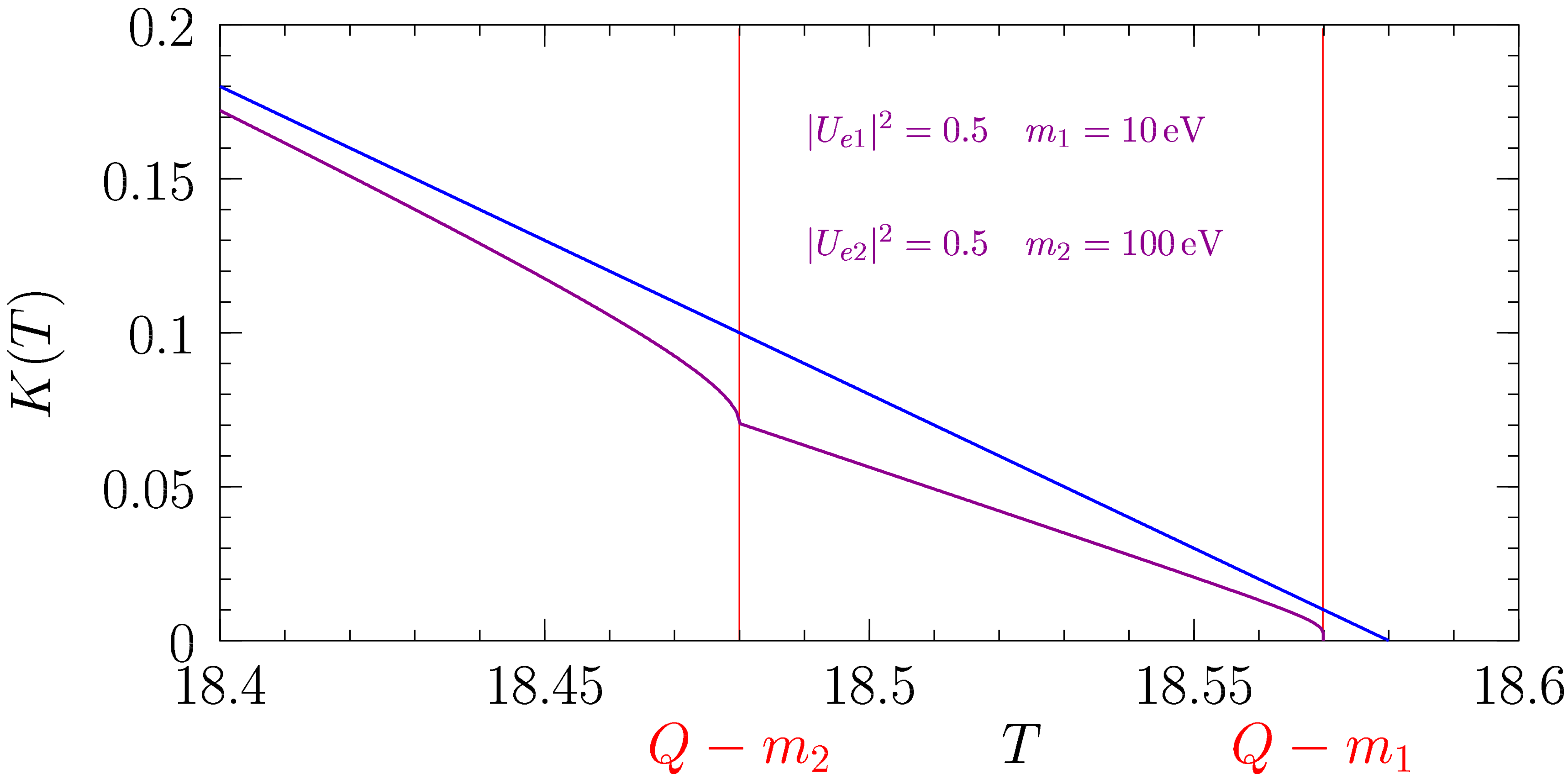
Mainz + Troitsk [Fogli et al., hep-ph/0408045]

future: KATRIN

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2 - 0.3\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow$ $K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case

$2N - 1$ parameters = N masses + $N - 1$ mixing matrix elements $\left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses ($m_k \ll Q - T$) $\Longrightarrow$ effective mass

$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

[McKellar, PLB 97 (1980) 93] [Shrock, PLB 96 (1980) 159]

[Kobzarev, Martemyanov, Okun, Shchepkin, SJNP 32 (1980) 823]

[Holzschuh, RPP 55 (1992) 1035] [Weinheimer et al., PLB 460 (1999) 219]

[Vissani, NPB PS 100 (2001) 273 (NOW 2000)] [Farzan, Smirnov, PLB 557 (2003) 224]

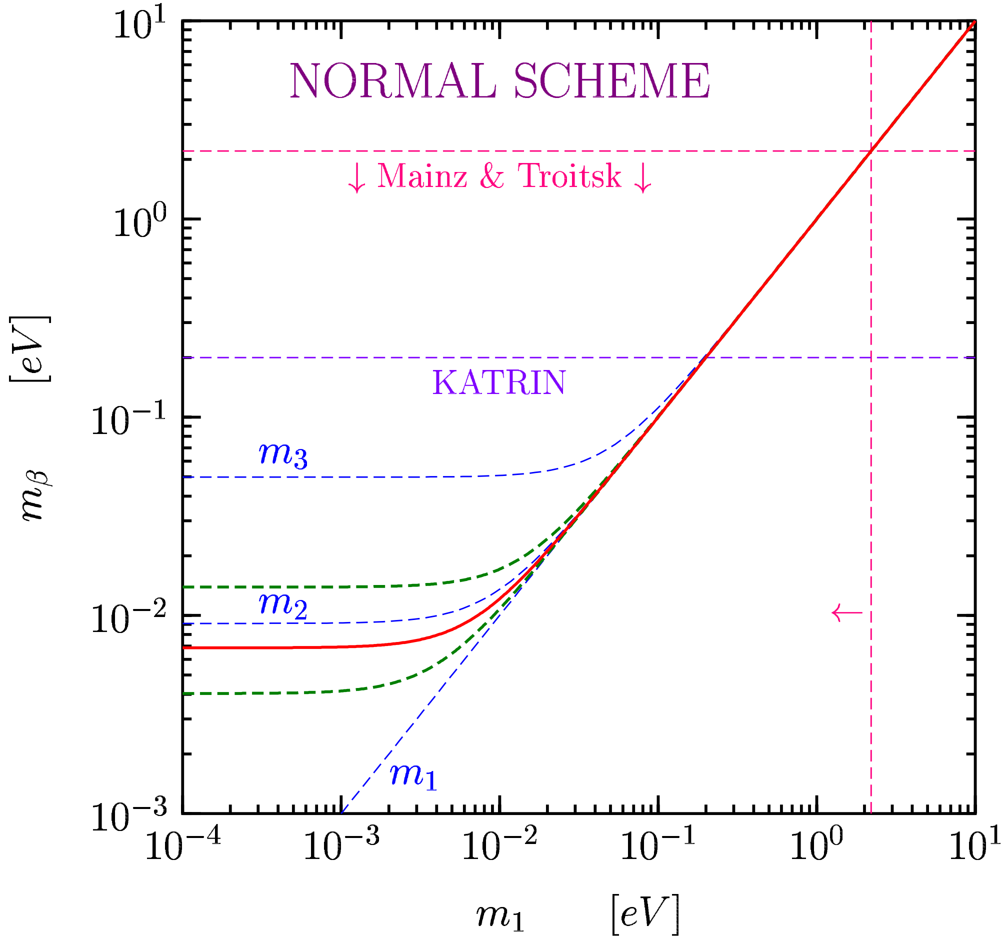
$$K^2 = (Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}}$$

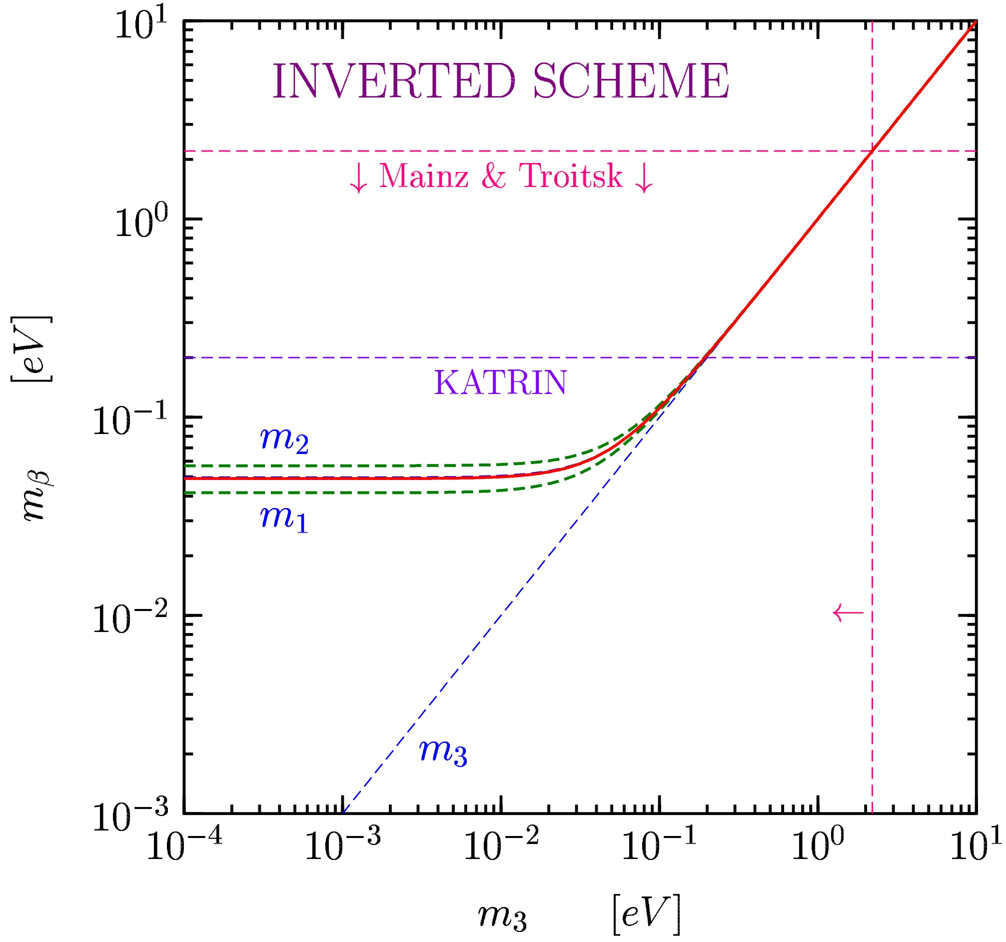
$$\simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right] = (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right]$$

$$\simeq (Q-T)^2 \sqrt{1 - \frac{m_\beta^2}{(Q-T)^2}} = (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

$m_{\nu_e} < 2.2\,\text{eV}$ (95% C.L.) $\Longrightarrow$ $m_\beta < 2.2\,\text{eV}$ (95% C.L.)

Three-Neutrino Mixing $\Longrightarrow$ $m_\beta^2 = c_{12}^2\, c_{13}^2\, m_1^2 + s_{12}^2\, c_{13}^2\, m_2^2 + s_{13}^2\, m_3^2$





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SUN}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$
$$m_\beta^2 \simeq m_1^2 + |U_{e2}|^2\, \Delta m_{\text{SUN}}^2 + |U_{e3}|^2\, \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_\beta^2 \simeq m_3^2 + \left(|U_{e1}|^2 + |U_{e2}|^2\right) \Delta m_{\text{ATM}}^2$$

Quasi Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

neutrinos are in equilibrium in the primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \qquad \nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \qquad \Longrightarrow \quad T_{\text{dec}} \sim 1\,\text{MeV}$$

neutrino decoupling

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \simeq 1.676 \times 10^{-4}\,\text{eV}$ $(T_\gamma = 2.725 \pm 0.001\,\text{K})$

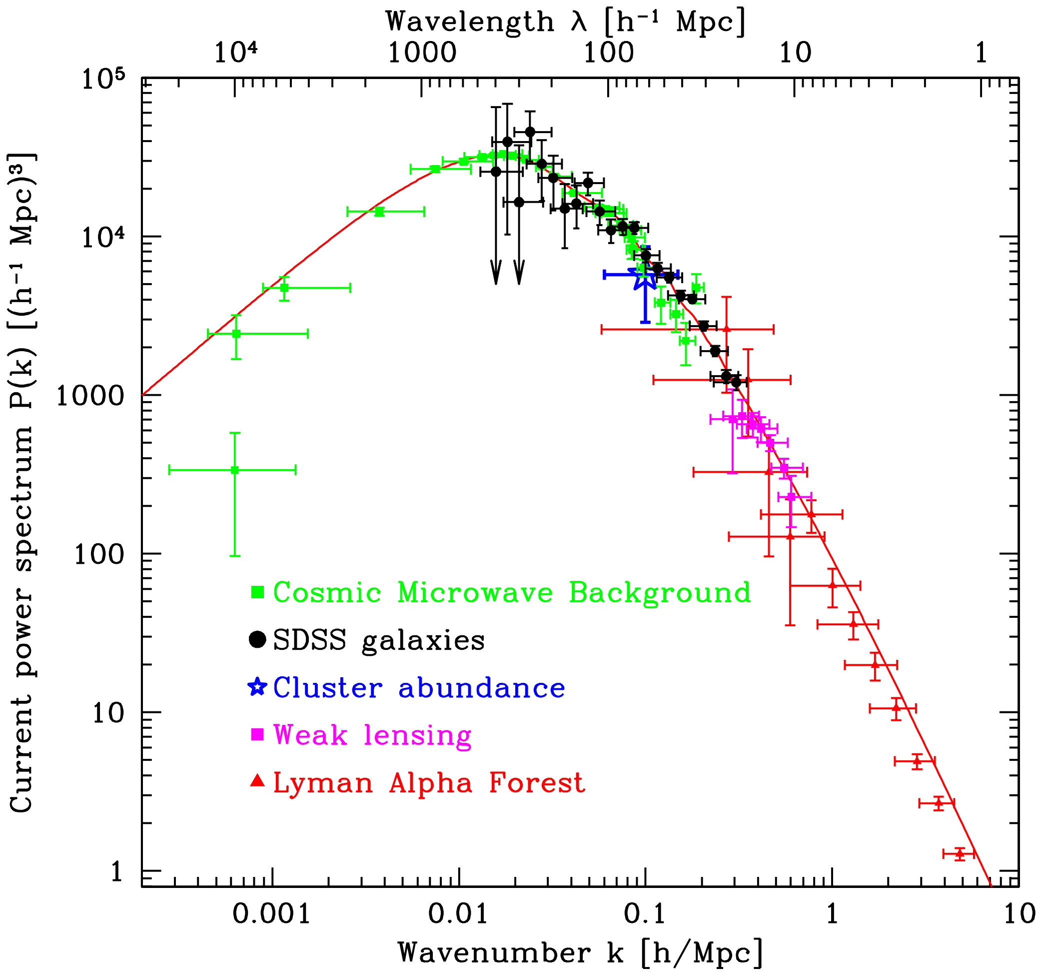
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$ $\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 1 \qquad \Longrightarrow \qquad \sum_k m_k \lesssim 46\,\text{eV}$$

Power Spectrum of Density Fluctuations



[SDSS, astro-ph/0310725]

hot dark matter prevents early galaxy formation

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for $k \gtrsim k_{\text{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\,\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

Reviews: [Dolgov, Phys. Rept. 370 (2002) 33], [Kainulainen, Olive, hep-ph/0206163], [Sarkar, hep-ph/0302175], [Hannestad, NJP 6 (2004) 108]

WMAP, AJ SS 148 (2003) 175

CMB (WMAP, CBI, ACBAR) + LSS (2dFGRS) + Lyα + HST + SN-Ia

$$\Lambda\text{CDM:} \quad \begin{cases} T_0 = 13.7 \pm 0.1\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}, \\ \Omega_{\text{tot}} = 1.02 \pm 0.02 \qquad \Omega_b h^2 = 0.0224 \pm 0.0009 \qquad \Omega_m h^2 = 0.135^{+0.008}_{-0.009} \end{cases}$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% confidence)} \quad \Longrightarrow \quad \sum_k m_k < 0.71\,\text{eV}$$

Hannestad, JCAP 0305 (2003) 004

$\sum_k m_k < 1.01\,\text{eV}$ (95% confidence) WMAP+CBI+2dFGRS+HST+SN-Ia

$\sum_k m_k < 1.20\,\text{eV}$ (95% confidence) WMAP+CBI+2dFGRS

$\sum_k m_k < 2.12\,\text{eV}$ (95% confidence) WMAP+2dFGRS

Elgaroy and Lahav, JCAP 04 (2003) 004

$\sum_k m_k < 1.1\,\text{eV}$ (95% confidence) WMAP+2dFGRS+HST

SDSS, PRD 69 (2004) 103501

CMB(WMAP)+LSS(SDSS)+SN-Ia

$h = 0.70^{+0.04}_{-0.03}$ $\Omega_m = 0.30 \pm 0.04$ $\sum_k m_k < 1.7\,\text{eV}$ (95% confidence)

SDSS, astro-ph/0406594

CMB(WMAP)+LSS(SDSS)+bias(SDSS) $P_g(k) = b^2\, P_m(k)$

$\Omega_m = 0.25 \pm 0.03$ $\sum_k m_k < 0.54\,\text{eV}$ (95% confidence)

SDSS, astro-ph/0407372

CMB(WMAP)+LSS(SDSS)+bias(SDSS)+Lyα(SDSS)+SN-Ia

$\Omega_\Lambda = 0.72 \pm 0.02$ $\sum_k m_k < 0.42\,\text{eV}$ (95% confidence)

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045

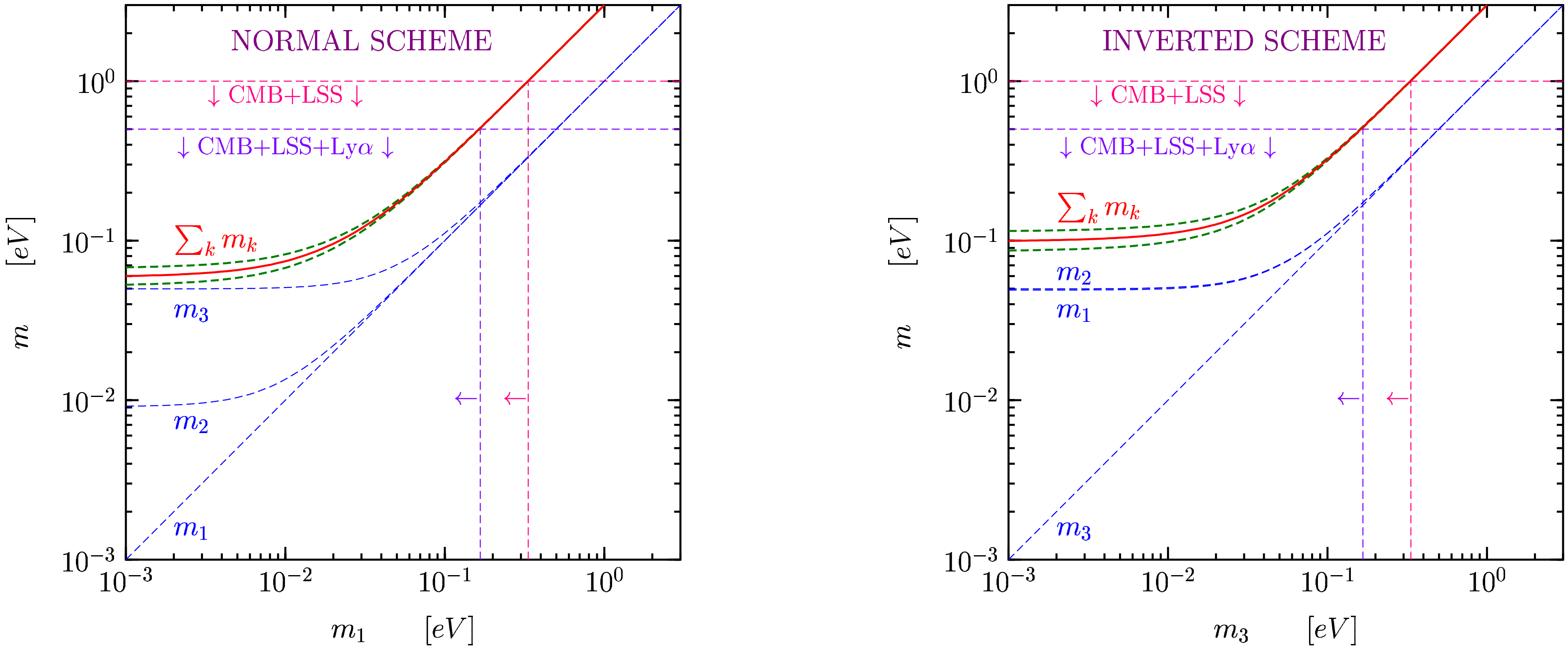
$\sum_k m_k < 1.4\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia

$\sum_k m_k < 0.47\,\text{eV}$ (2σ) CMB+LSS+HST+SN-Ia+Lyα(SDSS)

Approximate Estimate

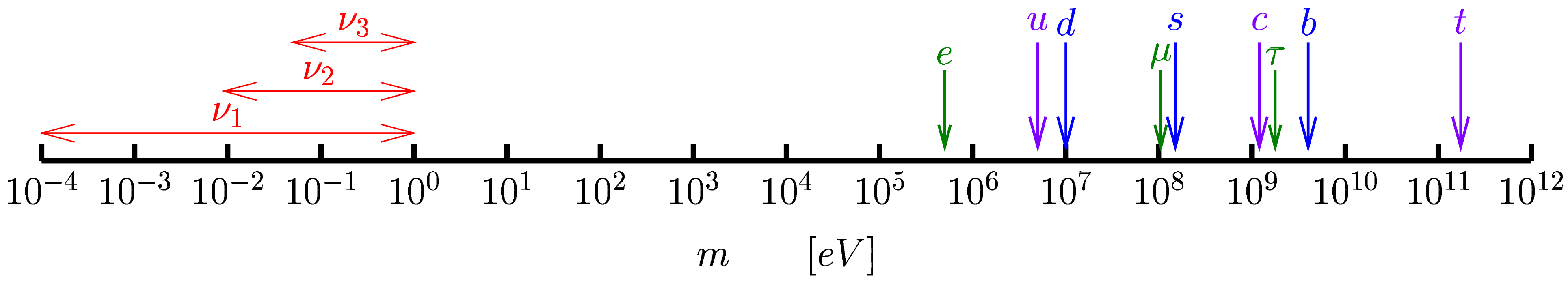
$$\sum_k m_k \lesssim 1\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia}$$

$$\sum_k m_k \lesssim 0.5\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+HST+SN-Ia+Ly}\alpha$$



FUTURE: IF $\sum_k m_k \lesssim 8 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Majorana Neutrino Mass?



known natural explanations of smallness of ν masses:

- ★ See-Saw Mechanism
- ★ 5-D Non-Renormalizable Effective Operator

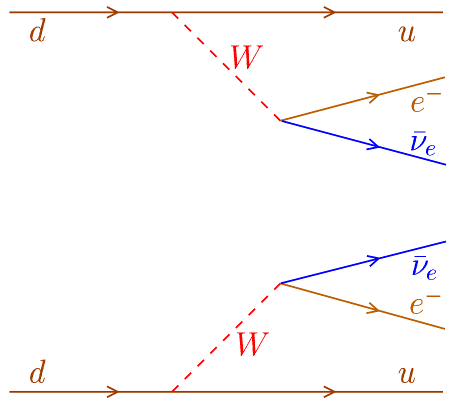
both imply

- ★ Majorana ν masses $\Longleftrightarrow$ $|\Delta L| = 2$ $\Longleftrightarrow$ $\beta\beta_{0\nu}$ decay
- ★ see-saw type relation $m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}}$
- ★ new high energy scale $\mathcal{M}$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

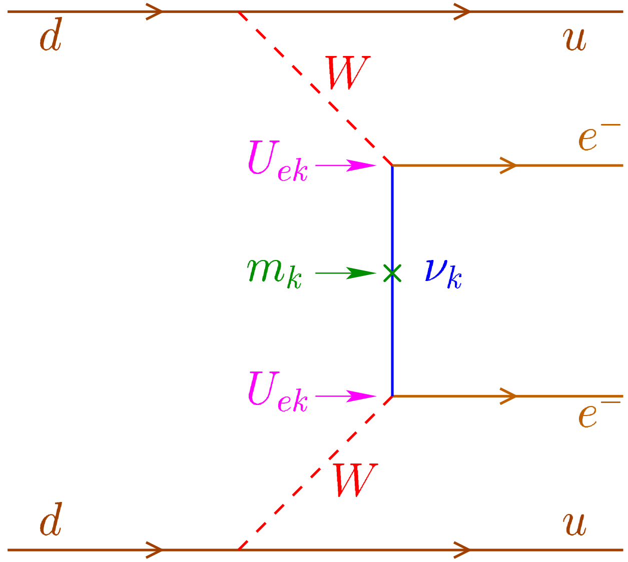


$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$



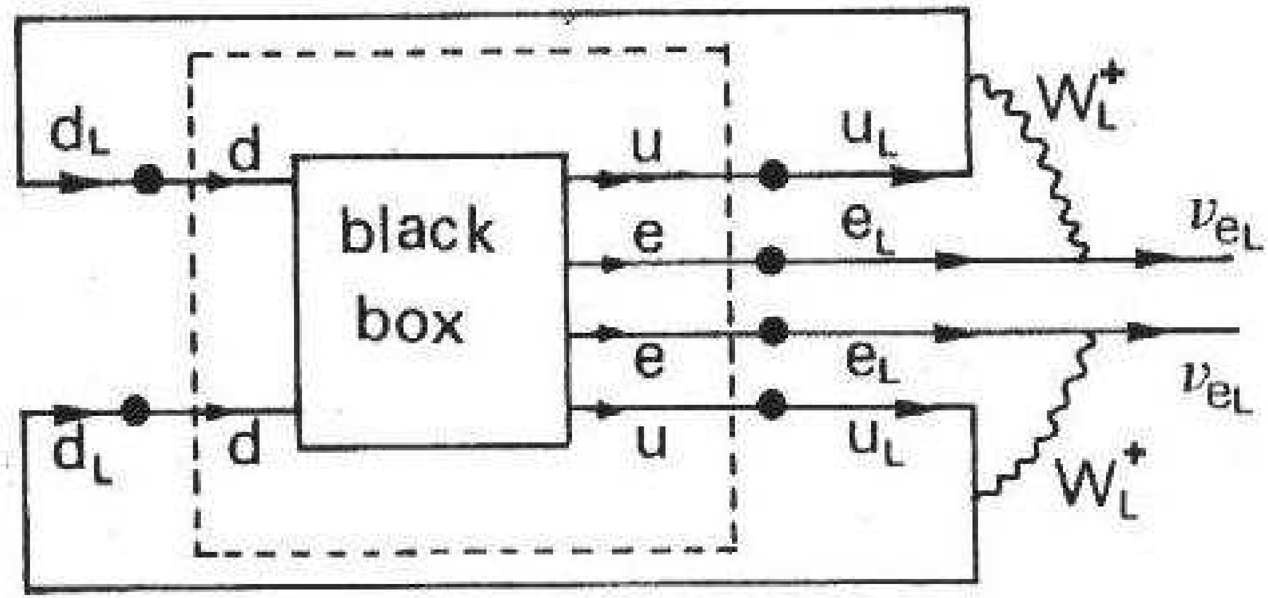
$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$$

Majorana Neutrino Mass $\Leftrightarrow$ $\beta\beta_{0\nu}$ Decay

[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]



Majorana Mass Term: $\mathcal{L}^{M}_{eL} = -\frac{1}{2}\, m \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right) = \frac{1}{2}\, m \left(\nu^T_{eL}\, \mathcal{C}^\dagger\, \nu_{eL} + \nu^\dagger_{eL}\, \mathcal{C}\, \nu^*_{eL} \right)$

two conditions:
- u, d, e are massive
- standard left-handed weak interaction exists

cancellations with other diagrams are very unlikely (unstable under perturbations)

The Problem of Calculation of Nuclear Matrix Elements

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2 \qquad m_{\beta\beta} = \sum_k U_{ek}^2\,m_k$$

Theoretically evaluated $\beta\beta(0\nu)$ half-lives (units of 10^{28} years for $\langle m_\nu \rangle = 10$ meV).

Isotope	[10]	[11]	[12]	[13]	[14]	[15]
^{48}Ca	3.18	8.83	-	-	-	2.5
^{76}Ge	1.7	17.7	14.0	2.33	3.2	3.6
^{82}Se	0.58	2.4	5.6	0.6	0.8	1.5
^{100}Mo	-	-	1.0	1.28	0.3	3.9
^{116}Cd	-	-	-	0.48	0.78	4.7
^{130}Te	0.15	5.8	0.7	0.5	0.9	0.85
^{136}Xe	-	12.1	3.3	2.2	5.3	1.8
^{150}Nd	-	-	-	0.025	0.05	-
^{160}Gd	-	-	-	0.85	-	-

10. W.C. Haxton and G.J. Stephenson Jr., Progr. Part. Nucl. Phys. 12(1984) 409.
11. E. Caurier et al., Nucl. Phys. A 654 (1999) 973.
12. J. Engel et al., Phys. Rev. C 37 (1988) 731.
13. A. Staudt et al., Europhys. Lett 13 (1990) 31.
14. A. Faessler and F. Simkovic, J. Phys. G 24 (1998) 2139.
15. G. Pantis et al., Phys. Rev. C 53 (1996) 695.

[Cremonesi, NPB PS 118 (2003) 287]

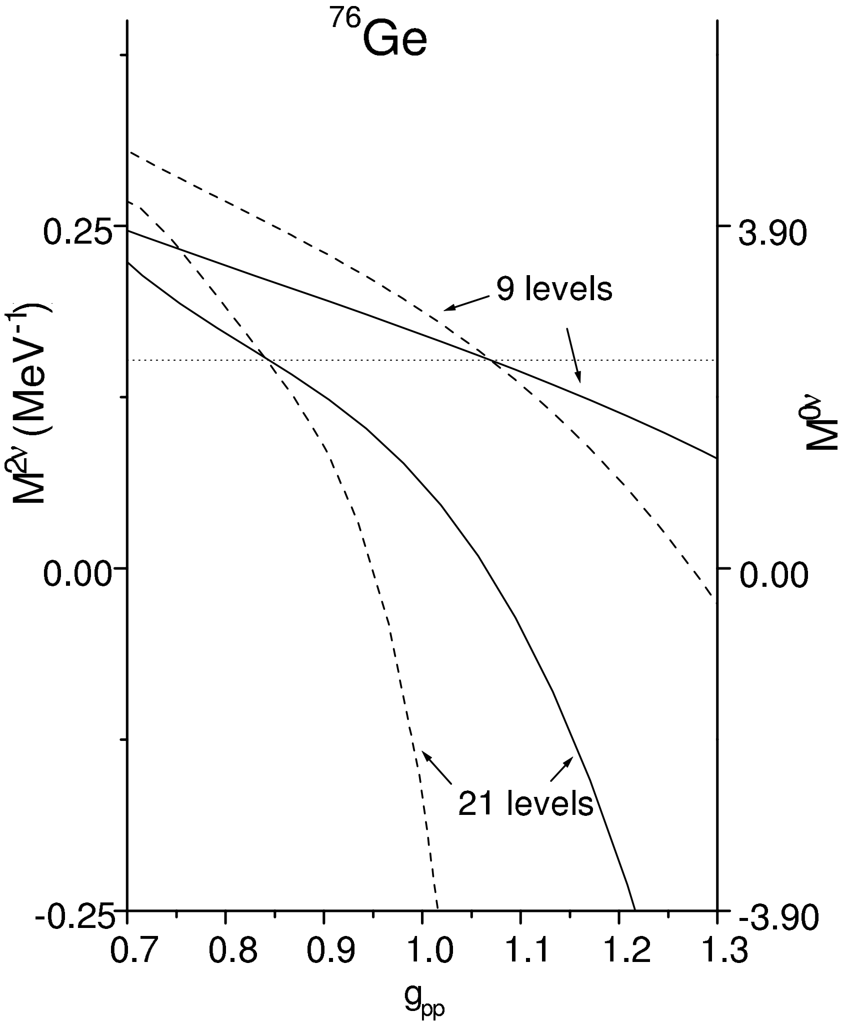
traditional estimated range for $|\mathcal{M}_{0\nu}|^2$: about one order of magnitude

$\Downarrow$

factor of 3 range for $|m_{\beta\beta}|$

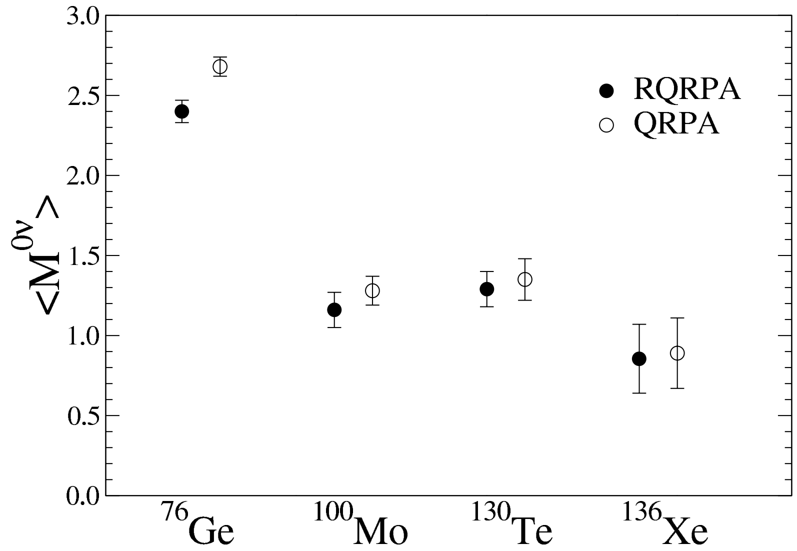
Recent QRPA calculation of $\mathcal{M}_{0\nu}$

[Rodin, Faessler, Simkovic, Vogel, PRC 68 (2003) 044302]



Nucleus	$\lvert\mathcal{M}_{0\nu}\rvert$
^{76}Ge	2.40 ± 0.07 ($\sim 3\%$ unc.)
^{100}Mo	1.16 ± 0.11 ($\sim 9\%$ unc.)
^{130}Te	1.29 ± 0.11 ($\sim 9\%$ unc.)

see also [Bilenky, Faessler, Simkovic, PRD 70 (2004) 033003]



very small uncertainties!

$\beta\beta_{0\nu}$ Decay Phenomenology, 21-23 April 2004, Durham, UK

- ★ Rodin-Faessler-Simkovic-Vogel uncertainties may be too optimistic

  (R)QRPA? – intermediate nuclear levels of $\beta\beta_{0\nu} \neq$ those of $\beta\beta_{2\nu}$

- ★ uncertainty may be estimated excluding unrealistic calculations

FUTURE

- ★ important to improve shell model and QRPA calculations to reach (hopefully) convergence
- ★ important to test and constrain model with all available data ($\beta^{\pm}$ decay of intermediate nucleus, μ capture $\mu^- + \mathcal{N}(A,Z) \to \mathcal{N}(A,Z-1) + \nu_\mu$, etc.)

no method can guarantee rightness of matrix element,
but important to increase confidence

^{76}Ge

C_{mm} [y^{-1}]	$\lvert m_{\beta\beta}\rvert$ [eV]	Method	Reference
1.42×10^{-14}	1.24	QRPA with np pairing	Pantis et al. (1999)
1.90×10^{-14}	1.07	Large-scale shell model	Caurier et al. (1996)
$(1.8 - 2.2) \times 10^{-14}$	1.0 – 1.1	QRPA	Bobyk et al. (2001)
2.75×10^{-14}	0.89	Full RQRPA	Simkovic et al. (1997)
3.63×10^{-14}	0.78	RQRPA with forbidden	Rodin et al. (2003)
$(2.21 - 8.83) \times 10^{-14}$	0.50 – 1.00	RQRPA	Stoica and Klapdor-K. (2001)
4.53×10^{-14}	0.70	QRPA with forbidden	Rodin et al. (2003)
$(1.85 - 12.5) \times 10^{-14}$	0.42 – 1.09	QRPA	Stoica and Klapdor-K. (2001)
$(3.36 - 8.54) \times 10^{-14}$	0.51 – 0.81	Full RQRPA	Stoica and Klapdor-K. (2001)
$(5.5 - 6.3) \times 10^{-14}$	0.59 – 0.63	RQRPA	Bobyk et al. (2001)
6.19×10^{-14}	0.60	RQRPA with forbidden	Simkovic et al. (1999)
6.97×10^{-14}	0.56	QRPA	Suhonen et al. (1992)
7.33×10^{-14}	0.55	QRPA	Pantis et al. (1996)
7.51×10^{-14}	0.54	Number-projected QRPA	Suhonen et al. (1992)
$(6.50 - 9.21) \times 10^{-14}$	0.49 – 0.58	Second QRPA	Stoica and Klapdor-K. (2001)
8.27×10^{-14}	0.52	QRPA	Barbero et al. (1999)
8.29×10^{-14}	0.51	RQRPA	Faessler and Simkovic (1998)
8.36×10^{-14}	0.51	QRPA	Civitarese and Suhonen (2003)
1.03×10^{-13}	0.46	RQRPA	Simkovic et al. (1999)
1.12×10^{-13}	0.44	QRPA	Muto et al. (1989), Staudt et al. (1990)
1.18×10^{-13}	0.43	QRPA	Tomoda (1991)
1.33×10^{-13}	0.41	QRPA	Aunola and Suhonen (1998)

from [Elliott, Engel, J. Phys. G 30 (2004) R183]

$$C_{mm} = \frac{m_e^2}{|m_{\beta\beta}|^2 \, T^{0\nu}_{1/2}} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, m_e^2$$

$$(T^{0\nu}_{1/2})^{-1} = C_{mm} \frac{|m_{\beta\beta}|^2}{m_e^2}$$

$$|m_{\beta\beta}| = \frac{m_e}{\sqrt{C_{mm} \, T^{0\nu}_{1/2}}}$$

unrealistic calculations excluded

$$1.42 \times 10^{-14}\,\mathrm{y}^{-1} \lesssim C_{mm} \lesssim 1.33 \times 10^{-13}\,\mathrm{y}^{-1} \quad \Longrightarrow \quad \log\left(\frac{C_{mm}}{\mathrm{y}^{-1}}\right) = -13.36 \pm 0.49 \quad \text{(EE range)}$$

about one order of magnitude range $\Longrightarrow$ about factor of 3 range for $|m_{\beta\beta}|$

3σ uncertainty = EE range

$$\log\left(\frac{C_{mm}}{\mathrm{y}^{-1}}\right) = -13.36 \pm 0.97 \quad \text{(B } 3\sigma\text{)}$$

[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

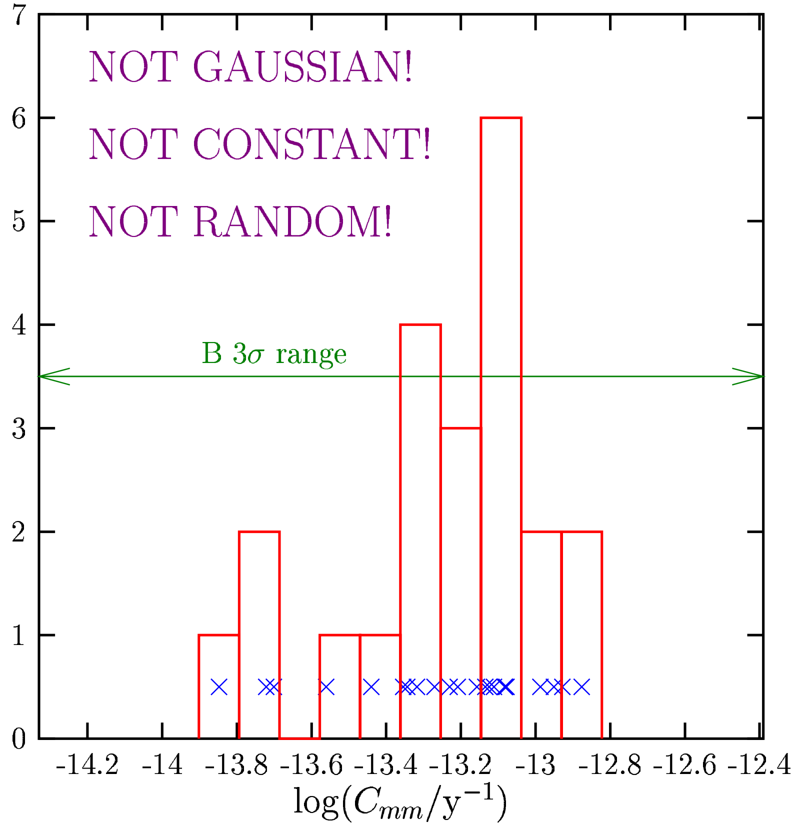
about two orders of magnitude range

$\Downarrow$

one order of magnitude range for $|m_{\beta\beta}|$

($\sim$ three times traditional range)

seems very conservative!



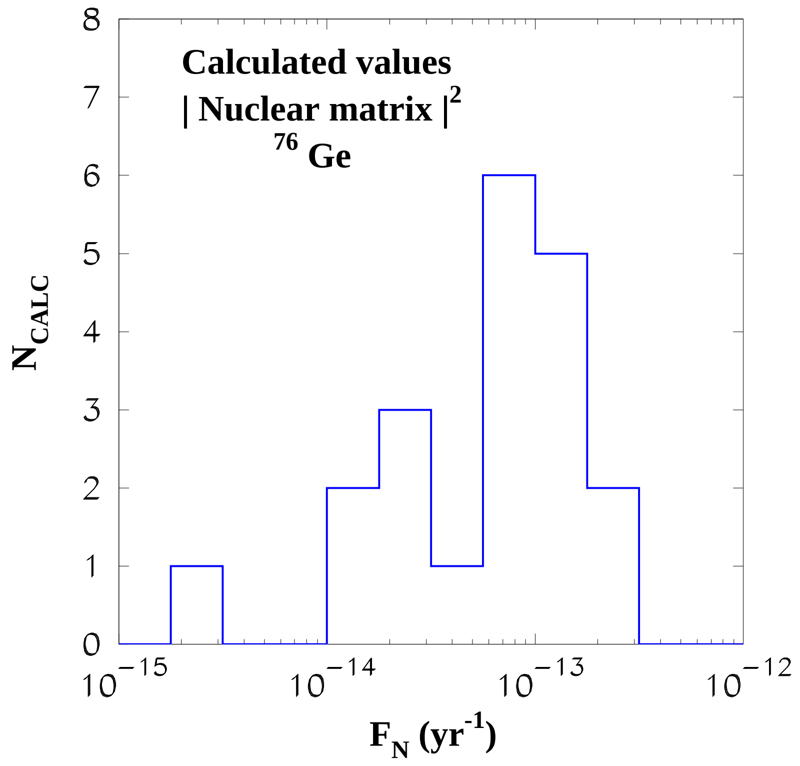
all published calculations

$$\log\left(\frac{C_{mm}}{y^{-1}}\right) = -13.55 \pm 1.02 \quad \begin{cases} (\text{P } 3\sigma \text{ actual}) \\ (\text{P } 3\sigma \text{ gaussian}) \\ (\text{P } 3\sigma \text{ constant}) \end{cases}$$

[Bahcall, Murayama, Peña-Garay, PRD 70 (2004) 033012]

almost the same as the B 3σ range

seems very conservative!



[Bahcall, Murayama, Peña-Garay, PRD 70 (2004) 033012]

$$F_N = C_{mm}$$

in any case, uncertainty quite arbitrary $\Longrightarrow$ results must be taken with caution

Indication of $\beta\beta_{0\nu}$ Decay at Quasi Degenerate Mass Scale

[Klapdor-Kleingrothaus, Dietz, Harney, Krivosheina, Mod. Phys. Lett. A16 (2001) 2409] [Klapdor-Kleingrothaus, Dietz, Krivosheina, Found. Phys. 32 (2002) 1181]

[Klapdor-Kleingrothaus, Dietz, Chkvorez, Krivosheina, NIMA 522 (2004) 371] [Klapdor-Kleingrothaus, Krivosheina, Dietz, Chkvorets, PLB 586 (2004) 198]

$T_{1/2}^{0\nu\,\mathrm{bf}} = 1.19 \times 10^{25}\,\mathrm{y}$ $\quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\mathrm{y} \quad (3\sigma)$ $\quad 4.2\sigma$ evidence

the indication must be checked by other experiments

$C_{mm} = 4.53 \times 10^{-14}\,\mathrm{y}^{-1} \Longrightarrow |m_{\beta\beta}|_{\mathrm{bf}} = 0.70\,\mathrm{eV} \qquad 0.37\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 0.92\,\mathrm{eV} \quad (3\sigma)$

[Rodin, Faessler, Simkovic, Vogel, PRC 68 (2003) 044302]

if confirmed very exciting (Majorana ν and large mass scale)

$$\log\left(C_{mm}/\mathrm{y}^{-1}\right) = -13.36 \pm 0.49 \Longrightarrow 0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV} \quad \text{(EE range)}$$

$$\log\left(C_{mm}/\mathrm{y}^{-1}\right) = -13.55 \pm 1.02 \Longrightarrow 0.14\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 3.7\,\mathrm{eV} \quad (\mathrm{P}\ 3\sigma)$$

$$\log\left(\frac{|m_{\beta\beta}|}{\mathrm{eV}}\right) = \log\left(\frac{m_e}{\mathrm{eV}}\right) - \frac{1}{2}\log\left(\frac{C_{mm}}{\mathrm{y}^{-1}}\right) - \frac{1}{2}\log\left(\frac{T_{1/2}^{0\nu}}{\mathrm{y}^{-1}}\right)$$

allows to add uncertainties in quadrature

$$\log\left(\frac{|m_{\beta\beta}|}{\mathrm{eV}}\right) = -0.23 \pm 0.53 \quad (\mathrm{B}\ 3\sigma) \Longrightarrow 0.17\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 2.0\,\mathrm{eV} \quad (\mathrm{B}\ 3\sigma)$$

[Fogli et al., hep-ph/0408045]

Best limits for $\beta\beta_{0\nu}$ Decay

Heidelberg-Moscow $^{76}\mathrm{Ge}$ [EPJA 12 (2001) 147]

$T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\mathrm{y}$ (90% C.L.) $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\mathrm{eV}$ (90% C.L. $\otimes$ EE range)

IGEX $^{76}\mathrm{Ge}$ [PRD 65 (2002) 092007]

$T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\mathrm{y}$ (90% C.L.) $\Longrightarrow$ $|m_{\beta\beta}| \lesssim 0.35 - 1.1\,\mathrm{eV}$ (90% C.L. $\otimes$ EE range)

FUTURE EXPERIMENTS

NEMO3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES $|m_{\beta\beta}| \sim$ few $10^{-1}\,\mathrm{eV}$

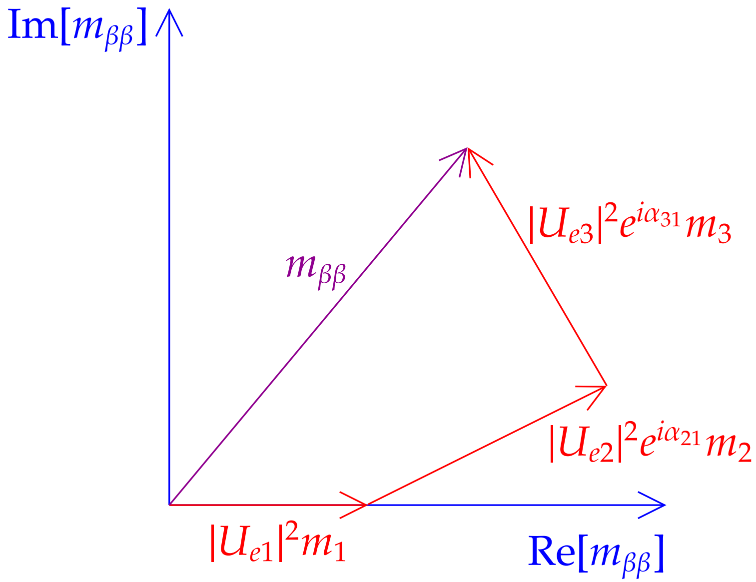
EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA $|m_{\beta\beta}| \sim$ few $10^{-2}\,\mathrm{eV}$

see [Zdesenko, RMP 74 (2002) 663], [Elliott,Vogel, Ann. Rev. Nucl. Part. Sci. 52 (2002) 115], [Elliott, Engel, J. Phys. G 30 (2004) R183]

Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

complex U_{ek} $\Rightarrow$ possible cancellations



$$\begin{aligned} m_{\beta\beta} &= |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{2i\lambda_{21}} \, m_2 + |U_{e3}|^2 \, e^{2i(\lambda_{31}-\delta)} \, m_3 \\ &= |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3 \end{aligned}$$

conserved CP $\Longrightarrow$ $\delta = 0 \quad \lambda_{kj} = \dfrac{\alpha_{kj}}{2} = 0 \, , \, \dfrac{\pi}{2}$ $\Longrightarrow$ $e^{2i\lambda_{kj}} = e^{i\alpha_{kj}} = \pm 1$

$\delta \neq 0$ $\Longrightarrow$ calling “Majorana phases” λ_{kj} or $\alpha_{kj}/2$ is a convention

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ \ldots & \ldots & s_{23}c_{13} \\ \ldots & \ldots & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ \ldots & \ldots & s_{23}c_{13}e^{i\delta} \\ \ldots & \ldots & c_{23}c_{13}e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Normal Hierarchy $m_1 \ll m_2 \ll m_3$

$$|m_{\beta\beta}| = \left| \sum_k U_{ek}^2 \, m_k \right| \simeq \left| U_{e2}^2 \, m_2 + U_{e3}^2 \, m_3 \right| = \left| |U_{e2}|^2 \, m_2 + |U_{e3}|^2 \, e^{i(\alpha_{31} - \alpha_{21})} \, m_3 \right| \leq |U_{e2}|^2 \, m_2 + |U_{e3}|^2 \, m_3$$

$$m_2 \simeq \sqrt{\Delta m^2_{21}} \simeq \sqrt{\Delta m^2_{\text{SUN}}} \qquad m_3 \simeq \sqrt{\Delta m^2_{31}} \simeq \sqrt{\Delta m^2_{\text{ATM}}}$$

$$1.8 \times 10^{-3} \lesssim |U_{e2}|^2 \, m_2 \lesssim 3.6 \times 10^{-3} \qquad |U_{e3}|^2 \, m_3 \lesssim 2.9 \times 10^{-3}$$

ν_2 contribution $|U_{e2}|^2 \, m_2$ may be dominant! (no cancellation $\Longrightarrow$ lower limit for $|m_{\beta\beta}|$)

[Giunti, PRD 61 (2000) 036002]

overlap of allowed ranges for $|U_{e2}|^2 \, m_2$ and $|U_{e3}|^2 \, m_3$ $\Longrightarrow$ strong cancellation is possible

in any case: $|m_{\beta\beta}| \lesssim 6 \times 10^{-3} \, \text{eV}$

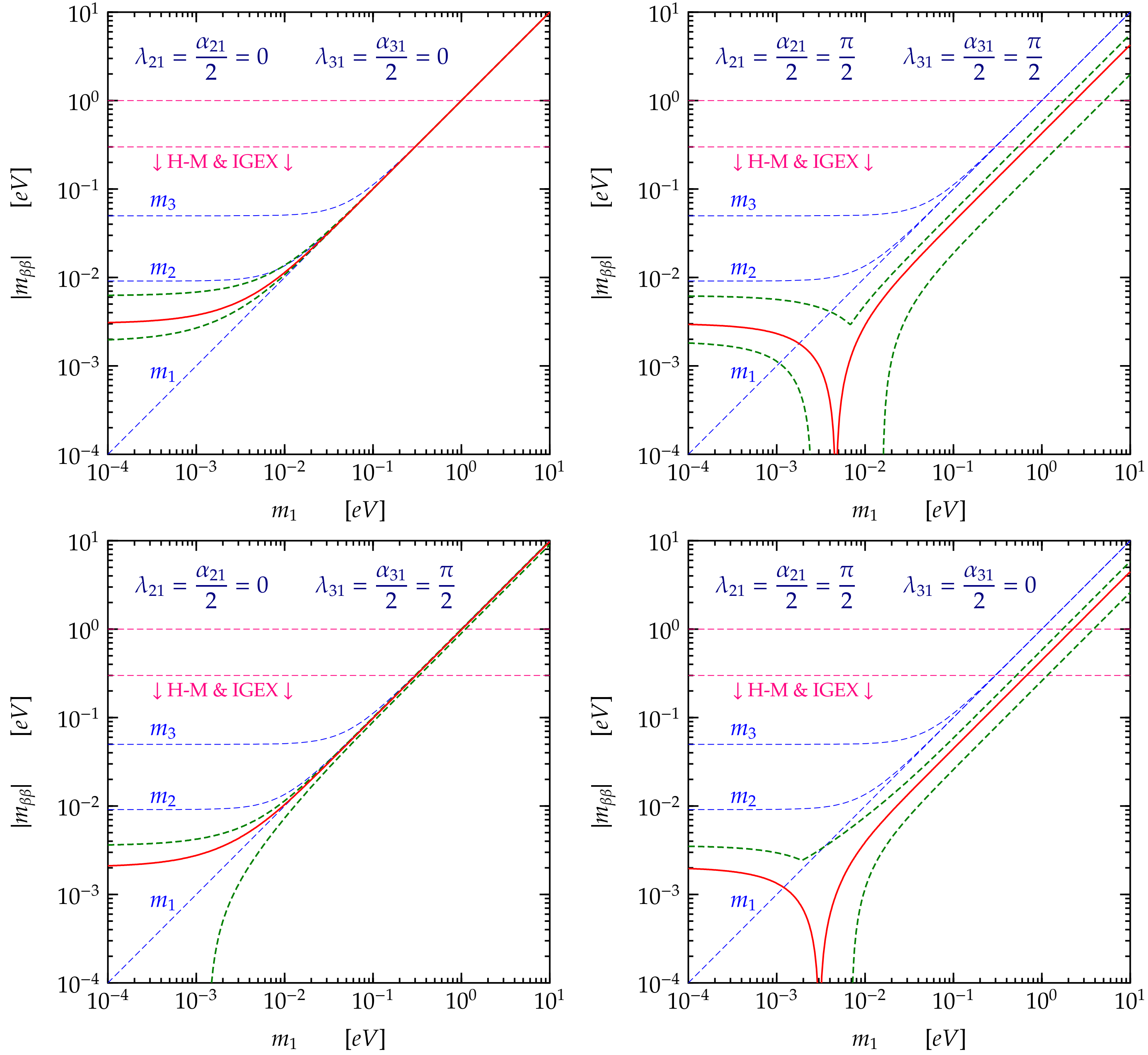
[Vissani, JHEP 06 (1999) 022]
[Bilenky et al, PLB 465 (1999) 193]

the difference $\alpha_{31} - \alpha_{21}$ of the two Majorana phases is potentially measurable

$$|m_{\beta\beta}|^2 \simeq |U_{e2}|^4 \, \Delta m^2_{\text{SUN}} + |U_{e3}|^4 \, \Delta m^2_{\text{ATM}} + 2 \, |U_{e2}|^2 \, |U_{e3}|^2 \, \sqrt{\Delta m^2_{\text{SUN}}} \, \sqrt{\Delta m^2_{\text{ATM}}} \, \cos(\alpha_{31} - \alpha_{21})$$

[Bilenky, Pascoli, Petcov, PRD 64 (2001) 053010]

CP Conservation: Normal Scheme



$$m_{\beta\beta} \simeq |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} \sqrt{m_1^2 + \Delta m^2_{\text{SUN}}} + |U_{e3}|^2 e^{i\alpha_{31}} \sqrt{m_1^2 + \Delta m^2_{\text{ATM}}}$$

$$\Delta m^2_{\text{SUN}} \simeq (7.4 - 9.3) \times 10^{-5} \text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq (1.8 - 3.2) \times 10^{-3} \text{eV}^2$$

$$|U_{e1}|^2 \simeq 0.59 - 0.77$$

$$|U_{e2}|^2 \simeq 0.21 - 0.37$$

$$|U_{e3}|^2 \simeq 0.00 - 0.05$$

Inverted Hierarchy $m_1 \simeq m_2 \gg m_3$

$$
\begin{aligned}
|m_{\beta\beta}| &= \left| \sum_k U_{ek}^2 \, m_k \right| \simeq \left| U_{e1}^2 \, m_1 + U_{e2}^2 \, m_2 \right| \\
&\simeq \left| U_{e1}^2 + U_{e2}^2 \right| \sqrt{\Delta m^2_{\text{ATM}}} \\
&= \left| |U_{e1}|^2 + |U_{e2}|^2 \, e^{i\alpha_{21}} \right| \sqrt{\Delta m^2_{\text{ATM}}}
\end{aligned}
$$

$$0.59 \lesssim |U_{e1}|^2 \lesssim 0.77 \qquad 0.21 \lesssim |U_{e2}|^2 \lesssim 0.37$$

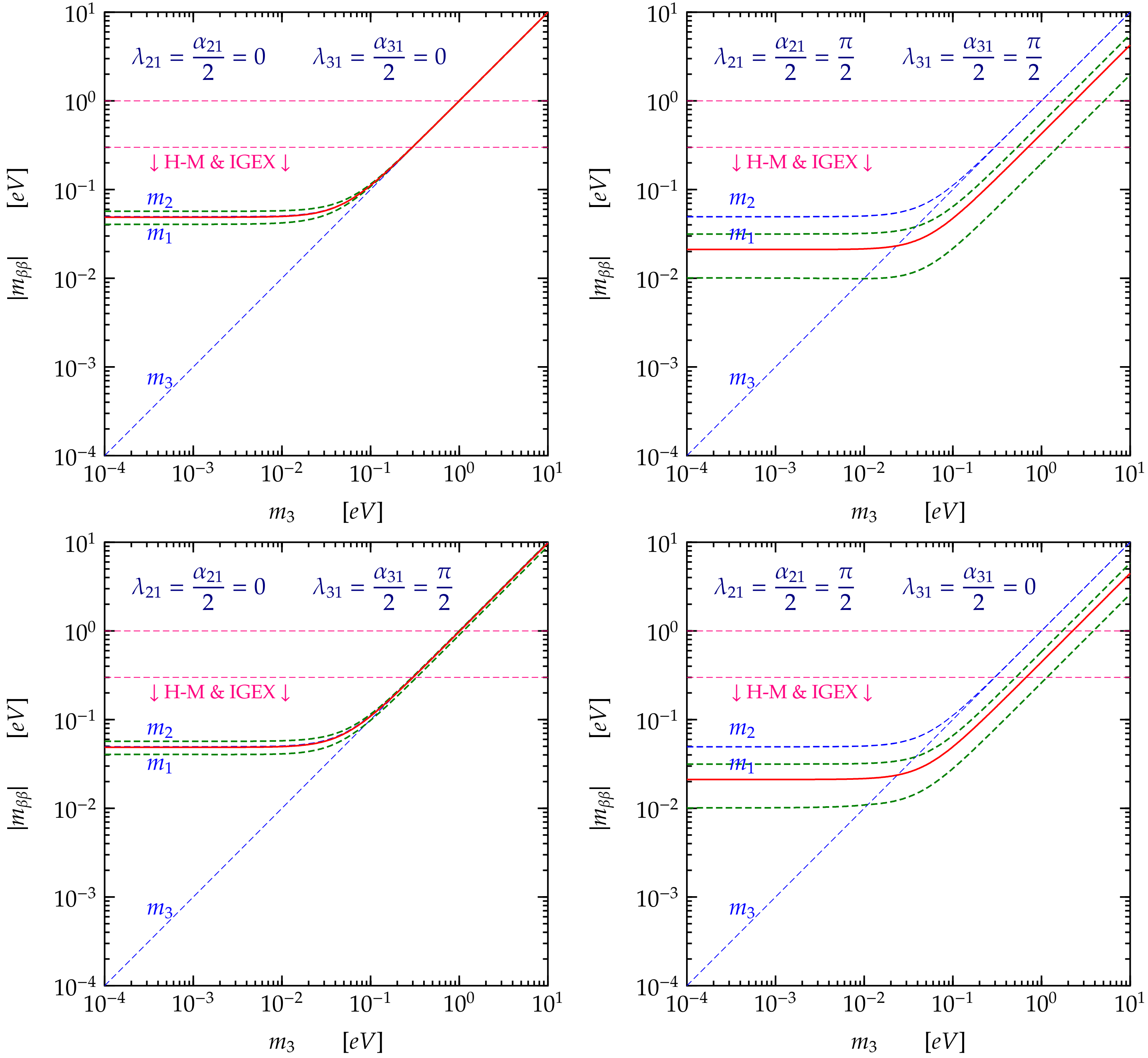
no overlap of allowed ranges for $|U_{e1}|^2$ and $|U_{e2}|^2$ $\Longrightarrow$ complete cancellation is not possible!

$$|m_{\beta\beta}| \sim \sqrt{\Delta m^2_{\text{ATM}}}$$

the Majorana phase α_{21} is potentially measurable [Bilenky et al, PRD 54 (1996) 4432]

$$\frac{|m_{\beta\beta}|^2}{\Delta m^2_{\text{ATM}}} \simeq |U_{e1}|^4 + |U_{e2}|^4 + 2\,|U_{e1}|^2\,|U_{e2}|^2 \; \cos\alpha_{21}$$

CP Conservation: Inverted Scheme



$$m_{\beta\beta} \simeq \left(|U_{e1}|^2 + |U_{e2}|^2 e^{i\alpha_{21}}\right) \times \sqrt{m_3^2 + \Delta m^2_{\text{ATM}}} + |U_{e3}|^2 e^{i\alpha_{31}}\, m_3$$

$$\Delta m^2_{\text{ATM}} \simeq (1.8 - 3.2) \times 10^{-3}\,\text{eV}^2$$

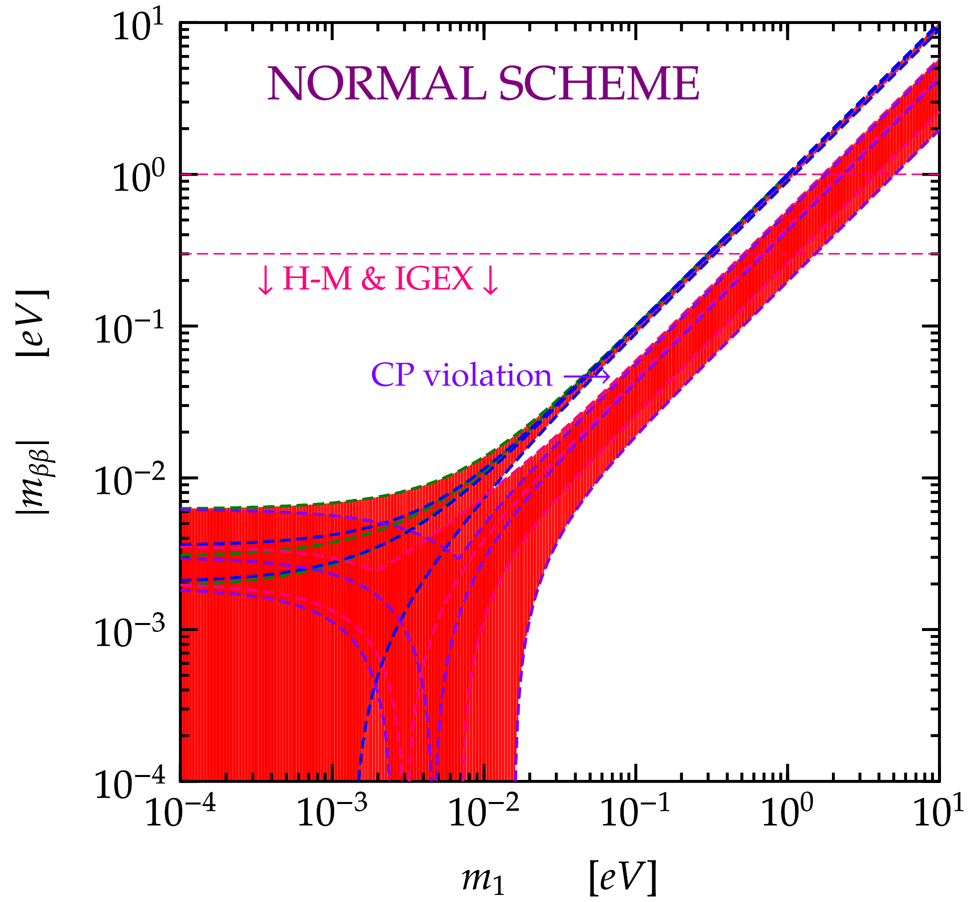
$$|U_{e1}|^2 \simeq 0.59 - 0.77$$

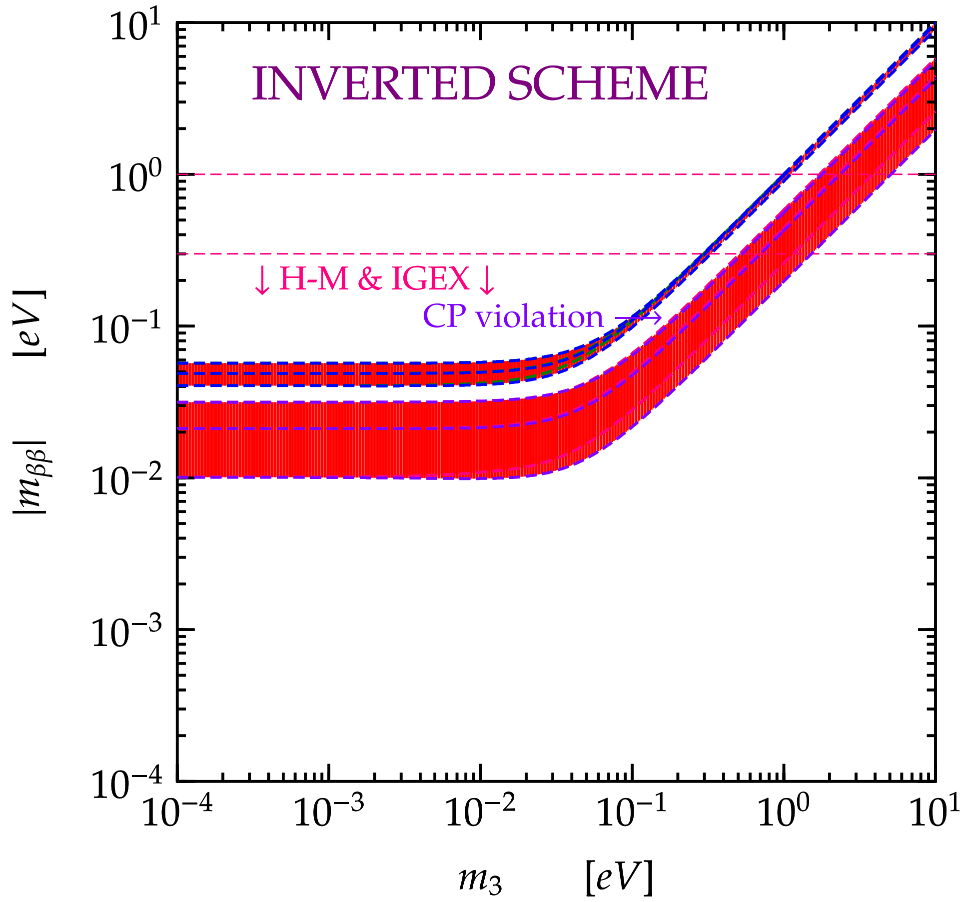
$$|U_{e2}|^2 \simeq 0.21 - 0.37$$

$$|U_{e3}|^2 \simeq 0.00 - 0.05$$

General Neutrino Oscillations Bounds for $\beta\beta_{0\nu}$ Decay

[Vissani, JHEP 06 (1999) 022] [Pascoli et al, PLB 549 (2002) 177] [Czakon et al, PRD65 (2002) 053008] [Elliott, Vogel, ARNPS 52 (2002) 115] [Joaquim, PRD68 (2003) 033019] [Giunti, Laveder, hep-ph/0310238] [Feruglio et al, NPB659 (2003) 359] [Pascoli, Petcov, PLB 580 (2004) 280] [Bilenky et al, PRD70 (2004) 033003] [Bahcall et al, PRD 70 (2004) 033012] [Petcov, NJP 6 (2004) 109]

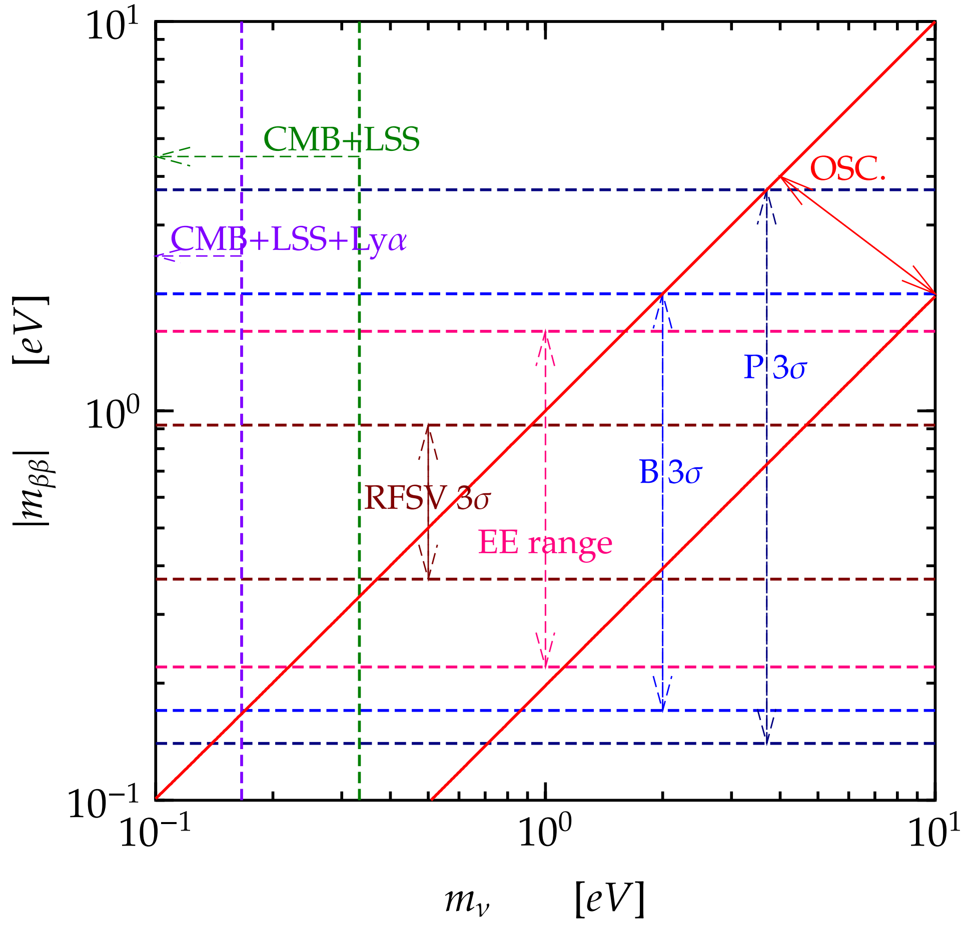


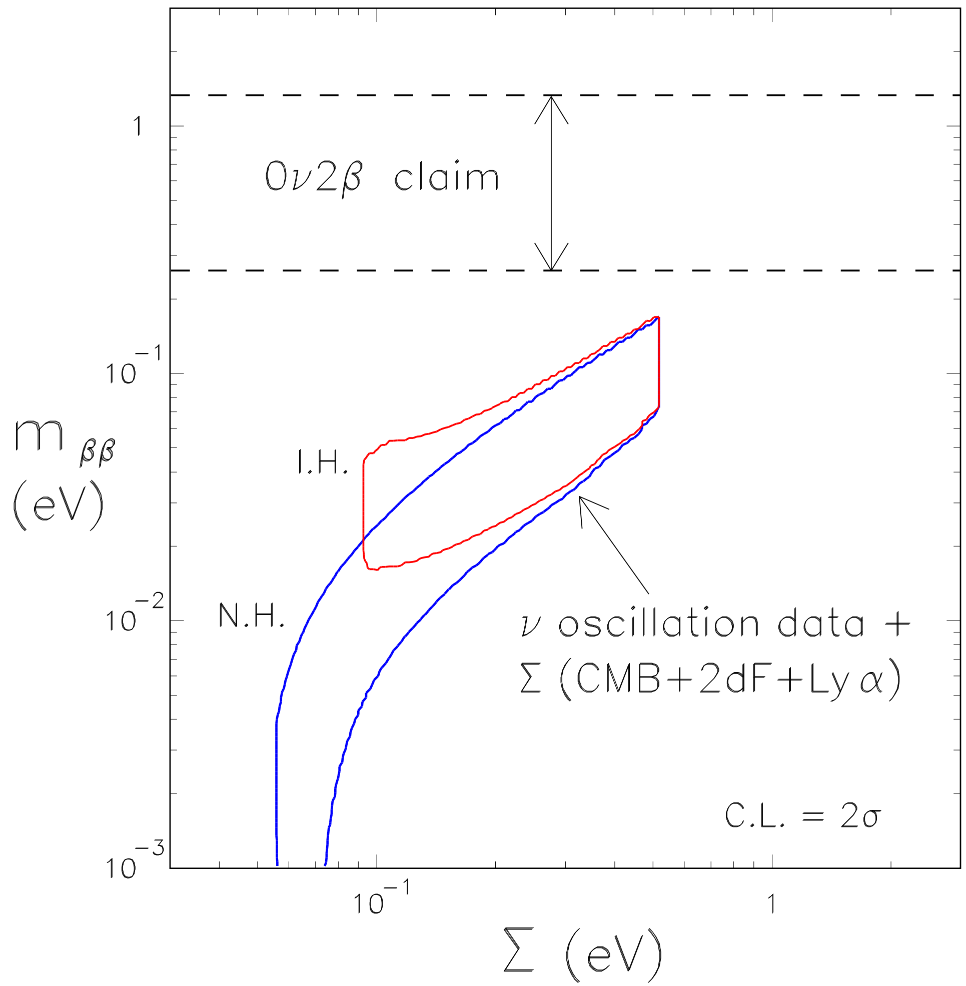


Quasi Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies |m_{\beta\beta}| \simeq \left| |U_{e1}|^2 + |U_{e2}|^2 e^{i\alpha_{21}} + |U_{e3}|^2 e^{i\alpha_{31}} \right| m_\nu$

FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

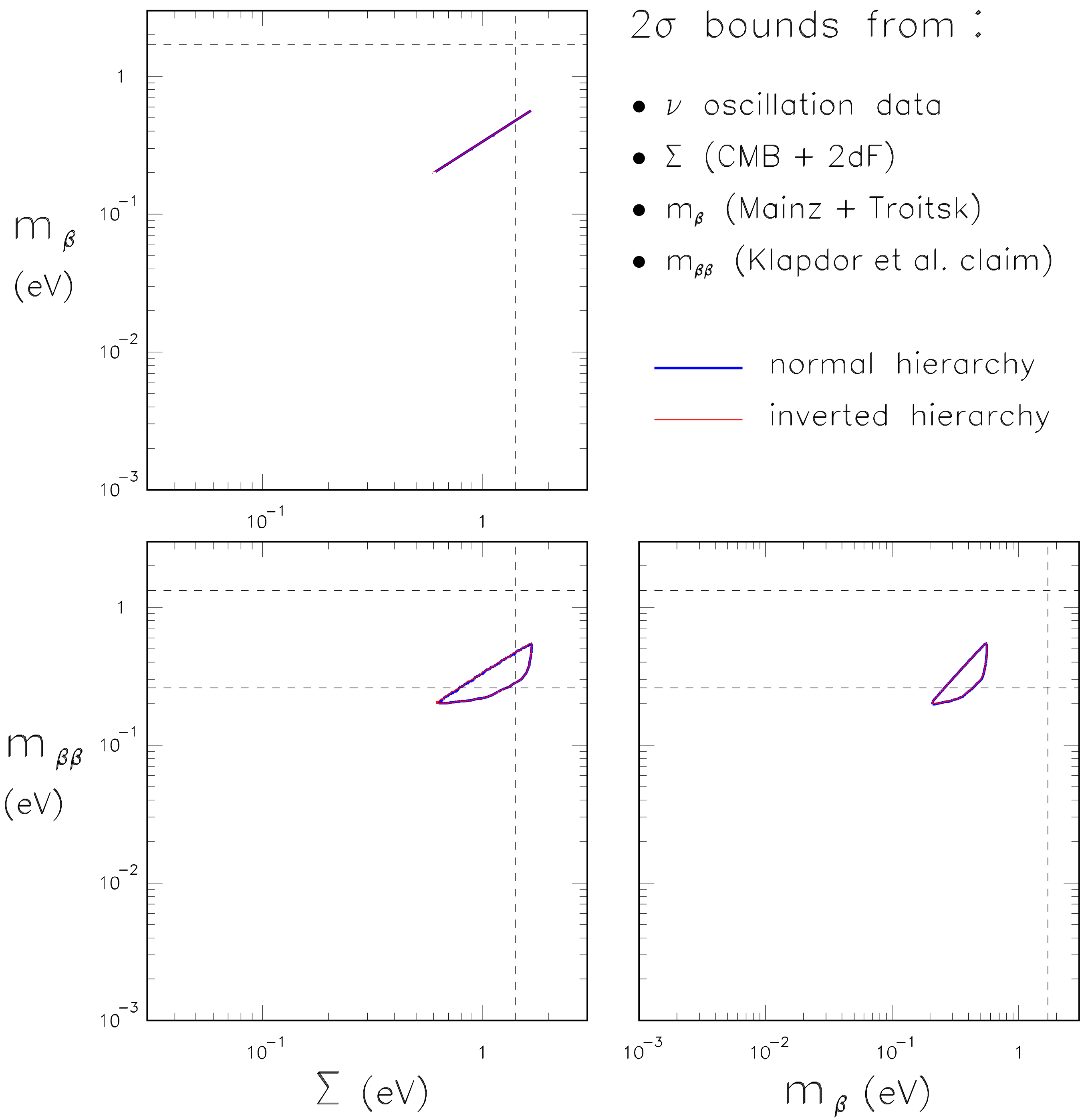
Indication of $\beta\beta_{0\nu}$ Decay at Quasi Degenerate Mass Scale





[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

tension among oscillation data, CMB+LSS+Lyα and $\beta\beta_{0\nu}$ signal



[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, hep-ph/0408045]

Conclusions

- Important open fundamental questions: nature of neutrinos (Dirac or Majorana), absolute scale of neutrino masses, pattern of neutrino masses (Normal Hierarchy, Inverted Hierarchy, Quasi Degenerate)?
- Most reliable determination of absolute scale of neutrino masses: β decay (kinematical measurement). Present upper bound $\sim 2\mathrm{eV}$. Future: KATRIN sensitive to $\sim 0.2 - 0.3\mathrm{eV}$.
- Cosmology: very powerful for $\sum_k m_k$, but plagued with assumptions and systematic uncertainties. Very good for indication, but must be checked by direct measurement.
- Best method to determine Majorana nature of neutrinos: $\beta\beta_{0\nu}$ decay. Present sensitivity $\sim 0.5 - 1\mathrm{eV}$. Many experiments plan to reach $\sim 10^{-1} - 10^{-2}\mathrm{eV}$. Very important to improve uncertainty of nuclear matrix element calculation!
- Very exciting indication of $\beta\beta_{0\nu}$ decay at Quasi Degenerate mass scale ($\sim 0.5 - 1\mathrm{eV}$). If confirmed by other experiments it would imply that detailed study of neutrino masses, CP violation and Majorana phases may be possible.

- Combination of different experimental results is a powerful tool to constrain ν properties.

- ν oscillation data $\Longrightarrow$ Bilarge Three-Neutrino Mixing $\Longrightarrow$ constraints on m_β, $\sum_k m_k$, $m_{\beta\beta}$, with unknown absolute mass scale.

- Present β decay, cosmological, $\beta\beta_{0\nu}$ decay data are sensitive to the Quasi Degenerate mass scale $\Longrightarrow$ upper bounds for the absolute scale of neutrino masses.

- Tension among oscillation data, CMB+LSS+Lyα and $\beta\beta_{0\nu}$ signal at Quasi Degenerate mass scale.

- If $\beta\beta_{0\nu}$ signal at Quasi Degenerate mass scale is not confi rmed by other experiments, it is important to reach sensitivity to $m_\beta \lesssim 4 \times 10^{-2}\,\mathrm{eV}$, $\sum_k m_k \lesssim 8 \times 10^{-2}\,\mathrm{eV}$, $|m_{\beta\beta}| \lesssim 10^{-2}\,\mathrm{eV}$ $\Longrightarrow$ maybe possible to discriminate between Normal and Inverted Hierarchy.