Leptogenesis and Neutrino Masses

Enrico Nardi

INFN – Laboratori Nazionali di Frascati, Italy

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Baryogenesis: explaining one single experimental number

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\[ Y_{\Delta B} \equiv \frac{n_B - \bar{n}_B}{s} = (8.75 \pm 0.23) \times 10^{-11} \]

[WMAP, BAO, SN-IA]

\[ 4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \]

\[ 0.017 \times \leq \Omega_B h^2 \leq 0.024 \]

[BBN: Light Elements Abundances]
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Particle physics models for baryogenesis relate \( Y_{\Delta B} \) to other observables.

**Leptogenesis**: is a class of scenarios where the Universe baryon asymmetry \( Y_{\Delta B} \) is produced from a lepton asymmetry \( Y_{\Delta L} \) generated in the decays of the heavy \( SU(2) \) singlet seesaw Majorana neutrinos.

**Baryon Asymmetry \( \Leftrightarrow \) Neutrino Physics**
Minimal extension of the SM: add $n = 3$ singlet neutrinos

$$-\mathcal{L} = \frac{1}{2} M_{N_i} \bar{N}^c_i N^c_i + \lambda_{i\alpha} \bar{N}_i \ell_\alpha \tilde{H}^\dagger + h_\alpha \bar{e}_\alpha \ell_\alpha H^\dagger + h.c.$$  

Basis: $M_N = \text{diag}(M_1, M_2, M_3)$; diagonal charged lepton Yukawas $h_\alpha$

This explains nicely the suppression of $\nu$ masses:

$$\mathcal{M}_\nu = -\lambda T \frac{\langle H \rangle^2}{M_N} \lambda$$
The seesaw model has 18 independent parameters (3 $M_i$ plus 3 + 3 from complex angles in $R$; 3 $m_{\nu_i}$ plus 3 angles and 3 phases in $U$). 3+6 parameters can be measured (in principle) at low energy, 3+6 are confined to high energy.
1. $\mathcal{L}$: Majorana masses $M_N$ imply lepton number violation ($\Delta L \neq 0$)

$\mathcal{B}$: EW-Sphalerons are SM processes that at $T \gg M_W$ violate $B + L$ (conserving $\Delta_\alpha = B/3 - L_\alpha$) and convert part of $\Delta L$ into $\Delta B$. 
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2. **$\mathbb{CP}$**: The complex Yukawa couplings $\lambda_{i\alpha}$ induce CP violation in the interference between tree level and loop decay amplitudes.
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   interference between tree level and loop decay amplitudes.

3. Deviations from thermal equilibrium: If $\tau_N \sim t_U(T \sim M_N)$ the $N$'s decay
   out-of-equilibrium. And since $t_U \sim H^{-1}$ the condition is: $\Gamma_N \sim H\bigg|_{T \sim M_N}$. 

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Whether leptogenesis can explain the baryon asymmetry of the Universe, is basically a quantitative question.
No asymmetry can be generated in thermal equilibrium


Consider the one-family SM:

\[ Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u, \quad d, \quad \ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e, \quad H, \quad N \]

We can have \( 6 \) chemical potentials:

\[ Q \equiv \mu_Q = \mu_{u_L} = \mu_{d_L}; \quad u \equiv \mu_{u_R}; \quad \ldots \]

since for Majorana neutrinos the chempot vanishes:

\[ M_N \neq 0 \Rightarrow \mu_N = 0 \]

Yukawa reactions can give \( 3 \) chemical equilibrium conditions:

\[ Q + H = u \quad Q - H = d \quad \ell - H = e \]

Plus \( 1 \) from sphaleron chemical equilibrium (effective operator \( O_{EW} = QQQ\ell \))

\[ (B + L)_{SU(2)} = 0 \quad \Rightarrow \quad 3Q + \ell = 0 \]

Plus \( 1 \) constraint from hypercharge conservation (global neutrality):

\[ \mathcal{Y}_{\text{tot}} = \sum \phi \Delta n_{\phi} y_{\phi} = \text{const} \quad \Rightarrow \quad \sum_f g_{\phi} \mu_{\phi} y_{\phi} = 0 \]
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Adding 1 Yukawa chemical equilibrium:
\[ \ell + H = 0 \Rightarrow Q, u, d, \ell, e, H = 0 ! \]
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\]

Chemical equilibrium \( \Leftrightarrow \) conservation law:

\[
\begin{align*}
h_e &\to 0 \quad \Leftrightarrow \quad \Delta n_e = 0 \\
\Gamma_{sphal} &\to 0 \quad \Leftrightarrow \quad \Delta B = 0
\end{align*}
\]

At each temperature, one chempot (\( \ell \)) is sufficient to describe the asymmetries.
Leptogenesis can only proceed at temperatures $T \gg 10^8 \text{ GeV}$ where:

$\Gamma_{m\tilde{g}} \sim \frac{m_{\tilde{g}}^2}{T} \ll H \Rightarrow m_{\tilde{g}} \rightarrow 0 \Rightarrow \tilde{g} \neq 0$,

$\Gamma_{\mu} \sim \frac{\mu^2}{T} \ll H \Rightarrow \mu_{H_uH_d} \rightarrow 0 \Rightarrow H_u + H_d \neq 0$. 

$U(1)_R$ 

$U(1)_{PQ}$
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Both these new symmetries have mixed $SU(2)$ and $SU(3)$ anomalies:

$$\mathcal{O}_{\text{EW}} \Rightarrow \tilde{\mathcal{O}}_{\text{EW}} = \Pi_{\alpha} (QQQ\ell_{\alpha}) \tilde{H}_u \tilde{H}_d \tilde{W}^4$$

$$\mathcal{O}_{\text{QCD}} \Rightarrow \tilde{\mathcal{O}}_{\text{QCD}} = \Pi_i (QQu^c d^c)_i \tilde{g}^6$$

[Open parenthesis: Supersymmetric Leptogenesis]

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$$O_{EW} \Rightarrow \tilde{O}_{EW} = \Pi_\alpha (QQQ\ell_\alpha) \tilde{H}_u \tilde{H}_d \tilde{W}^4 \quad A(R_3) = A(R - 3PQ) = 0$$

$$O_{QCD} \Rightarrow \tilde{O}_{QCD} = \Pi_i (QQu^c d^c)_i \tilde{g}^6 \quad A(R_2) = A(R - 2PQ) = 0$$

We end up with a leptogenesis picture quite different from the usual one:

- Particle sparticle non-superequilibration: $\mu_{\tilde{\psi}} = \mu_{\psi} \pm \tilde{g}$
- A new global charge neutrality condition $(R = \frac{5}{3}B - L + R_2) \quad \Delta R = 0$
- The sneutrino density asymmetry
  
  The sneutrino density asymmetry $\Delta_{\tilde{N}} = n_{\tilde{N}} - n_{\tilde{N}^*}$

  joins the leptonic asymmetries $\Delta_\alpha = \frac{B}{3} - L_\alpha$ as a new independent quantity

  [... admittedly, with no striking numerical consequences ...]
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\[ \Gamma_N = \frac{M}{16\pi} (\lambda \lambda^\dagger)_{11} \]

by rescaling

\[ \tilde{m} \equiv 16\pi \frac{v^2}{M^2} \times \Gamma_N = \frac{v^2}{M} (\lambda \lambda^\dagger)_{11} \]

\[ H = \sqrt{\frac{8\pi G_N \rho}{3}} \approx 1.7 \sqrt{g_* \frac{T^2}{M_P}} \]

\[ m_* \equiv 16\pi \frac{v^2}{M^2} \times H(M) \approx 10^{-3} \text{eV} \]

\[ \tilde{m}(\geq m_1) \approx \sqrt{\Delta m^2_{\odot}}, \sqrt{\Delta m^2_{\oplus}} \]

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Coming back to neutrino masses …

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Experimental confirmation of $m_\nu \neq 0$
in the correct mass range for LeptoG:
\[ \implies \text{burst of lepto-papers around Y2K.} \]
Do we have a limit on $m_\nu$ from LeptoG? The DI bound:

[S. Davidson & A. Ibarra, PLB 535 (2002)]

[W. Buchmüller, P. Di Bari & M. Plümacher; S. Blanchet & P. Di Bari;]
[T. Hambye, Y. Lin, A. Notari, M. Papucci & A. Strumia; ...]

Computation of $\epsilon_\alpha = \frac{\Gamma_\ell_\alpha - \Gamma_{\ell\alpha}}{\Gamma_N} \ (\text{vertex} + \text{self-energy})$ yields:

$$
\epsilon_\alpha = \frac{-1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{j \neq 1} \text{Im} \left\{ \lambda_{j\alpha} \lambda_{1\alpha}^* \left[ \frac{3M_1}{2M_j} (\lambda\lambda^\dagger)_{j1} + \frac{M_2^2}{M_j} (\lambda\lambda^\dagger)_{1j} + \frac{5M_3^3}{6M_j^3} (\lambda\lambda^\dagger)_{j1} + \ldots \right] \right\}
$$

$L$: $D_5 = (\ell\phi)^2$
$L$: $D_6 = (\ell\phi^*) \phi (\ell\phi)$
$L$: $D_7 = (\ell\phi) \partial^2 (\ell\phi)$

$D_5 \Rightarrow$ neutrino mass operator;  $D_6 \Rightarrow$ non unitarity in lepton mixing;  $D_7 \Rightarrow$ spoils the DI bound.
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\[ \text{DI: } |\epsilon^{(D_5)}| = \left| \sum_\alpha \epsilon^{(D_5)}_\alpha \right| \leq \frac{3}{16\pi} \frac{M_1}{v^2} (m_3 - m_1) \xrightarrow{m_3 \approx m_1} |\epsilon^{(D_5)}| \leq \frac{3}{16\pi} \frac{\Delta m^2_{\odot}}{2v^2} \frac{M_1}{m_3} \]
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- Holds only for large hierarchies $M_1 \gg M_{2,3}$. ($D_7$ can dominate when $m_3 - m_1 \approx 0$).
- Applies only in the unflavored regime $T \gtrsim 10^{12}$ GeV. (No DI for flavored $\epsilon_\alpha$.)
- Applies only if leptogenesis is $N_1$ dominated. (No DI for the heavier sneutrinos $\epsilon_{2,3}$.)

Still, if $m_\nu^{\text{obs}} > m_\nu^{\text{max}}$ (cosmology?) one of the above conditions is not realized.
What is the Limit? – (CP asymmetry and collision diagrams)

L.A. Muñoz, EN & J. Noreña, unpublished

Decay

$\Delta L = 2$

$\Delta L = 1_{\text{top}}$

$\Delta L = 1_{\text{gauge}}$

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Network of (unflavored) Boltzmann equations

1. \( \dot{Y}_N = - \left( \frac{Y_N}{Y^eq_N} - 1 \right) \left( \gamma_D + 2\gamma_{Ss} + 4\gamma_{St} \right), \)

2. \( \dot{Y}_{\Delta L} = \left( \frac{Y_N}{Y^eq_N} - 1 \right) \varepsilon_1 \gamma_D - \left[ 2y_\ell + (y_t - yQ_3) \left( \frac{Y_N}{Y^eq_N} + 1 \right) \right] \gamma_{St} - \left( \frac{Y_N}{Y^eq_N} y_\ell + y_t - yQ_3 \right) \gamma_{Ss} - 2(y_\ell + y_H)(\gamma_{Ns} + \gamma_{Nt}) + \dot{Y}_{EW} \)

3. \( \dot{Y}_{\Delta B} = \dot{Y}_{EW} \quad \text{But sphalerons conserve } B - L : \quad \dot{Y}_{\Delta B} - \dot{Y}_{EW} = 0 \)

Eliminate the sources \( Y_{EW} \) subtracting 2. from 3. and express all asymmetries in terms of \( B - L \):

\[ y_\ell \equiv -c_\ell \frac{Y_{B-L}}{Y^eq} ; \quad y_H \equiv -c_H \frac{Y_{B-L}}{Y^eq} \quad \text{using also: } \quad y_t - yQ_3 = \frac{yH}{2} \]

\[ \dot{Y}_{B-L} = - \left( \frac{Y_N}{Y^eq_N} - 1 \right) \left[ \varepsilon_1 \gamma_D + \left( c_\ell \gamma_{Ss} + \frac{c_H}{2} \gamma_{St} \right) \frac{Y_{B-L}}{Y^eq} \right] - \left[ (2c_\ell + c_H) \left( \gamma_{St} + \frac{1}{2} \gamma_{Ss} \right) + 2(c_\ell + c_H) \left( \gamma_{Ns} + \gamma_{Nt} \right) \right] \frac{Y_{B-L}}{Y^eq} \]
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\[ \left[ \left( 2c_\ell + c_H \right) \left( \gamma_{St} + \frac{1}{2} \gamma_{Ss} \right) + 2 \left( c_\ell + c_H \right) \left( \gamma_{N_s} + \gamma_{N_t} \right) \right] \frac{Y_{B-L}}{Y_{eq}} \]
The leptogenesis limit on $m_{\nu_3}$. (Relevance of Higgs effects)

- Vertical axis: the lightest heavy neutrino mass $M_1$ (GeV);
- Horizontal axis: the “washout parameter” $\tilde{m}_1 = v^2 \frac{\langle \lambda \lambda^\dagger \rangle_{11}}{M_1}$ (GeV).

$M_1-\tilde{m}_1$ values yielding successful leptogenesis, for different values of $m_{\nu_3}$ (3-σ)

- **Right picture:** Effects of the Higgs asymmetry neglected ($c_H = 0$).
  Small, medium, large points: $m_{\nu_3} = 0.161, 0.162, 0.163$ eV.

- **Left picture:** Effects of the Higgs asymmetry included ($c_H = -1/3$).
  Small, medium, large points: $m_{\nu_3} = 0.130, 0.131, 0.132$ eV.

\[ m_{\nu_3}^{\text{max}} = 0.13 \text{ eV} \quad \tilde{m}_1^{\text{max}} = 0.28 \text{ eV} \]
Recap: Mass limits in Basic Leptogenesis (Seesaw type I):

- **The One Flavor Regime** ($T \gtrsim 10^{12}$ GeV): Constraints
  - If $N$’s are strongly hierarchical, the DI limit on the maximum CP asymmetry for $N_1$ holds, and $m_{\nu}^{\text{max}} = 0.13$ eV.
  - If light $N$’s are only mildly hierarchical or degenerate, there is NO BOUND on $m_{\nu}$ from the requirement of successful leptogenesis!

- **Leptogenesis with flavors**:
  - Additional sources of CP violation: it can easily be $\epsilon_\alpha > \epsilon$.
  - We can have successful leptogenesis also for degenerate light neutrinos and for a wider range for the washout parameter $\tilde{m}_1$.
  - There is NO BOUND on absolute scale of light neutrinos.

- **Leptogenesis with heavy flavors** $N_2$ and $N_3$ can be successful with:
  - $N_1$ in the decoupled regime $\epsilon_1 \approx 0$, $\tilde{m}_1 \ll m_*$. $\epsilon_{2,3}$ dominate.
  - $N_1$ in a strongly coupled regime, if $\ell_{2,3}$ are strongly misaligned with $\ell_1$.
  - In both cases there is NO BOUND on absolute scale of light neutrinos.
Beyond SM + type 1 seesaw, and beyond the seesaw

• SUSY Leptogenesis
  ✷ The SUSY seesaw model gives a qualitatively different (but quantitatively similar) realization of leptogenesis.
  ✷ Alternative mechanisms: Soft Leptogenesis can be successful at much lower scale, because has new sources of CP.
  ✷ Alternative mechanisms: Affleck-Dine

• Different types of Seesaw:
  ✷ Type I seesaw (standard: $SU(2)_L$ singlets Majorana neutrinos)
  ✷ Type II seesaw ($SU(2)_L$ scalar triplet)
  ✷ Type III seesaw ($SU(2)_L$ fermion triplet)

• Dirac Leptogenesis
  ✷ Leptogenesis without lepton number violation
Leptogenesis: proving vs. disproving.

Direct tests: Produce $N$’s and measure the $CP$ asymmetry in their decays

$$m_\nu \sim \frac{\lambda^2 v^2}{M_N} \sim \left( \frac{\lambda}{10^{-6}} \right)^2 \left( \frac{1 \text{ TeV}}{M_N} \right) \sqrt{\Delta m_{atm}^2}$$

Not possible!
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A direct proof: At $T \gtrsim \Lambda_{EW}$ sphalerons relate $B$ and $L$: $\Delta L \approx -2 \times \Delta B$

Baryogenesis: $\Delta B \Rightarrow \Delta L$ thus necessarily $\Delta L_e = \Delta L_\mu = \Delta L_\tau$

Leptogenesis. $\Delta L \Rightarrow \Delta B$: almost unavoidably $\Delta L_e \neq \Delta L_\mu \neq \Delta L_\tau$ ($T \gg m_\nu$)

However, for non-relativistic Majorana neutrinos the $\Delta L$ information is lost, and since today $T_\nu \sim 10^{-4} \text{ eV} \ll \Delta m_{\text{atm, sol}}^2$...

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Not possible!

Indirect tests: Reconstruct the complete seesaw model

18 parameters vs. 9 observables: $3m_\nu + 3\theta_{ij} + \delta, \alpha_1, \alpha_2$ Not possible!
Can theory help?  

*yes… if nature is kind to us*

- Neutrinos: The hierarchy is milder than for charged fermions  
  (the spectrum could be quasi-degenerate)
- Two mixing angles are large and one maybe maximal.
- Are these hints for a non-Abelian flavor symmetry in the $\nu$ sector?
Can theory help? yes... if nature is kind to us

- Neutrinos: The hierarchy is milder than for charged fermions (the spectrum could be quasi-degenerate)
- Two mixing angles are large and one maybe maximal.
- Are these hints for a non-Abelian flavor symmetry in the $\nu$ sector?

Non-Abelian flavor symmetry

⇓

Large reduction in the number of (seesaw) parameters

⇓

New connections between LE observables and HE quantities

⇓

New information on crucial HE leptogenesis parameters

[See S. Morisi talk (Monday, Branch V)]

Recent works: Jenkins & Manohar; E. Bertuzzo, P. Di Bari, F. Feruglio, EN; Hagedorn, Molinaro & Petcov; D. Aristizabal Sierra, F. Bazzocchi, I. de Medeiros Varzielas, L. Merlo, S. Morisi,; Gonzalez Felipe & Serodio.
by proving that (some of) the Sakharov conditions are (likely to be) satisfied:

1. **$L$ violation**: Is provided by the Majorana nature of the $N$’s: $\ell_\alpha \phi \leftrightarrow N \leftrightarrow \bar{\ell}_\beta \bar{\phi}$

Experimentally: we hope to see $0\nu 2\beta$ decays (requires IH or quasi degenerate $\nu$’s)

[Iachiello & Giuliani talks, and afternoon’s Branch I]

If $m_\nu$ is measured @ 0.2 eV (Cosmology? - Cooray, Melchiorri) and $0\nu 2\beta$ is not seen?

Leptogenesis would be strongly disfavored (or ruled out)
About future experiments? 

We can hope for circumstantial evidences... 

by proving that (some of) the Sakharov conditions are (likely to be) satisfied:

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2. $C$ & $CP$ violation: Experimentally, we hope to see $\mathbf{CP}_L$ (Dirac phase only)

If $\mathbf{CP}_L$ is observed: Circumstantial evidence for LG (not a final proof)

If $\mathbf{CP}_L$ is not observed: LG is not disproved: Small $\delta$ phase, small $\theta_{13}$, etc...
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[Iachiello & Giuliani talks, and afternoon’s Branch I]

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2. **$C$ & $CP$ violation**: Experimentally, we hope to see $CP_L$ (Dirac phase only)

If $CP_L$ is observed: Circumstantial evidence for LG (not a final proof)
If $CP_L$ is not observed: LG is not disproved: Small $\delta$ phase, small $\theta_{13}$, etc…

3. **Out of equilibrium dynamics in the early Universe**: (apparently the most difficult)

We have seen that can be satisfied for $\tilde{m}_1 \sim 10^{-3} \div 10^{-1}$ eV (optimal values)
This could well be the first circumstantial evidence!
Conclusions and Outlook

- Leptogenesis is a very attractive scenario to explain $Y_{\Delta B}$.
- Recent developments have shown that quantitative and qualitative estimates of $Y_{\Delta B}$ have to take into account lepton flavors and the heavier Majorana neutrinos.
- Implications for neutrino masses ($m_{\nu} \lesssim 0.13$ eV) established in the one-flavor regime and for hierarchical $N$’s do not hold in general.
Conclusions and Outlook

- Leptogenesis is a very attractive scenario to explain $Y_{\Delta B}$.
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- Implications for neutrino masses ($m_\nu \lesssim 0.13 \text{ eV}$) established in the one-flavor regime and for hierarchical $N$’s do not hold in general.
- Experimental detection of $0\nu 2\beta$ decays and/or $\mathcal{CP}_L$ in the lepton sector will strengthen the case for leptogenesis – but still not prove it.
- Failure of revealing $\mathcal{CP}_L$ will not disprove LG.
- If $m_\nu \gtrsim 0.1 \text{ eV}$ is established, failure of revealing $0\nu 2\beta$-decays will seriously endanger the Majorana $\nu$ hypothesis and strongly disfavor LG.
Conclusions and Outlook

• Leptogenesis is a very attractive scenario to explain $Y_{\Delta B}$.

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• Failure of revealing $\mathcal{CP}_L$ will not disprove LG.

• If $m_\nu \gtrsim 0.1 \text{ eV}$ is established, failure of revealing $0\nu 2\beta$-decays will seriously endanger the Majorana $\nu$ hypothesis and strongly disfavor LG.

• Finally, LHC + EDM experiments will be able to establish or falsify EWB. This will indirectly determine the relevance of future LG studies.
Under what conditions low & high eng. CP can be connected?


Generically, only under rather unnatural and/or *ad hoc* conditions
Under what conditions low & high eng. $CP$ can be connected?

[Generically, only under rather unnatural and/or \textit{ad hoc} conditions]

\textbf{Casas-Ibarra parameterization for the $N$ Yukawa couplings} \cite{NPB618 (2001)}

$$
\lambda_{\alpha K} = \frac{1}{v} \left( U^\dagger \sqrt{m_\nu} \cdot R \sqrt{M_N} \right)_{\alpha K} \quad \text{;} \quad R = \frac{v}{\sqrt{m_\nu}} \cdot U^T \lambda \cdot \frac{1}{\sqrt{M_N}}
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The flavor asymmetry $\epsilon_\alpha$ is prop. to the imaginary part of:

$$\lambda_{\alpha 1}^* \lambda_{\alpha K} \left( \lambda^\dagger \lambda \right)_{1K} = \frac{M_1 M_K}{v^4} \left( \sum_i m_{\nu_i} R_{i1}^* R_{iK} \right) \left( \sum_{i,j} \sqrt{m_{\nu_j} m_{\nu_i}} R_{j1}^* R_{iK} U_{j\alpha} U_{i\alpha}^* \right)$$

The total asymmetry $\epsilon \propto \text{Im}:$

$$(\lambda^\dagger \lambda)^2_{1K} = \frac{M_1 M_K}{v^4} \left( \sum_i m_{\nu_i} R_{i1}^* R_{iK} \right)^2$$
Under what conditions low & high eng. $CP$ can be connected?

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\begin{align*}
\lambda_{\alpha K} & = \frac{1}{v} \left[ U^\dagger \sqrt{m_\nu} \cdot R \sqrt{M_N} \right]_{\alpha K}; \\
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\text{The total asymmetry } \epsilon & \propto \text{Im:} \left( \lambda^\dagger \lambda \right)_{1K}^2 = \frac{M_1 M_K}{v^4} \left( \sum_i m_{\nu_i} R_{i1}^* R_{iK} \right)^2
\end{align*}

Assuming that $R$ is real

1: $\epsilon_\alpha$ depends only on the $\nu$-mix-matrix $U$ !

2: [$\epsilon = 0$, but $\epsilon_\alpha \neq 0$, and thus $Y_{\Delta B} \neq 0$]

Dedicated studies within this scenario: Branco \textit{et al.;} Pastore \textit{et al.}
To simplify: neglect $N_{2,3}$ except for their effects in the loops ($CP$ asymmetry)

$$-\mathcal{L}_{\text{Yukawa}} = \lambda^*_{\alpha_1} \bar{\ell}_\alpha N_1 H_u + h_{\alpha \beta} \bar{\ell}_\alpha e_\beta H_d + h.c.$$
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This can be written in a more simple way by choosing a specific basis

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$$-\mathcal{L}_{\text{Yukawa}} = \lambda^*_1 \bar{\ell}_1 N_1 H_u + h_{i\alpha} \bar{\ell}_i e_\alpha H_d \quad (i=1, \perp_1, \perp_2)$$

Different bases give different results. The approx. solution of the BE for LG:

$$Y_{\Delta B} \approx 10^{-3} \times \eta_\ell \cdot \epsilon_\ell$$

$$\eta_\ell \sim \frac{m_\star}{\tilde{m}_\ell} \quad \text{(strong washout)}; \quad \tilde{m}_\ell \propto \lambda^*_{\ell 1} \lambda_{\ell 1}$$

$$Y_{\Delta B} \approx 10^{-3} \times \left\{ \sum \eta_\alpha \cdot \epsilon_\alpha \right\}$$

$$\sum \eta_\alpha \cdot \sum \epsilon_\alpha \equiv \eta \cdot \epsilon$$

**flavor regime**

**one flavor approximation**
To simplify: neglect $N_{2,3}$ except for their effects in the loops ($CP$ asymmetry)

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This can be written in a more simple way by choosing a specific basis

$$-\mathcal{L}_{\text{Yukawa}} = \lambda_{\alpha 1}^{*} \bar{\ell}_{\alpha} N_1 H_u + h_{\alpha} \bar{\ell}_{\alpha} e_{\alpha} H_d \quad \text{when } T \lesssim 10^{12} \text{ GeV}$$  

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$$Y_{\Delta B} \approx 10^{-3} \times \eta_{\ell} \cdot \epsilon_{\ell} \quad \eta_{\ell} \sim \frac{m_{\ast}}{m_{\ell}} \text{ (strong washout); } \tilde{m}_{\ell} \propto \lambda_{1}^{*} \lambda_{1}$$  

$$Y_{\Delta B} \approx 10^{-3} \times \left\{ \sum \eta_{\alpha} \cdot \epsilon_{\alpha} \right. \quad \text{flavor regime}$$  

$$\left. \sum \eta_{\alpha} \cdot \sum \epsilon_{\alpha} \right\} \equiv \eta \cdot \epsilon \quad \text{one flavor approximation}$$  

The physical basis is determined dynamically at each $T$ by the $h$-reaction rates.
More in detail: Lepton Flavor Effects

$$-\mathcal{L}_{\text{Yukawa}} = \lambda_1 \bar{N}_1 \ell_1 H_u + \text{h.c.}$$

$T \gg 10^{12}$ GeV, no charged lepton Yukawa scattering has occurred yet \hspace{1cm} (n_f = 1)$
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\[ -\mathcal{L}_{\text{Yukawa}} = \lambda_{\alpha 1} \tilde{N}_1 \ell_{\alpha} H_u + h^*_\alpha \overline{e}_\alpha \ell_{\alpha} H_d + \text{h.c.} \]

\( T \gg 10^{12} \text{ GeV}, \) no charged lepton Yukawa scattering has occurred yet \( (n_f = 1) \)

\( T < 10^{12} \text{ GeV}, \) \( \tau \)-Yukawa scatterings in equilibrium; \quad \text{Basis: } (\ell_\tau, \ell_{\perp \tau}) \quad (n_f = 2) \)
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\[ T < 10^{9} \text{ GeV}, \ \mu\text{-Yukawa in equilibrium;} \quad \text{Basis: } (\ell_\tau, \ell_\mu, \ell_e = \ell_\perp\tau\mu) \quad (n_f = 3) \]
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\( T < 10^{9} \, \text{GeV} \), \( \mu \)-Yukawa in equilibrium; \quad \text{Basis: } (\ell_\tau, \ell_\mu, \ell_e = \ell_\perp \tau \mu) \quad (n_f = 3) \)

The \( \ell_1 (\ell'_1) \) flavor content becomes important:

\[ P_\alpha = | \langle \ell_\alpha | \ell_1 \rangle |^2 \quad (\bar{P}_\alpha = | \langle \bar{\ell}_\alpha | \bar{\ell}'_1 \rangle |^2) \]
More in detail: Lepton Flavor Effects

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The $\ell_1$ ($\bar{\ell}'_1$) flavor content becomes important: $P_{\alpha} = |\langle \ell_\alpha | \ell_1 \rangle|^2$ \quad ($\bar{P}_{\alpha} = |\langle \bar{\ell}_\alpha | \bar{\ell}'_1 \rangle|^2$)

- With flavor $CP$ asymmetries: $\epsilon_{\alpha} = \frac{\Gamma(N_1 \to \ell_\alpha H) - \bar{\Gamma}(N_1 \to \bar{\ell}_\alpha \bar{H})}{\Gamma_{N_1}} = P_{\alpha} \epsilon$
- and flavor dependent washouts: $\tilde{m}_\alpha \sim P_{\alpha} \tilde{m}_1$
- the asymmetry is enhanced: $Y_{\Delta L} \propto \sum \frac{m_{*}}{\tilde{m}_\alpha} \epsilon_{\alpha} \approx n_f \left( \frac{m_{*}}{\tilde{m}_1} \epsilon \right)$
More in detail: Lepton Flavor Effects

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- With flavor $CP$ asymmetries: $\epsilon_\alpha = \frac{\Gamma(N_1 \rightarrow \ell_\alpha H) - \bar{\Gamma}(N_1 \rightarrow \bar{\ell}_\alpha \bar{H})}{\Gamma_{N_1}} = P_\alpha \epsilon + \frac{\Delta P_\alpha}{2}$
- and flavor dependent washouts: $\tilde{m}_\alpha \sim P_\alpha \tilde{m}_1$
- the asymmetry is enhanced: $Y_{\Delta L} \propto \sum \frac{m_*}{\tilde{m}_\alpha} \epsilon_\alpha \approx n_f \left( \frac{m_*}{\tilde{m}_1} \epsilon \right) + \frac{m_*}{\tilde{m}_1} \sum \frac{\Delta P_\alpha}{2P_\alpha}$

The most interesting effects are due to the different flavor composition of $\ell_1$, $\bar{\ell}_1'$:

\[CP(\bar{\ell}_1') \neq \ell_1 \Rightarrow \Delta P_\alpha \equiv P_\alpha - \bar{P}_\alpha \neq 0\]
Two-flavor case: $\ell_\tau, \ell_\perp_\tau$ ($10^9 \text{GeV} < T < 10^{12} \text{GeV}$): $|Y_{\Delta(B-L)}|$ versus $P_T^0$

$|Y_{\Delta(B-L)}|$ (units of $10^{-5}|\epsilon|$) as a function of $P_T^0 \equiv |\langle \ell_\tau | \ell_1 \rangle|^2$ in the 2-flavor regime. **Dashed**: special case in which $P_T = \bar{P}_T$. **Solid**: typical behavior when $P_T \neq \bar{P}_T$. The value of $\epsilon_1^T / \epsilon_1$ (that can be $> 1$) is marked on the upper $x$-axis.
Casas-Ibarra parameterization for the $N$ Yukawa couplings [NPB618 (2001)]

$$\lambda_{\alpha K} = \frac{1}{v} \left[ U^\dagger \sqrt{m_\nu} \cdot R \sqrt{M_N} \right]_{\alpha K}; \quad R = \frac{v}{\sqrt{m_\nu}} \cdot U^T \cdot \lambda \cdot \frac{1}{\sqrt{M_N}}$$
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The flavor asymmetry $\epsilon_{\alpha}$ (leading term) $\propto$ the imaginary part of:

$$\lambda^*_\alpha \lambda_{\alpha K} \left( \lambda^\dagger \lambda \right)_{1K} = \frac{M_1 M_K}{v^4} \left( \sum_i m_\nu_i R^*_i R^{}_{iK} \right) \left( \sum_{i,j} \sqrt{m_\nu_j m_\nu_i} R^*_j R^{}_{iK} \ U_{j\alpha} U^*_{i\alpha} \right)$$

The total asymmetry $\epsilon \propto \text{Im}$:

$$\left( \lambda^\dagger \lambda \right)^2_{1K} = \frac{M_1 M_K}{v^4} \left( \sum_i m_\nu_i R^*_i R^{}_{iK} \right)^2$$
Purely Flavored Leptogenesis ($\epsilon = 0$): SM+seesaw

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The total asymmetry $\epsilon \propto \text{Im}: \quad \left( \lambda^\dagger \lambda \right)_{1K}^2 = \frac{M_1 M_K}{v^4} \left( \sum_i m_{\nu_i} R_{i1}^* R_{iK} \right)^2$

Assuming that $R$ is real implies surprising results:

1: $\epsilon = 0$, but $\epsilon_\alpha \neq 0$, and thus $Y_{\Delta B} \neq 0$

2: $\epsilon_\alpha$ depends only on the $\nu$-mix-matrix $U$!

Recent studies of this scenario: Pastore et al.; Branco et al.;
Purely Flavored Leptogenesis: Beyond the SM+seesaw

[D. Aristizabal, M. Losada, EN, PLB659 (2008)]

Assume a $U(1)_F$ (flavor) symmetry that forbids a direct $\bar{\ell}NH$ coupling, and that the flavor symmetry is still unbroken during LG: $\langle S \rangle = 0$.

\[
\tilde{\lambda}_{\alpha K} = \left( h\frac{\langle S \rangle}{M_F} \lambda^\dagger \right)_{\alpha K};
\]

\[
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\[ \epsilon = \sum_{\alpha} \epsilon_{\alpha} = 0 \]

By decoupling $\epsilon_{\alpha}$ from $\tilde{m}_{\alpha}$, $m_\nu$ the LG scale can be lowered: $M_N \sim \text{few TeV}$. 