SELF-INDUCED SPECTRAL SPLITS IN SUPERNova NEUTRINOS: THREE-FLAVOR EFFECTS

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In the region just above the neutrino-sphere the neutrino density exceeds the ordinary electron background. Neutrinos themselves form a background medium.

\[ V = \sqrt{2G_FN_e} \sim R^{-3} \]

\[ \mu = \sqrt{2G_Fn_\nu} < 1 - \cos \theta_{pq} \sim R^{-2} \times R^{-2} = R^{-4} \]

Lesson: self-interactions (\( \mu \)) can induce large, non-MSW flavor change at small radii, despite large matter density \( V \)

**TALK BY G. RAFFELT**
The evolution equation of the density matrix for each mode $p$

$$i\hbar \partial_t \rho_p = [H_p, \rho_p]$$

Diagonal elements related to flavor content

$$\rho_{\alpha\alpha} = \frac{F_{\nu\alpha} (E, r)}{F(E, r)}$$

Off-diagonal elements responsible for flavor conversions

● “Single-angle” Hamiltonian

$$H_p = \Omega_p + V + \mu \int \frac{d^3q}{(2\pi)^3} (\rho_p - \bar{\rho}_p)$$

Matter term

$\nu - \nu$ interaction term

$$\Omega_p = \left(-\frac{\Delta m_{sol}^2}{2}, \frac{\Delta m_{sol}^2}{2}, \Delta m_{atm}^2\right)/2|p|$$

Vacuum oscillations
- **Mixing parameters:** \( U = U(\theta_{12}, \theta_{13}, \theta_{23}) \) as for CKM matrix
- **Mass spectrum**

\[
\begin{align*}
\Delta m_{\text{atm}} &= 2.0 \times 10^{-3} \text{eV}^2 \\
\Delta m_{\text{sol}} &= 8.0 \times 10^{-5} \text{eV}^2 \\
\theta_{12} &= 0.6 \\
\theta_{23} &= \pi / 4 \\
\theta_{13} &\leq 0.1
\end{align*}
\]
Neutrino mass hierarchy (and $\theta_{13}$) set initial condition and fate

With only initial $\nu_e$ and $\bar{\nu}_e$:

• **Normal hierarchy**

  Pendulum starts in $\sim$ downward (stable) positions and stays nearby. No significant flavor change.

• **Inverted hierarchy**

  Pendulum starts in $\sim$ upward (unstable) positions and eventually falls down. Significant flavor changes.

$\theta_{13}$ sets initial misalignment with vertical. Specific value not much relevant.

Which mass hierarchy?

With only initial $\nu_\mu$ and $\bar{\nu}_\mu$ large flavor conversions in NH. The unstable case is when the initial ensemble consists of that flavor which is dominated by the heavier mass eigenstate.

[Hannestad, Raffelt, Sigl, Wong, astro-ph/0608695, Duan, Carlson, Fuller, Qian, astro-ph/0703776]
**SPECTRAL SPLITS IN 2 FLAVORS (H-SYSTEM)**

[Dasgupta, Dighe, Raffelt, Smirnov, 0904.3542]

\[ F_{ve}:F_{\bar{v}_e}:F_{\nu_x} = 0.85:0.75:1.00 \]

(typical in cooling phase)

Spectral splits can develop around the crossing points of the original neutrino spectra.

A given crossing point is unstable if

- \( d(F_e - F_x)/dE < 0 \) for inverted mass hierarchy
- \( d(F_e - F_x)/dE > 0 \) for normal mass hierarchy
THREE-FLAVOR EFFECTS

Inverted Hierarchy, 2 flavors, $\nu_e$

Inverted Hierarchy, 2 flavors, $\bar{\nu}_e$

Inverted Hierarchy, full 3 flavors, $\nu_e$

Inverted Hierarchy, full 3 flavors, $\bar{\nu}_e$

Alexander Friedland, arXiv:1001:0996
SPECTRAL SPLITS IN 2 FLAVORS (L-SYSTEM)

We would expect a behavior similar to that of the H-system in NH. However, no flavor conversion occurs. Two reasons:

- insufficient growth of instability \( [\tau \approx (\omega \mu)^{-1/2}] \)
- lack of adiabaticity: \( \omega < d \ln \mu /dr \)

Some initial disturbance helps to kick-start swaps

Strength of instability given by the off-diagonal terms in the density matrix. We put as initial seeds

\[ \rho_{ex} = \epsilon |\rho_{ee} + \rho_{xx}| \]
We work in the rotated basis

\[
\begin{pmatrix}
\nu_e, \nu_x, \nu_y
\end{pmatrix} = R^T(\theta_{23}) \begin{pmatrix}
\nu_e, \nu_\mu, \nu_\tau
\end{pmatrix}
\]

Let's split the density matrix and the Hamiltonian in \((0) + (1)\) parts

\[
\rho^{(0)} = \begin{pmatrix}
\rho_{ee} & \rho_{ex} & 0 \\
\rho^*_{ex} & \rho_{xx} & 0 \\
0 & 0 & \rho_{yy}
\end{pmatrix}
\]

\[
\rho^{(1)} = \begin{pmatrix}
0 & 0 & \rho_{ey} \\
0 & 0 & \rho_{xy} \\
\rho^*_{ey} & \rho^*_{xy} & 0
\end{pmatrix}
\]

e-x block. Oscillations in the \(L= (\Delta m^2_{\text{sol}}, \theta_{12})\) sector

e-y block. Oscillations in the \(H= (\Delta m^2_{\text{atm}}, \theta_{13})\) sector

For \(\theta_{13} \neq 0\), H and L sectors are coupled

\(H^{(1)}\) produces the **off-diagonal components** first in \(\rho^{(1)}\) and subsequently in \(\rho^{(0)}\) by \(\theta_{13}\) effects.

The growth is speeded up by \(H^{(1)}\) which induces oscillations \(\Delta m^2_{\text{atm}}\)-dependent at the leading order.
The coupling between the L and H sectors induces $3^\nu$ effects.

The initial kick, associated with $\Delta m^2_{\text{atm}}$, is necessary to trigger the instability in the L system.
The two mass differences $\Delta m^2_{\text{atm}} < 0$ and $\Delta m^2_{\text{sol}} > 0$ process complementary parts of the $\nu$ spectra. Their effects do not interfere in the same energy range.

$\nu_e$ and $\nu_\gamma$ swap part of their spectra unstable under the action of $\Delta m^2_{\text{atm}}$.

$\nu_e$ and $\nu_x$ swap part of their spectra unstable under the action of $\Delta m^2_{\text{sol}}$. 
3ν SPECTRAL SPLITS IN INVERTED HIERARCHY

- Atmospheric swaps (e,y)
- Solar swaps (e,x)
- Higher energy split is transferred from e to x
- Non-adiabatic effects (especially for anti-nu)
• Almost same as 2-flavors (e-\(\nu\)).
• Solar driven conversions are too slow to compete.

In this case 3 and 2\(\nu\) instabilities act in the same regions of the \(\nu\) spectrum leading only to minor departures from the 2\(\nu\) evolution.
CONCLUSIONS

- Neutrino-neutrino interactions induce peculiar spectral swaps among different neutrino flavors.

- The development of these features is associated with instabilities in the flavor space.

- In inverted hierarchy, during the cooling phase, the presence of $\Delta m^2_{\text{sol}} > 0$ gives rise to instabilities in regions of the neutrino energy spectra that were stable under the two-flavor evolution governed by $\Delta m^2_{\text{atm}} < 0$ and $\theta_{13}$. The combinations of these two instabilities would produce a wash-out of the high-energy splitting spectral features in $\nu_e$.

- In normal hierarchy the three-flavor instabilities and the two-flavor ones act in the same regions of the neutrino energy spectrum leading only to minor departures from the two-flavor evolution.
SPECTRAL SPLITS IN THE ACCRETION PHASE

\[ F_{\nu e} : F_{\bar{\nu} e} : F_{\nu x} = 2.4 : 1.6 : 1.0 \]

We should have seen 4 splits, but we see 2 only, because the inner swap is exponentially narrower.