Coherence of Supernova Neutrinos

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Based on work done in collaboration with A.Yu. Smirnov
Interesting Effects with Supernova Neutrinos

- Oscillations inside the Earth
- MSW and collective effects inside the supernova

Is coherence preserved in these cases?
Coherence of Supernova Neutrinos

- Coherent 2-flavor oscillation probability
  \[ P(\nu_e \rightarrow \nu_e) = |\cos^2 \theta + \sin^2 \theta e^{i\phi}|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \]

- Oscillation phase \( \phi = -\frac{\Delta m^2 L}{2E} \)

- Mass eigenstates have different velocities
  \( \leadsto \) Wave packets cease to overlap

- Probability is incoherent sum
  \[ P(\nu_e \rightarrow \nu_e) = |\cos^2 \theta|^2 + |\sin^2 \theta e^{i\phi}|^2 = \cos^4 \theta + \sin^4 \theta \]

\( \leadsto \) No oscillations between supernova and Earth
Wave packets overlap up to coherence length $L_{\text{coh}} \sim \frac{\sigma}{\Delta \nu}$

- Depends on
  - Size of wave packets $\sigma$
  - Velocity difference $\Delta \nu$ of mass eigenstates
Size of Wave Packets

- Determined by length of production process
- Electron capture $pe^- \rightarrow n\nu_e$
- Time scale $\sim$ time electron needs to cross proton, $\tau \sim \sigma_e/\nu_e$
- Temperature $\sim$ 5 MeV $\Rightarrow$ electron relativistic, $\nu_e \sim 1$
- Size of electron wave packet $\sigma_e \sim$ mean free path
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Size of electron wave packet $\sigma_e \sim$ mean free path

Result: $\sigma \sim \sigma_e \sim (4\pi\alpha^2 n)^{-1/3} \sim 10^{-11}\text{ cm}$

For comparison:
- Atmospheric neutrinos: $\sigma \sim 1\text{ cm}$
- Reactor neutrinos: $\sigma \sim 10^{-8}\text{ cm}$

$\sigma_E \sim 1/\sigma \sim 1\text{ MeV}$ not much smaller than $E \sim 10\text{ MeV}$
Coherence Length of Supernova Neutrinos

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  - Size of wave packets $\sigma \sim 10^{-11} \text{ cm}$
  - Mass difference $\Delta m^2_{\text{atm}} \sim 10^{-3} \text{ eV}^2$ or $\Delta m^2_{\text{sol}} \sim 10^{-5} \text{ eV}^2$
  - Neutrino energy $E \sim 10 \text{ MeV}$
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- Mixing angle $\theta_{13} \sim 9^\circ$ or $\theta_{12} \sim 30^\circ$
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Supernova, inner region:
$L_{\text{coh}} \sim 100 \text{ km} \lesssim \text{length scale of collective effects}$

Earth: $L_{\text{coh}} \sim 1000 \text{ km}$
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  - $L_{\text{coh}} \sim 100$ km $\ll$ length scale of collective effects
- Earth: $L_{\text{coh}} \sim 1000$ km

Supernova neutrinos are special
Very short wave packets $\sim$ short coherence length
Sometimes Decoherence Is Irrelevant

\[ P(\nu_e \rightarrow \nu_e) = |\cos^2 \theta + \sin^2 \theta e^{i\phi}|^2 \]

\[ = \cos^4 \theta + \sin^4 \theta + 2 \cos^2 \theta \sin^2 \theta \cos \phi \]

- Measured probability: average over energy resolution \( \Delta E \) of detector
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- \( \Delta E \) too large \( \sim \) interference term vanishes
- \( \sim \) Need sufficient energy resolution to observe oscillations

\[ \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c} 
\hline
E/MeV & 0 & 5 & 10 & 15 & 20 & 25 & 30 \\
\hline
P(\nu_e \rightarrow \nu_e) & 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\hline
\end{array} \]
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- Corresponding uncertainty of arrival time \( \Delta t \sim 1/\Delta E \)
- Coherence preserved for wave packets arriving within \( \Delta t \), even if they are spatially separated
  \( \sim \) Detector restores coherence
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- Always the case in vacuum and matter with slowly changing density (adiabatic case)
- Does this change in non-adiabatic case?
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two

\[ \rho \quad \rho' \quad \rho'' \]

\[ \nu_1 \]
Adiabaticity Violation

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\[ \rho \quad \rho' \quad \rho'' \]

\[\nu_1 \quad \nu'_1 \quad \nu'_1 \]

\[\nu_2 \quad \nu'_2 \quad \nu'_2 \]
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two

\[
\begin{align*}
\rho & \quad \rho' & \quad \rho'' \\
\nu_1 & \quad \nu'_1 & \quad \nu''_1 \\
\nu_2 & \quad \nu'_2 & \quad \nu''_2
\end{align*}
\]
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two

\[ \rho \quad \rho' \quad \rho'' \]

\[ \nu_1' \quad \nu_1' \quad \nu_1'' \]

\[ \nu_2' \quad \nu_2' \quad \nu_2'' \]
Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two
- Different oscillation phase in each layer
- Complete coherence:

\[ P = \left| a + b e^{i\phi_1} + c e^{i\phi_2} + \ldots \right|^2 \]
Adiabaticity Violation

- Simplest case: density step
- Each wave packet **splits up** into two
- Different oscillation phase in each layer
- Complete coherence:

\[ P = \left| a + b e^{i\phi_1} + c e^{i\phi_2} + \ldots \right|^2 \]

- Complete decoherence:

\[ P = a^2 + b^2 + c^2 + \ldots \]

- Energy resolution good enough to resolve all oscillation features
  \( \Rightarrow \) detector restores coherence as before
Incomplete Averaging

\[ P = \left| a + b e^{i\phi_1} + c e^{i\phi_2} + \ldots \right|^2 \]

\[ = a^2 + b^2 + c^2 + \cdots + 2ab \cos \phi_1 + 2bc \cos(\phi_2 - \phi_1) \]

Energy resolution may be

- **too bad** to observe \( \cos \phi_1 \) term
- **good enough** to observe \( \cos(\phi_2 - \phi_1) \) term if \( \phi_1 \approx \phi_2 \)
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But only if wave packets overlap (coherent case)?
Wave packets don’t need to separate forever:
Wave packets don’t need to separate forever:

\[ \rho' \]

\[ \nu_1' \]

\[ \nu_2' \]

\[ \Delta x \]

\[ \rho'' \]

\[ \nu_1'' \]

\[ \nu_2'' \]

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Wave packets don’t need to separate forever:
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\[ \rho'' \]

\[ \Delta x \]
Wave Packet Catch-Up

- Wave packets don’t need to separate forever:
  \[ \Delta E \text{ good enough to observe } \cos(\phi_2 - \phi_1) \text{ term} \]
- Wave packets catch up \[\iff\] Decoherence and energy averaging are equivalent
Wave Packet Catch-Up

- Wave packets don’t need to separate forever:
  - Wave packets catch up $\iff \Delta E$ good enough to observe $\cos(\phi_2 - \phi_1)$ term
- Decoherence and energy averaging are equivalent
- Open questions:
  - How general is this?
  - Are collective effects in a supernova different?
Finite $\sigma_E \sim$ symmetric broadening of energy spectrum $f(E)$

Change relevant for $\sigma_E^2 \gtrsim f/f''$
Effect of Wave Packet Size on Spectrum

- Finite $\sigma_E \sim$ symmetric broadening of energy spectrum $f(E)$
- Change relevant for $\sigma_E^2 \gtrsim f/f''$
- Example:
  - $f(E) \propto E^\alpha \exp[-(\alpha + 1) E/\bar{E}]$
  - Mean energy $\bar{E} = 10 \text{ MeV}$
  - $\alpha = 3$
  - $\sigma_E = 2 \text{ MeV}$
- Potentially observable deformation of spectrum for $E \lesssim 3 \text{ MeV}$ and around $E \simeq 5 \text{ MeV}$
Supernova Neutrinos

- Extremely small size of wave packets $\sigma \sim 10^{-11}$ cm
- No experimental consequences in simple examples
- Possibly relevant for
  - collective effects in a supernova
  - energy spectrum