Exotic charged current interactions in tritium beta decay

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Outline of the talk

I. Direct neutrino mass experiments
II. Physics of tritium beta decay
III. Exotic CC interactions in tritium beta decay
IV. Summary and conclusions
I. Direct neutrino mass experiments
Neutrino mass bounds from direct neutrino mass experiments

- Bounds from direct neutrino mass experiments: Mainz and Troitsk experiment [hep-ex/0412056; 1108.5034]: Precise study of the endpoint of the tritium beta spectrum.
  \[ \rightarrow m_\nu < 2 \text{ eV}. \]

Direct neutrino mass experiments are important to confirm/challenge cosmological observations and \( m_{\beta\beta} \) bounds.

Generic method of direct neutrino mass experiments: Measurement of endpoint of \( \beta \)-spectrum or electron capture spectrum:

- \( A \rightarrow B + e^- + \bar{\nu}_e, \)
- \( A + e^- \rightarrow B + \nu_e. \)
Direct neutrino mass experiments

Ideal isotopes: Q-value as small as possible $\rightarrow m_\nu \neq 0$ has largest effect on spectrum.

- **Tritium:** $^3H \rightarrow ^3He^+ + e^- + \bar{\nu}_e \quad (Q = 18.6 \text{ keV}),$
- **Holmium:** $^{163}\text{Ho}^+ + e^- \rightarrow ^{163}\text{Dy} + \nu_e \quad (Q = 2.8 \text{ keV})$

**Experiments:** See talk by Loredana Gastaldo.

- **KATRIN** (see talk by Guido Drexlin): tritium beta spectrum,
- **Project 8:** tritium beta spectrum,
- **PTOLEMY** (see talk by Alfredo Cocco): tritium beta spectrum,
- **NuMECS, HOLMES, ECHo:** Holmium electron capture spectrum.
What can we learn from direct neutrino mass experiments?

- Study of the endpoint of the energy spectrum: Bounds on the absolute neutrino mass scale,
- search for eV-scale sterile neutrinos, . . . → see talk by Guido Drexlin.

What else?

Example: The KATRIN tritium source has an activity of about $10^{11}$ decays per second

→ If experimental setup can be upgraded for measuring the whole spectrum: TRISTAN (see talks by Guido Drexlin and Thierry Lasserre):

→ Ultra-high statistics and ultra-high precision measurement of the tritium beta spectrum.
What can we learn from extensions of KATRIN-like experiments?

Ultra-high statistics and ultra-high precision measurement of the tritium beta spectrum.

This can even lead to bounds on (or discovery of) new physics provided that

the standard model physics involved in beta decay is sufficiently well understood.

For a review on this issue see Susanne Mertens et al. [1409.0920] and the talk by Thierry Lasserre.

For the research project presented in this talk: Assumed that sufficient understanding is given.
What can we learn from extensions of KATRIN-like experiments?

In our paper we studied three effects:

- **Difference** relativistic / non-relativistic treatment of the spectrum,
- spectral distortion from new charged-current (CC) interactions
- for light active and keV sterile neutrinos
II. Physics of tritium beta decay
Kinematics of $\beta$ decay

\[ A \rightarrow B + e^- + \nu_e \]

\[ |\nu_e\rangle = \sum_{j=1}^{3+n_s} U_{ej} |\nu_j\rangle \]

Fully relativistic treatment of 3-body decay gives

\[
\left( \frac{d\Gamma}{dE_e} \right)_{\nu_j} = \frac{1}{64\pi^3 m_A} \int_{E_j^-}^{E_j^+} dE_j \, |\mathcal{M}(A \rightarrow B + e^- + \nu_j)|^2
\]

- $E_j^\pm$ min. and max. neutrino energy (function of particle masses and $E_e$).
- $|\mathcal{M}(A \rightarrow B + e^- + \nu_j)|^2$ squared matrix element (unpolarized): function of particle masses and $E_e$ and $E_j$. 
Theoretical framework

- Lorentz invariance,
- only tree-level interactions,
- only effective four-fermion interactions,
- no assumption about the Lorentz structure → all types of interactions allowed (scalar, vector, axial vector, ...)

Under these assumptions the amplitude $\mathcal{M}$ has the form

$$\mathcal{M} = [\bar{u}_e \mathcal{O} v_j] [\bar{u}_B \mathcal{O}' u_A].$$

If operators $\mathcal{O}$ and $\mathcal{O}'$ independent of particle momenta, $|\mathcal{M}|^2$ contains only terms of the form

$$(p \cdot p') \quad \text{or} \quad (p \cdot p')(p'' \cdot p''').$$
Theoretical framework

\[ \mathcal{M} \sim (p \cdot p') \quad \text{or} \quad (p \cdot p')(p'' \cdot p''') \]

→ Energy-momentum conservation: \( p_A = p_B + p_e + p_j \)

⇒ \( p \cdot p' \) depends only on \( E_e \) (electron energy) and \( E_j \) (neutrino energy).

\[
|\mathcal{M}(A \rightarrow B + e^- + \bar{\nu}_j)|^2 = A + B_1 E_e + B_2 E_j + C E_e E_j + D_1 E_e^2 + D_2 E_j^2,
\]

⇒ The energy spectrum in our framework can be parameterized by six parameters!

In a given model \( A, B_1, B_2, C, D_1 \) and \( D_2 \) depend on the particle masses and coupling constants only!
Relativistic electron energy spectrum

\[
\left( \frac{d\Gamma}{dE_e} \right)_{\bar{\nu}_j} = \frac{1}{64\pi^3 m_A} \int_{E_j^-}^{E_j^+} dE_j \left| \mathcal{M}(A \rightarrow B + e^- + \bar{\nu}_j) \right|^2
\]

\[
= \frac{1}{64\pi^3 m_A} \times \left\{ (A + B_1 E_e + D_1 E_e^2)(E_{j+} - E_{j-}) + \frac{1}{2}(B_2 + C E_e)(E_{j+}^2 - E_{j-}^2) + \frac{1}{3}D_2(E_{j+}^3 - E_{j-}^3) \right\}.
\]
Effect of relativistic/non-relativistic spectrum

- Effect on spectral endpoint position:

\[(E_e^{\text{max}})_{\text{NR}} = m_A - m_B - m_j,\]
\[(E_e^{\text{max}})_{\text{R}} = \frac{m_A^2 + m_e^2 - (m_B + m_j)^2}{2m_A}.\]

**Difference for tritium decay:** \(\approx 3.4\) eV.

- **Whole spectrum:** Difference is of the order of \(\approx 10^{-4} \div 10^{-3}\).

In the following: Show “textbook example”: Standard model expression relativistic/non-relativistic.
Example: Standard model: relativistic/non-relativistic

\[
\mathcal{L} = -\frac{G_F}{\sqrt{2}} V_{ud} \left( \bar{e} \gamma^\mu (1 - \gamma^5) \nu_e \right) \left( \bar{B} \gamma_\mu (g_V 1 - g_A \gamma^5) A \right) + \text{H.c.}
\]

\[
\Rightarrow
\]

\[
A = \frac{\gamma}{2} m_A m_B (g_V^2 - g_A^2)(m_A^2 - m_B^2 + m_e^2 + m_j^2),
\]

\[
B_1 = \frac{\gamma}{2} m_A \left\{ (g_V - g_A)^2 (m_A^2 - m_B^2 + m_e^2 - m_j^2) - 2m_A m_B (g_V^2 - g_A^2) \right\},
\]

\[
B_2 = \frac{\gamma}{2} m_A \left\{ (g_V + g_A)^2 (m_A^2 - m_B^2 - m_e^2 + m_j^2) - 2m_A m_B (g_V^2 - g_A^2) \right\},
\]

\[
C = 0,
\]

\[
D_1 = -\gamma m_A^2 (g_V - g_A)^2,
\]

\[
D_2 = -\gamma m_A^2 (g_V + g_A)^2,
\]

\[
\gamma \equiv 16 G_F^2 |V_{ud}|^2 |U_{ej}|^2.
\]
Standard model $\beta$-spectrum for tritium decay:
Example: Standard model: relativistic/non-relativistic

Non-relativistic approximation is the leading term in the expansion \( m_A \):

\[
\rightarrow \left( \frac{d\Gamma}{dE_e} \right)_{\text{NR}}, m_j=0 = \frac{2 G_F^2 |V_{ud}|^2}{\pi^3} |\vec{p}_e| E_e (m_A - m_B - E_e)^2.
\]

Deviation relativistic/non-relativistic expression: \( \Delta \equiv \frac{(d\Gamma/dE_e) - (d\Gamma/dE_e)_{\text{NR}}}{(d\Gamma/dE_e)_{\text{NR}}} \).

![Graph showing deviation \( \Delta \) as a function of \( E_e - m_e \) in keV]

Solid: \( m_j = 0 \); dashed: \( m_j = 5 \text{ keV} \).

**Deviation** \( \sim 10^{-4} \div 10^{-3} \)
III. Exotic CC interactions in tritium beta decay
Lagrangian

**Effective operator approach:** All possible Lorentz-invariant interactions. Use notation of Cirigliano et al. [1303.6953]. \( L = 1 - \gamma^5 \). Add RH \( \nu_s \).

\[
\mathcal{L}_{CC} = - \frac{G_F V_{ud}}{\sqrt{2}} \left\{ (1 + \delta_\beta)(\bar{e}L_\mu \nu_e)(\bar{u}L^\mu d) + \sum_j (\sim) \epsilon_j (\bar{e} O_j \nu_e)(\bar{u} O'_j d) \right\} + \text{H.c.}
\]

<table>
<thead>
<tr>
<th>( \epsilon_j )</th>
<th>( O_j )</th>
<th>( O'_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_L )</td>
<td>( \gamma_\mu (1 - \gamma^5) )</td>
<td>( \gamma^\mu (1 - \gamma^5) )</td>
</tr>
<tr>
<td>( \tilde{\epsilon}_L )</td>
<td>( \gamma_\mu (1 + \gamma^5) )</td>
<td>( \gamma^\mu (1 - \gamma^5) )</td>
</tr>
<tr>
<td>( \epsilon_R )</td>
<td>( \gamma_\mu (1 - \gamma^5) )</td>
<td>( \gamma^\mu (1 + \gamma^5) )</td>
</tr>
<tr>
<td>( \tilde{\epsilon}_R )</td>
<td>( \gamma_\mu (1 + \gamma^5) )</td>
<td>( \gamma^\mu (1 + \gamma^5) )</td>
</tr>
<tr>
<td>( \epsilon_S )</td>
<td>( 1 - \gamma^5 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( \tilde{\epsilon}_S )</td>
<td>( 1 + \gamma^5 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( -\epsilon_P )</td>
<td>( 1 - \gamma^5 )</td>
<td>( \gamma^5 )</td>
</tr>
<tr>
<td>( -\tilde{\epsilon}_P )</td>
<td>( 1 + \gamma^5 )</td>
<td>( \gamma^5 )</td>
</tr>
<tr>
<td>( \epsilon_T )</td>
<td>( \sigma_{\mu\nu} (1 - \gamma^5) )</td>
<td>( \sigma^{\mu\nu} (1 - \gamma^5) )</td>
</tr>
<tr>
<td>( \tilde{\epsilon}_T )</td>
<td>( \sigma_{\mu\nu} (1 + \gamma^5) )</td>
<td>( \sigma^{\mu\nu} (1 + \gamma^5) )</td>
</tr>
</tbody>
</table>

**Two basic types of couplings:** \( \epsilon \): left-handed \( \nu_s \), \( \tilde{\epsilon} \): right-handed \( \nu_s \).
Numerical analysis

Bounds for the coefficients $\epsilon$ and $\tilde{\epsilon}$ from Cirigliano et al. [1303.6953].

<table>
<thead>
<tr>
<th>parameter</th>
<th>best 90 % CL upper bound</th>
<th>used for our estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>\text{Re},\epsilon</td>
</tr>
<tr>
<td>$\epsilon_L$</td>
<td>$5 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\tilde{\epsilon}_L$</td>
<td>$6 \times 10^{-2}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\epsilon_R$</td>
<td>$5 \times 10^{-4}$</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
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<td>$5 \times 10^{-3}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\epsilon_S$</td>
<td>$8 \times 10^{-3}$</td>
<td>$1 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\tilde{\epsilon}_S$</td>
<td>$1.3 \times 10^{-2}$</td>
<td>$1.3 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\epsilon_P$</td>
<td>$4 \times 10^{-4}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\tilde{\epsilon}_P$</td>
<td>$2 \times 10^{-4}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\epsilon_T$</td>
<td>$1 \times 10^{-3}$</td>
<td>$1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\tilde{\epsilon}_T$</td>
<td>$3 \times 10^{-3}$</td>
<td>$3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Procedure for numerical analysis

- Fixed neutrino masses $m_j = 0.5 \text{ eV}$ (light active neutrinos) or $m_j = 5 \text{ keV}$ (heavy sterile right-handed neutrino), see talk by Alex Merle,
- one $\epsilon$ or $\bar{\epsilon}$ set to 1, the others set to 0,
- relevant mixing matrix element set to 1.

$\rightarrow$ Computed the values for the six coefficients $A, \ldots, D_2$.

$\rightarrow$ Tables in paper.

Paper gives prescription how to scale the tabulated values for other values of $\epsilon$ and other mixing matrix elements.

$\Rightarrow$ Can directly obtain the beta spectrum for any new CC interaction.
Signal to be expected in a high-precision experiment

Mixing active/sterile (left/right) is of the order of

\[ \frac{m_{\text{light}}}{m_{\text{heavy}}} \lesssim \frac{\text{eV}}{\text{keV}} \sim 10^{-3}. \]

⇒ “heavy neutrinos/\(\epsilon\)” and “light neutrinos/\(\bar{\epsilon}\)”: strongly suppressed by mixing matrix → no observable effect.

- \(\epsilon\): studied light neutrinos (\(m_j = 0.5\) eV),
- \(\bar{\epsilon}\): heavy neutrinos (\(m_j = 5\) keV).

Used upper bounds for \(\epsilon\) and \(\bar{\epsilon}\) from literature.

→ Plots of:

\[ \Delta^{(\sim)}(\epsilon_j) \equiv \frac{\text{test spectrum (NP)} - \text{reference spectrum (no NP)}}{\text{reference spectrum (no NP)}}. \]
New physics effects for light neutrinos with $m_j = 0.5\, \text{eV}$
New physics effects for heavy neutrinos with $m_j = 5$ keV
Upgraded KATRIN (or KATRIN-like) experiment may have access to the full energy spectrum of tritium decay. → Ultra-high statistics and ultra-high precision!

Fully relativistic calculation of the $\beta$ spectrum. Generic spectrum depends on six process-dependent parameters.

Department from usual non-relativistic approximation: $\approx 10^{-4} \div 10^{-3}$.

New physics in $\beta$ decay: Studied all possible new CC interactions in effective operator framework.

Both light (sub-eV) and heavy (keV) neutrinos considered: Effect on endpoints negligible, full spectrum can show sizable distortions at the permille level.
Conclusions

- Accessibility of the new-physics effects by a future KATRIN-like experiment: Example: keV scale right-handed neutrinos: Sensitivity estimate (Mertens et al. [1409.0920]): $\sim 10^{-7}$.

- modified KATRIN-like setup sensitive to
  - $\epsilon_L, \epsilon_R, \epsilon_S, \epsilon_T$ in case of light left-handed (even almost massless) neutrinos.
    Different new-physics scenarios can be distinguished by the shape of the spectral distortion;
  - $\tilde{\epsilon}_L, \tilde{\epsilon}_R, \tilde{\epsilon}_S, \tilde{\epsilon}_T$ in case of keV-scale right-handed neutrinos.
    New physics effects are not easily distinguishable (shapes are quite similar).

- If systematic effects in the experiment and all Standard Model contributions are under control, an extended KATRIN-like setup may significantly improve the bounds on new CC interactions in $\beta$ decay.
Thank you for your attention!
Backup slides
Current bounds on neutrino masses

- **Bounds from cosmology:** Bounds on the sum of the three active neutrino masses:

\[
\sum m_\nu < 0.72 \text{ eV} \quad \text{Planck TT+lowP},
\]
\[
\sum m_\nu < 0.21 \text{ eV} \quad \text{Planck TT+lowP+BAO},
\]
\[
\sum m_\nu < 0.49 \text{ eV} \quad \text{Planck TT, TE, EE+lowP},
\]
\[
\sum m_\nu < 0.17 \text{ eV} \quad \text{Planck TT, TE, EE+lowP+BAO}.
\]

(Planck Collaboration [1502.01589]).

Dependent on which data taken into account. In any case **bound stronger than current direct neutrino mass bounds**.

- **Bounds from** \((\beta\beta)_{0\nu}\)-**searches:** Bounds on

\[
m_{\beta\beta} = \left| \sum_{k=1}^{3} U_{ek}^2 m_k \right| \rightarrow \text{bounds on absolute mass scale}
\]
Current bounds on neutrino masses

Dell’Oro et al. [1601.07512]

\[
\begin{align*}
136\text{Xe} & (\text{KamLAND-Zen+EXO-200}) \\
130\text{Te} & (\text{Cuoricino+CUORE-0}) \\
76\text{Ge} & (\text{IGEX+HdM+GERDA-I}) \\
\end{align*}
\]

\[
m_{\nu} \lesssim \mathcal{O}(1 \text{eV})
\]
Minimal and maximal neutrino energy

\[ E_{j\pm} = \frac{-(m_A - E_e)(E_e m_A - \alpha) \pm \tilde{p}_e \sqrt{(E_e m_A - \alpha + m_j^2)^2 - m_B^2 m_j^2}}{m_A^2 - 2m_A E_e + m_e^2}, \]

\[ \alpha = \frac{1}{2} \left( m_A^2 - m_B^2 + m_e^2 + m_j^2 \right). \]
Kurie plots

\[ K(E_e) \equiv \frac{1}{m_A - m_B} \sqrt{\frac{d\Gamma/dE_e}{G_0(E_e)}}, \]

where

\[ G_0(E_e) \equiv \frac{2G_F^2|V_{ud}|^2}{\pi^3} |\vec{p}_e| E_e F(Z, E_e). \]

\( m_j = 2.0, 1.0, 0.5 \text{ and } 0 \text{ eV}, \quad 10, 5, 3 \text{ and } 0 \text{ keV.} \)
Momentum-dependent operators $O, O'$

In our study this case appears only for the weak magnetism correction:

$$
\langle p(p_p)|\bar{u}\gamma_\mu d|n(p_n)\rangle = \bar{u}_p(p_p) \left[ g_V(q^2)\gamma_\mu - i g_{\text{WM}}(q^2) \frac{1}{2M_N} \sigma_{\mu\nu} q^\nu \right] u_n(p_n) + O((q/M_N)^2)
$$

$$
q = p_A - p_B
$$

Gives contribution to $|M|^2$ of the form

$$
\frac{(p \cdot q)(p' \cdot q)(p'' \cdot p''')}{M_N^2} \quad \text{or} \quad \frac{(p \cdot p')(p'' \cdot p''')(q \cdot q)}{M_N^2}.
$$

For tritium decay suppressed by

$$
\frac{q^2}{M_A^2} \lesssim 10^{-10}.
$$
Corrections from Standard Model physics

See Susanne Mertens et al. [1409.0920] and the talk by Thierry Lasserre:

- **Excited final states**: the effect on the spectrum is very large—larger than 10\% close to the endpoint. Far from the endpoint $\sim 1\%$.

- Coulomb interaction between the outgoing electron, the daughter nucleus (→ *Fermi function* $F(Z, E_e)$) and the left behind orbital electron of the former $^3\text{H}_2$-molecule.

- The nuclear recoil: automatically taken into account by using the exact relativistic expression for $d\Gamma/dE_e$.

- The daughter nucleus $^3\text{He}^{2+}$ is not pointlike → modifies the Coulomb field acting on the emitted electron.

- Radiative corrections: The dominant radiative corrections will be QED-corrections. → Of the order of $\sim 1\%$.

- **Hadronic matrix elements**.

Moreover, source is a gas at finite temperature (30K).
Corrections from Standard Model physics

Hadronic matrix elements: See Cirigliano et al. [1303.6953].

\[ \langle p(p_p)|\bar{u}\gamma_{\mu}d|n(p_n)\rangle = \bar{u}_p(p_p) \left[ g_V(q^2)\gamma_{\mu} - \frac{i g_{WM}(q^2)}{2M_N} \sigma_{\mu\nu} q^\nu \right] u_n(p_n) + \mathcal{O}((q/M_N)^2), \]

\[ \langle p(p_p)|\bar{u}\gamma_{\mu}\gamma_5d|n(p_n)\rangle = g_A(q^2) \bar{u}_p(p_p)\gamma_{\mu}\gamma_5 u_n(p_n) + \mathcal{O}((q/M_N)^2), \]

\[ \langle p(p_p)|\bar{u}d|n(p_n)\rangle = g_S(q^2) \bar{u}_p(p_p) u_n(p_n), \]

\[ \langle p(p_p)|\bar{u}\gamma_5d|n(p_n)\rangle = g_P(q^2) \bar{u}_p(p_p)\gamma_5 u_n(p_n) = \mathcal{O}(q/M_N), \]

\[ \langle p(p_p)|\bar{u}\sigma_{\mu\nu}d|n(p_n)\rangle = g_T(q^2) \bar{u}_p(p_p)\sigma_{\mu\nu} u_n(p_n) + \mathcal{O}(q/M_N). \]

Written here for proton and neutron. The same structure for tritium\(^3\)He.

Values for form factors from measurements or lattice.
Choose (for numerical estimates) neutrino mass spectrum:
light neutrinos with \( m_j = 0.5 \text{ eV} \) and heavy neutrinos with \( m_j = 5 \text{ keV} \)
→ see talk by Alex Merle.