Bound states in dark-matter phenomenology

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Long-range interactions mediated by massless or light particles

Bound states
Long-range interactions

Motivation

- Self-interacting DM
- DM explanations of astrophysical signals, e.g. galactic positrons, IceCube neutrinos
- Little hierarchy problem, e.g. twin Higgs models
- Sectors with stable particles in String Theory

Hidden sector DM
- Self-interacting DM
- DM explanations of astrophysical signals, e.g. galactic positrons, IceCube neutrinos
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- Sectors with stable particles in String Theory

- WIMP DM with $m_{DM} >$ few TeV! [Hisano et al. 2002]

- Particular scenarios of sub-TeV WIMPs

- Motivation:
  - Minimal DM [Cirelli et al.]
  - LHC implications for SUSY
  - Direct/Indirect detection constraints

Co-annihilating with / produced by the decays of charged/coloured particles [cMSSM]
**Asymmetric DM → Stable bound states**
- Kinetic decoupling of DM from radiation, in the early universe
- DM self-scattering in halos: Screening
  [KP, Pearce, Kusenko (2014)]
- Indirect detection signals: Radiative level transitions
  [Pearce, Kusenko (2013); Cline et al. (2014); Detmold, McCullough, Pochinsky (2014); Pearce, KP, Kusenko (2015)]
- Direct detection signals: Screening, inelastic scattering

**Symmetric / Self-conjugate DM → Unstable bound states**
Formation + Decay = Extra annihilation channel
- Relic abundance
  [von Harling, KP (2014); Ellis et al. (2015)]
- Indirect detection
  [Cirelli, Panci, KP, Sala, Taoso, (in preparation)]
A. **Confining theories**

“Non-perturbative non-perturbative bound states”, e.g. hadrons

Cosmologically, they definitely form. May leave a remnant weakly coupled long(-ish)-range interaction.

B. **Weakly coupled theories**

“Perturbative non-perturbative bound states”, e.g. atoms.

Formation efficiency depends on the details:

(i) *bound-state formation cross-section*, and
(ii) *thermodynamic environment*

*(early universe, DM halos, interior of stars)*
Outline

- This talk: **Hidden-sector Abelian model**
  - Effect of bound states on the relic density.
  - Indirect detection signals.

- You can ask me about: **Relevance to WIMPs**
  - TeV scale: relic density & indirect detection
  - Sub-TeV scale: relic density
Relic density of symmetric DM with contact interactions

- Early universe:
  DM kept in chemical equilibrium via annihilations, $\chi + \chi \leftrightarrow f + \bar{f}$.
  DM density $n_\chi = n_\chi(T)$

- As universe expands and cools
  $\Rightarrow$ Density decreases
  $\Rightarrow$ Annihilations become inefficient
  $\Rightarrow$ Exponential decrease of $n_\chi(T)$ stalls: **freeze-out**
  $\Rightarrow$ Relic density

\[
\Omega_\chi \simeq 0.26 \times \left[ \frac{3 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle} \right]
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**Assumption:**
$$\sigma_{\text{ann}} v_{\text{rel}} \approx \sigma_0 + \sigma_1 v_{\text{rel}}^2 + \ldots$$
Valid for contact interactions.
Relic density of symmetric DM with long-range interactions

Toy model: Dark QED

Dirac fermions \( (\chi, \bar{\chi}) \) of mass \( m \), coupled to a massless dark photon \( \gamma \), with dark fine-structure constant \( \alpha \).

Very important parameter:

\[ \zeta = \frac{\alpha}{v_{\text{rel}}} \]
Annihilation

\[ \chi + \bar{\chi} \rightarrow \gamma + \gamma \]

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\[ \sigma_{\text{ann}} \nu_{\text{rel}} = \sigma_0 S_{\text{ann}}(\zeta) \]

\[ \sigma_0 = \pi \frac{\alpha^2}{m^2} \]

\[ S_{\text{ann}}(\zeta) = \frac{2\pi \zeta}{1 - e^{-2\pi \zeta}} \]

\( S_{\text{ann}}(\zeta \ll 1) \approx 1 \)

\( S_{\text{ann}}(\zeta \gg 1) \approx 2\pi \zeta \)

Processes

[Hisano et al. (2002);
Cirelli et al. (2005 ... 2016);
Feng et al. (2009, 2010);
Slatyer et al. (2013);
Beneke et al. (2014 ... 2016); ... ]
Relic density of symmetric DM with long-range interactions

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Toy model: Dark QED

**Processes**

**Bound state formation and decay**

\[ \chi + \bar{\chi} \rightarrow (\chi \bar{\chi})_{\text{bound}} + \gamma \]

\[ (\chi \bar{\chi})_{\text{bound}} \rightarrow 2\gamma \text{ or } 3\gamma \]

\[ \sigma_{\text{BSF}} \nu_{\text{rel}} = \sigma_0 S_{\text{BSF}}(\zeta) \]

\[ \sigma_0 = \pi \alpha^2 / m^2 \]

\[ S_{\text{BSF}}(\zeta) = \frac{2^9}{3 e^4 \arccot(\zeta)} \frac{\zeta^4}{(1 + \zeta^2)^2} \]

\[ \frac{2 \pi \zeta}{1 - e^{-2 \pi \zeta}} \]

\[ S_{\text{BSF}}(\zeta \ll 1) \approx \frac{2^9 \zeta^4}{3} \ll 1 \]

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\zeta = \frac{\alpha}{v_{\text{rel}}}
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**Annihilation**

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Relic density of symmetric DM with long-range interactions

\[ \zeta = \alpha / \nu_{\text{rel}} \]

BSF dominates over annihilation everywhere the Sommerfeld effect is important (\( \zeta > 1 \)!)
Relic density of symmetric DM with long-range interactions

\[
\frac{d n_{\chi}}{d t} + 3H n_{\chi} = - \left( n_{\chi}^2 - n_{\chi}^{\text{eq}} \right) \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle - n_{\chi}^2 \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle + (n_{\uparrow \downarrow} + n_{\uparrow \uparrow}) \Gamma_{\text{ion}}
\]

\[
\frac{d n_{\uparrow \downarrow}}{d t} + 3H n_{\uparrow \downarrow} = + \frac{1}{4} n_{\chi}^2 \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle - n_{\uparrow \downarrow} (\Gamma_{\text{ion}} + \Gamma_{\text{decay}, \uparrow \downarrow})
\]

\[
\frac{d n_{\uparrow \uparrow}}{d t} + 3H n_{\uparrow \uparrow} = + \frac{3}{4} n_{\chi}^2 \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle - n_{\uparrow \uparrow} (\Gamma_{\text{ion}} + \Gamma_{\text{decay}, \uparrow \uparrow})
\]

\[
(\chi \bar{\chi})_{\uparrow \downarrow} \rightarrow 2 \gamma : \quad \Gamma_{\text{decay}, \uparrow \downarrow} = \alpha^5 (m/2)
\]

\[
(\chi \bar{\chi})_{\uparrow \uparrow} \rightarrow 3 \gamma : \quad \Gamma_{\text{decay}, \uparrow \uparrow} = \frac{4 \left( \pi^2 - 9 \right)}{9 \pi} \alpha^6 (m/2)
\]

\[
(\chi \bar{\chi})_{\uparrow \downarrow} \text{ or } \uparrow \uparrow + \gamma \rightarrow \chi + \bar{\chi} : \quad \Gamma_{\text{ion}}(T) = \frac{2}{(2\pi)^3} 4 \pi \int_0^\infty d\omega \frac{\omega^2}{e^{\omega/T} - 1} \sigma_{\text{ion}}(\omega)
\]

[von Harling, KP (2014)]
To obtain the observed DM density, we need:
Relic density of symmetric DM with long-range interactions
[von Harling, KP (2014)]

Effect on DM density, coupling, mass

Much larger than the experimental uncertainty of 1%.

\[
\frac{\Omega_{SE \, \text{ann}}}{\Omega_{DM}} \approx 4 \quad \text{at 140 TeV}
\]

\[
\frac{\Omega_{SE \, \text{ann}}}{\Omega_{DM}} \approx 2 \quad \text{at 15 TeV}
\]

\[
\Omega_{SE \, \text{ann}} / \Omega_{DM} \approx 4
\]

\[
\Omega_{SE \, \text{ann}} / \Omega_{DM} \approx 2
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\Omega_{SE \, \text{ann}} / \Omega_{DM} \approx 2
\]

\[
\frac{\delta m}{m} \approx 10\%
\]

\[
\frac{\delta m}{m} \approx 10\%
\]

\[
\delta \alpha / \alpha
\]

\[
\delta m / m
\]

Larger than the experimental sensitivity (~ 10%).
Bound states of annihilating dark matter
Generalisations needed

Massive mediators

Different interactions, e.g. scalar mediator.

Non-Abelian non-confining theories, e.g. EW interactions.

Relic density

Indirect detection
Two parameters needed

$$\zeta = \alpha / v_{\text{rel}} \quad \text{and} \quad \xi = m_{\text{DM}} \alpha / (2m_\phi)$$

[velocity dependence] [model dependence]

- **At low enough velocities (large $\zeta$)**
  - $\sigma_{\text{ann}} v_{\text{rel}} \sim$ constant (saturation of $1/v_{\text{rel}}$ enhancement)
  - $\sigma_{\text{ann}} v_{\text{rel}} \sim v_{\text{rel}}^2$ (suppression)

- **Resonances at discrete $\xi$ values**, which are different for annihilation and BSF. Precise location:
  - Annihilation: $\zeta$ independent
  - BSF: Mild $\zeta$ dependence

**Important features for indirect detection!**
Vector mediator: $\xi$ values away from $\ell = 0$ and $\ell = 1$ resonances

Parameters:

$\zeta = \alpha / v_{\text{rel}}$

$\xi = m_{\text{DM}} \alpha / (2m_\phi)$

[ KP, Postma, de Vries (in preparation) ]
Parameters:
\[ \zeta = \frac{\alpha}{v_{\text{rel}}} \]
\[ \xi = \frac{m_{\text{DM}} \alpha}{2m_\varphi} \]

Vector mediator: Resonances

\[ \xi = m_{\text{DM}} \alpha / (2m_\varphi) \]
Vector mediator

ξ values near the $n = 2, \ell = 1$ resonance

ξ values near the $n = 2, \ell = 0$ resonance

Parameters:
\[ \zeta = \frac{\alpha}{v_{\text{rel}}} \]
\[ \xi = \frac{m_{\text{DM}}}{2m_{\varphi}} \alpha \]
Massive (vector or scalar) mediators

Take-home message

Combination of annihilation & BSF processes, different velocity dependence, and resonant structure

Rich phenomenology (indirect detection):

• Spectral features: High and low-energy radiation; correlated.
• Signal strength dependence on the velocity dispersion of the galaxy.
Conclusions

● Bound-state formation is the response of nature to long-range interactions, i.e. the way in which nature curtails their effect. Nevertheless, overall they imply rich(er) phenomenology.

● Connection between DM and electroweak physics
  ⇒ Fully compute the long-range properties of EW interactions. Implications for TeV-scale and sub-TeV WIMPs.