Solar and atmospheric neutrino oscillations in Super-Kamiokande

Yusuke Koshio
(Okayama university)
for Super-Kamiokande collaboration

Neutrino Oscillation Workshop 2016
5-10th of September, 2016, Otranto (Lecce, Italy)
Super-Kamiokande collaboration

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4 Boston University, USA
5 University of British Columbia, Canada
6 Brookhaven National Laboratory, USA
7 University of California, Irvine, USA
8 California State University, USA
9 Chonnam National University, Korea
10 Duke University, USA
11 Fukuoka Institute of Technology, Japan
12 Gifu University, Japan
13 GIST College, Korea
14 University of Hawaii, USA
15 Imperial College London, UK
16 KEK, Japan
17 Kobe University, Japan
18 Kyoto University, Japan
19 University of Liverpool, UK
20 Miyagi University of Education, Japan
21 ISEE, Nagoya University, Japan
22 National Center for NR, Poland
23 SUNY, Stony Brook, USA
24 Okayama University, Japan
25 Osaka University, Japan
26 University of Oxford, UK
27 Queen Mary University, UK
28 University of Regina, Canada
29 Seoul National University, Korea
30 University of Sheffield, UK
31 Shizuoka University of Welfare, Japan
32 Sungkyunkwan University, Korea
33 Tokai University, Japan
34 Tokyo Institute of Technology, Japan
35 University of Toronto, Canada
36 Kavli IPMU (WPI), University of Tokyo, Japan
37 University of Tokyo, Japan
38 TRIUMF, Canada
39 Tsinghua University, China

151 collaborators
39 institutions
8 countries

On 1st, Sep., 2016
Super-Kamiokande

50000 tons of Water Cherenkov detector

Kamioka mine

~1km

~3km

~2km

(2700 mwe)

Japan

<table>
<thead>
<tr>
<th>Phase</th>
<th>Period</th>
<th>Fiducial vol. (kton)</th>
<th># of PMTs</th>
<th>Energy thr. (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-I</td>
<td>1996.4 ~ 2001.7</td>
<td>22.5</td>
<td>11146 (40%)</td>
<td>4.5</td>
</tr>
<tr>
<td>SK-II</td>
<td>2002.10 ~ 2005.10</td>
<td>22.5 (&gt;5.5MeV)</td>
<td>5182 (20%)</td>
<td>6.5</td>
</tr>
<tr>
<td>SK-III</td>
<td>2006.7 ~ 2008.8</td>
<td>22.5 (&lt;5.5MeV)</td>
<td>11129 (40%)</td>
<td>4.5</td>
</tr>
<tr>
<td>SK-IV</td>
<td>2008.9 ~</td>
<td>22.5 (&gt;5.5MeV)</td>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.5 (4.5&lt;E&lt;5.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.9 (&lt;4.5MeV)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Running and improvements over 20 years!

Sep. 5-10, 2016

NOW2016
Super-Kamiokande

Multi-purpose detector
- Neutrino observation with wide energy range (from MeV to 100PeV)
  Solar, Supernova, Atmospheric, Accelerator, etc.
- Proton decay search
- Dark matter search etc.
Solar neutrinos
Solar neutrinos

**pp-chain**

\[
p + p \rightarrow ^2H + e^+ + \nu_e \\
p + e^- + p \rightarrow ^2H + \nu_e \\
^2H + p \rightarrow ^3He + \gamma \\
^3He + ^3He \rightarrow \alpha + 2p \\
^3He + \alpha \rightarrow ^7Be + \gamma \\
^7Be + ^7Be \rightarrow ^7Li + \nu_e \\
^7Li + p \rightarrow 2\alpha \\
^7Li + \alpha \rightarrow ^8B + \gamma \\
^8B \rightarrow ^8Be^* + e^+ + \nu_e \\
^8Be^* \rightarrow 2\alpha
\]

**CNO cycle**

\[
^12C \rightarrow ^13N + \gamma \\
^12C + p \rightarrow ^13N + e^+ + \nu_e \\
^13N \rightarrow ^13C + \gamma \\
^13C + p \rightarrow ^14N + e^+ + \nu_e \\
^14N \rightarrow ^14O + \gamma \\
^14O + p \rightarrow ^15O + e^+ + \nu_e \\
^15O \rightarrow ^15N + \gamma \\
^15N + p \rightarrow ^16O + e^+ + \nu_e \\
^16O \rightarrow ^16N + \gamma \\
^16N + p \rightarrow ^17N + e^+ + \nu_e \\
^17N \rightarrow ^17F + \gamma \\
^17N + p \rightarrow ^18F + e^+ + \nu_e \\
^18F \rightarrow ^17O + \gamma \\
^17O + p \rightarrow ^18O + e^+ + \nu_e \\
^18O \rightarrow ^18O + \gamma
\]

Solar neutrino energy spectrum

Serenelli, Haxton, Pena-Garay

**Super-K**

pp \(\pm 0.6\%\)

\(13\text{N} \pm 14\%\)

\(15\text{O} \pm 15\%\)

\(17\text{F} \pm 17\%\)

\(7\text{Be} \pm 7\%\)

\(7\text{Be} \pm 7\%\)

\(8\text{B} \pm 14\%\)

\(7\text{Li} \pm 14\%\)

\(8\text{Be} \pm 30\%\)
Solar neutrino observation in SK

Typical event

Super-Kamiokande

Run 1742 Event 102496
96-05-31 07:13:13
Inner: 199 hits, 123 PE
Outer: -1 hits, 0 PE (in-time)
Trigger ID: 0001
E = 9:086 GEN=0.77 COSINE= 0.949
Solar Neutrino

\[ \nu + e^- \rightarrow \nu + e^- \]

- Find solar direction
- Realtime measurements
  - day-night flux differences
  - seasonal variation
- Energy spectrum

Detector performance

<table>
<thead>
<tr>
<th></th>
<th>resolution (10 MeV)</th>
<th>information</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertex</td>
<td>55cm</td>
<td>hit timing</td>
</tr>
<tr>
<td>direction</td>
<td>23deg.</td>
<td>hit pattern</td>
</tr>
<tr>
<td>energy</td>
<td>14%</td>
<td># of hits.</td>
</tr>
</tbody>
</table>

\( E_e = 8.6 \text{ MeV (kin.)} \)
\( \cos \theta_{\text{sun}} = 0.95 \)

Sep. 5-10, 2016

NOW2016
Motivation of the measurement

See the neutrino oscillation MSW effect directly

Spectrum distortion

Day-Night flux asymmetry

Super-K can search for the spectrum “upturn” expected by neutrino oscillation MSW effect

Vacuum oscillation dominant

Matter oscillation dominant

Super-K

Regenerate $\nu_e$ by earth matter effect

Expected

(day-night)/((day+night)/2)

$\Delta m^2_{21}$ in eV$^2$

$0 < \sin^2(2\theta) < 1$

$0.1 < \sin^2(\theta) < 0.5$

Sep. 5-10, 2016

NOW2016
Recent updates

• Submitted to arXiv:1606.07538 (SK-IV 1664days)
• Lowering trigger threshold in May 2015. Efficiency in the lowest energy bin (3.5-4.0MeV kinetic) is ~99% from 84%. It makes systematic error of this energy bin reduced.
• Data set (SK-IV : 2365days, SK-I~IV : 5200days)
• Solar neutrino flux, Yearly flux to see any correlation with solar activity, Day/Night flux asymmetry, Energy spectrum, Oscillation analysis.
Observed solar neutrino signal

SK I ~ IV combined 5200 days

- Data
- Best fit
- Background

\[ { }^8\text{B} \text{ flux: } 2.355 \pm 0.033 \times 10^6 / \text{cm}^2 / \text{s} \]

Data/MC: \( 0.4486 \pm 0.0062 \)

Observation: 

- \( \sim 84k \) signal events are extracted

Consistent among different periods

Preliminary
Yearly neutrino flux

$^8$B flux vs sun spot

No correlation with 11 years solar activity is observed

$\chi^2 = 15.52/19$ (dof)

Prob. = 68.9%

Solar neutrino rate measurement in SK is fully consistent with a constant solar neutrino flux emitted by the Sun


Source: WDC-SILSO, Royal Observatory of Belgium, Brussels

Sep. 5-10, 2016

NOW2016
Day-Night flux differences

Assuming the expected time variation as a function of $\cos \theta_z$ like below, amplitude of $A_{DN}$ was fitted.

<table>
<thead>
<tr>
<th>Amplitude fit</th>
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</thead>
<tbody>
<tr>
<td>SK-I</td>
</tr>
<tr>
<td>-2.0$\pm$1.8$\pm$1.0%</td>
</tr>
<tr>
<td>SK-II</td>
</tr>
<tr>
<td>-4.4$\pm$3.8$\pm$1.0%</td>
</tr>
<tr>
<td>SK-III</td>
</tr>
<tr>
<td>-4.2$\pm$2.7$\pm$0.7%</td>
</tr>
<tr>
<td>SK-IV</td>
</tr>
<tr>
<td>-3.6$\pm$1.6$\pm$0.6%</td>
</tr>
</tbody>
</table>

combined: $-3.3\pm1.0\pm0.5\%$

non-zero significance: $3.0 \sigma$

$\Delta m_{21}^2=4.84\times10^{-5}$ eV$^2$

$\sin^2 \theta_{12}=0.311$

$\sin^2 \theta_{13}=0.025$

SK-I,II,III,IV best fit

SK-IV is 1664 days

arXiv 1606.07538

Recoil Electron Kinetic Energy (MeV)

Day/Night Asymmetry (%)

$\Delta m_{21} = \Delta m_{21}^2 = m_{2} - m_{1}$

$\sin^2 \theta_{12}$

$\sin^2 \theta_{13}$

$\cos \theta_z$
Recoil electron spectrum

SK-I Energy Spectrum

SK-II Energy Spectrum

SK-III Energy Spectrum

SK-IV Energy Spectrum
Recoil electron spectrum

All SK phase are combined without regard to energy resolution or systematics in this figure

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar+KamLAND</td>
<td>76.60</td>
</tr>
<tr>
<td>Solar global</td>
<td>73.86</td>
</tr>
<tr>
<td>quadratic fit</td>
<td>72.33</td>
</tr>
</tbody>
</table>

(total # of bins of SKI - IV is 83, dof 80)

MSW is slightly disfavored by $\sim2.0\sigma$ using the Solar+KamLAND best fit parameters,

(statistic error only)
Allowed survival probability

\[ P_{ee}(E_\nu) = c_0 + c_1 \left( \frac{E_\nu}{MeV} - 10 \right) + c_2 \left( \frac{E_\nu}{MeV} - 10 \right)^2 \]

- **Solar+KamLAND**
  - \( \sin^2 \theta_{12} = 0.308 \)
  - \( \Delta m^2_{21} = 7.49 \times 10^{-5}eV^2 \)

- **Solar**
  - \( \sin^2 \theta_{12} = 0.308 \)
  - \( \Delta m^2_{21} = 4.84 \times 10^{-5}eV^2 \)

- ✓ SK and SNO is complementary shape constraints
- ✓ MSW is consistent at 1 \( \sigma \)

Preliminary
Data set for global solar analysis

✓ SK:
- SK-I 1496 days, spectrum 4.5-19.5 MeV(kin.)+D/N: $E_{\text{kin}} > 4.5$ MeV
- SK-II 791 days, spectrum 6.5-19.5 MeV(kin.)+D/N: $E_{\text{kin}} > 7.0$ MeV
- SK-III 548 days, spectrum 4.0-19.5 MeV(kin.)+D/N: $E_{\text{kin}} > 4.5$ MeV
- SK-IV 2365 days, spectrum 3.5-19.5 MeV(kin.)+D/N (1664 days): $E_{\text{kin}} > 4.5$ MeV
✓ SNO:
- Parameterized analysis ($c_0$, $c_1$, $c_2$, $a_0$, $a_1$) of all SNO phased. (PRC88, 025501 (2013))
  (Note: the same method is applied to both SK and SNO with $a_0$ and $a_1$ to LMA expectation.)
✓ Radiochemical: Cl, Ga
✓ $^8$KamLAND reactor: Latest (3-flavor) analysis (PRD88, 3, 033001 (2013))
✓ $^8$B spectrum: Winter 2006 (PRC73, 73, 025503 (2006))
✓ B and hep flux free, if not mentioned.
SK I - IV combined

$\Delta m_{21}^2$ in $10^{-5}$ eV$^2$

- $\sin^2 \theta_{13} = 0.0219 \pm 0.0014$
- $8B$ flux is constraint by SNO NC data

- $\sqrt{2}$ SK favors LMA solution $> 3\sigma$
- $\sim 2\sigma$ tension with KamLAND in $\Delta m_{21}^2$

$\sin^2 \theta_{12}$

$\Delta m_{21}^2$

- Preliminary

$\sin^2 \theta_{12} = 0.316^{+0.034}_{-0.026}$
- $\Delta m_{21}^2 = 7.54^{+0.19}_{-0.18}$

$\sin^2 \theta_{12}$

$\Delta m_{21}^2$

- $\sin^2 \theta_{12} = 0.337^{+0.027}_{-0.023}$
- $\Delta m_{21}^2 = 4.74^{+1.40}_{-0.80}$

$\sin \theta_{12}$

$\Delta m_{21}^2$

- $\sin \theta_{12}^2 = 0.326^{+0.022}_{-0.019}$
- $\Delta m_{21}^2 = 7.50^{+0.19}_{-0.17}$

The unit of $\Delta m_{21}^2$ is $10^{-5}$ eV$^2$
Solar global

\[ \Delta m^2_{21} = 7.54^{+0.19}_{-0.18} \]
\[ \sin^2 \theta_{12} = 0.316^{+0.034}_{-0.026} \]

\[ \Delta m^2_{21} = 4.84^{+1.26}_{-0.60} \]
\[ \sin^2 \theta_{12} = 0.308 \pm 0.014 \]

The unit of \( \Delta m^2_{21} \) is \( 10^{-5} \text{ eV}^2 \)

\[ \sin^2 \theta_{13} = 0.0219 \pm 0.0014 \]
Atmospheric neutrino oscillations
Atmospheric neutrinos

Cosmic rays strike air nuclei and the decay of the out-going hadrons gives neutrinos.
✓ Flux measurement by SK (arXiv:1510.08127)
✓ Model calculation is consistent with data.

L=10~20 km

L~ up to 13000 km

Sep. 5-10, 2016
Atmospheric neutrino observation

Parent neutrino spectrum in SK

Event topologies

- **Fully-contained**
  - single ring
  - multi ring

- **e-like**
- **μ-like**

- **Partially-contained**
- **Upward-going muon**
  - stop
  - through

Average energies:
- **FC**: ~1 GeV
- **PC**: ~10 GeV
- **Upμ**: ~100 GeV
3 flavor neutrino oscillation analysis

- Consider all the sub-leading effects ($\Delta m^{2}_{21}$, matter)
- Matter effect on $\theta_{13}$ resolves mass hierarchy
- $\Delta m^2_{21}$ resolves octant $\theta_{23}$
- Interference is sensitive to $\delta_{CP}$

$\nu_e$ flux ratio with/w.o. oscillation

<table>
<thead>
<tr>
<th>For mass hierarchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutrino</td>
</tr>
<tr>
<td>normal</td>
</tr>
<tr>
<td>inverted</td>
</tr>
</tbody>
</table>

$\nu_e$ sample
- Multi-GeV e-like
- Multi-ring e-like

$\bar{\nu}_e$ sample
- Multi-GeV e-like
- Multi-ring e-like
- other

Statistically separation based on
- number of decay electrons
- number of rings
- transvers momentum

More $e$ for $\theta_{23}<\pi/2$

More $e$ for $\pi<\delta_{CP}<2\pi$
Recent updates

- Data set
  - SK-IV : 2519.9 days, SK-I~IV : 0.33Mtyr
  - PMT gain correction
- MC
  - Honda 11 (500yrs for each SK phase)
  - Neutrino interaction (NEUT)
  - Detector simulation
- Systematic error -> MC related, detector systematics
Standard: Analysis Data Update (SK-I+II+III+SK-IV [2519 days])

- Sub-GeV e-like 1-dcy e
- Sub-GeV e-like 0-dcy e
- Sub-GeV μ-like 0-dcy e
- Sub-GeV μ-like 1-dcy e
- Up Stop μ
- Sub-GeV π₀-like 1-R
- Multi-GeV e-like ν_e
- Multi-GeV e-like ν_e
- PC Stop
- Non-showering μ
- Sub-GeV μ-like 2-dcy e
- Multi-Ring e-like ν_e
- Multi-Ring e-like ν_e
- PC Thru
- Showering μ
- Sub-GeV π₀-like M-R
- Multi-GeV μ-like
- Multi-Ring μ-like
- Multi-Ring Unclassified

lepton momentum (MeV)  cos zenith  cos zenith  cos zenith  cos zenith  cos zenith
Parameter determination (only SK)

| Fit (517 dof) | $\chi^2$ | $\sin^2 \theta_{13}$ | $\delta_{CP}$ | $\sin^2 \theta_{23}$ | $|\Delta m^2_{32}|$ eV$^2$ |
|---------------|----------|----------------------|--------------|----------------------|-----------------|
| SK (IH)       | 576.08   | 0.0219 (fix)         | 4.189        | 0.575                | 2.5x10$^{-3}$   |
| SK (NH)       | 571.74   | 0.0219 (fix)         | 4.189        | 0.587                | 2.5x10$^{-3}$   |

$\Delta \chi^2 = \chi^2_{NH} - \chi^2_{IH} = -4.3$ (-3.1 expected)

✓ Probability for IH is 0.031 ($\sin^2 \theta_{23}=0.6$) and 0.007 ($\sin^2 \theta_{23}=0.4$), while for NH is 0.45 ($\sin^2 \theta_{23}=0.6$)
Mass hierarchy sensitivity sample

- Contribution to $\Delta \chi^2 = \chi^2_{\text{NH}} - \chi^2_{\text{iH}} = +0.42$
- $\nu_e$ multiplicity: $+0.35$
- $\bar{\nu}_e$ multiplicity: $-0.92$

**Diagram**

- Multi-GeV $\nu_e$ and $\bar{\nu}_e$
- Multi-Ring $\nu_e$ and $\bar{\nu}_e$
- Multi-Ring Other $\nu_e$ and $\bar{\nu}_e$
Parameter determination (SK+T2K)

| Fit (585 dof) | $\chi^2$  | $\sin^2 \theta_{13}$ | $\delta_{CP}$ | $\sin^2 \theta_{23}$ | $|\Delta m^2_{32}|$ eV$^2$ |
|---------------|----------|----------------------|---------------|-------------------|-------------------|
| SK+T2K (IH)   | 644.82   | 0.0219 (fix)         | 4.538         | 0.55              | 2.5x10^{-3}       |
| SK+T2K (NH)   | 639.61   | 0.0219 (fix)         | 4.887         | 0.55              | 2.4x10^{-3}       |

$\Delta \chi^2 = \chi^2_{NH} - \chi^2_{IH} = -5.2$

(-3.8 expected for SK best, -3.1 for combined best)

Probability for IH is 0.024 ($\sin^2 \theta_{23}=0.6$) and 0.001 ($\sin^2 \theta_{23}=0.4$), while for NH is 0.43 ($\sin^2 \theta_{23}=0.6$)
\( \tau \) appearance update

Hard to identify event by event but can be statistically seen

Multi-ring e-like sample

Search for events consistent with hadronic decays of tau lepton using neural network method

data=PDF(BG)+\( \alpha \times \text{PDF(} \tau \text{)}+\sum \varepsilon_i \times \text{PDF}_i \)

PDF of i-th sys. error shifting by 1\( \sigma \)

\( \tau \) fraction is found to be 1.42±0.32, which is 4.6\( \sigma \) from 0.

Assuming NH expected at 1 is 3.3\( \sigma \)
MH sensitivity using $\tau$

In the resonance region, $\nu_\tau$ CC causes BG with a large uncertainty (~25% for xsec). A better sensitivity on mass hierarchy using $\tau$ neural net with optimized binning is expected.
Sterile neutrino searches at SK are independent of the $\Delta m^2$ and the number of sterile neutrinos.
Other topics
Search for GW coincident event

- Submitted to arXiv 1608.08745.
- ±500sec time window for solar, supernova and atmospheric neutrino sample. (3.5MeV ~100PeV)
- Four candidates for GW150914 and no candidate for GW151226 are found. Remaining events are consistent with background expectation.
- Fluence limits are calculated from these results.
Super-K Gd

Inverse beta decay

\[ \nu_e \rightarrow p + e^+ \gamma (2.2\text{MeV}) \]

\[ \approx 8\text{MeV} \]

Delayed coincidence

Dissolve Gadolinium into Super-K

J. Beacom and M. Vagins,

First observation of neutrinos emitted from past supernovae
Super-K Gd


On June 27, 2015, the Super-Kamiokande collaboration approved the SK-Gd project which will enhance neutrino detectability by dissolving gadolinium in the Super-K water. T2K and SK will jointly develop a protocol to make the decision about when to trigger the SK-Gd project, taking into account the needs of both experiments, including preparation for the refurbishment of the SK tank and readiness of the SK-Gd project, and the T2K schedule including the J-PARC MR power upgrade. Given the currently anticipated schedules, the expected time of the refurbishment is 2018.
Summary

• Solar neutrinos
  • $\Delta m^2$ from SK spectrum and D/N data has more 2$\sigma$ tension with KamLAND data.
  • Yearly plot of $^8$B neutrino flux observed in SK is consistent with constant.

• Atmospheric neutrinos
  • Mass hierarchy: preference to NH
    $$\Delta \chi^2 = \chi^2(\text{NH}) - \chi^2(\text{IH}) = -4.3(\text{SK}), -5.2(\text{SK+T2K})$$
  • Tau neutrino appearance: significance of signal 4.6$\sigma$

SK keep running with continuous improvements to obtain more and better data, and hope new physics in future!
Back up
Spectrum predicted by non-standard models

**Sterile neutrino**


**Non-standard interaction**


**MaVaN**


**Unparticle**

Non-standard interactions

Modifying the Hamiltonian in the matter potential

$$H_{\text{mat}} = \sqrt{2} G_F n_e \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu}^* & \epsilon_{e\tau}^* \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}^* \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}$$

NSI parameterization

$$\epsilon_{11}^f = \epsilon_{ee}^f - \epsilon_{\tau\tau}^f \sin^2 \theta_{23} \quad \epsilon_{12}^f = -2 \epsilon_{e\tau}^f \sin \theta_{23} \quad \epsilon_{1j}^f = \sum_{f=\mu,\tau} \epsilon_{1j}^f n_f/n_e \quad (\text{where } j=1,2)$$

Apply NSI interactions of solar $\nu$ with d-quake in matter

SK NSI CL bands (90\%, 95\%, 99\%, 3$\sigma$) w/ 2 d.o.f.

SK+SN0 NSI CL bands (90\%, 95\%, 99\%, 3$\sigma$) w/ 2 d.o.f.

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NOW2016
Sterile neutrino oscillations

Hydrogen earth approximation
Turn off the sterile matter effect

Exclude region
Test of Lorentz Invariance

Adding LV term, $H_{\text{LV}}$, to the neutrino Hamiltonian

$$H = UMU^\dagger + V_e + H_{\text{LV}},$$

$$\begin{pmatrix}
0 & a_{e\mu}^T & a_{e\tau}^T \\
(a_{e\mu}^T)^* & 0 & a_{\mu\tau}^T \\
(a_{e\tau}^T)^* & (a_{\mu\tau}^T)^* & 0
\end{pmatrix} - \frac{4E}{3} \begin{pmatrix}
0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\
(c_{e\mu}^{TT})^* & 0 & c_{\mu\tau}^{TT} \\
(c_{e\tau}^{TT})^* & (c_{\mu\tau}^{TT})^* & 0
\end{pmatrix}$$

$a^{\tau\alpha \beta} : \text{dim}=3$ induces oscillation effects~L
$c^{TT \alpha \beta} : \text{dim}=4$ induces oscillation effects~LxE
Test of Lorentz Invariance

Constraints on Lorentz Invariance Violating Oscillations at 95% C.L.

- No indication of Lorentz invariance violation.
  - Limits placed on the real and imaginary parts of 6 parameters < O(10^{-23})
  - Established new limits in the \( \mu \tau \) sector for both \( a^T_{\alpha \beta} \) and \( c^{TT}_{\alpha \beta} \) coefficients.
  - Improvements on existing limits between 3 and 7 orders of magnitude.