where are neutrino mass-mixing parameters leading us?

Otranto, September 5th 2016

NOW 2016
Neutrino Oscillation Workshop

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GUTs: the open sea (once called the big desert)

RGE flow

flavour symmetries

radiative neutrino masses
starting point

[Marone, Neutrino 2016]

$\Delta \chi^2_{\text{IO-NO}} = 3.1$

CP-conservation disfavored at $\geq 2\sigma$

Normal Ordering slightly preferred

missing pieces

Dirac or Majorana?

mass ordering

$\Sigma_i m_i$ absolute mass scale

$\delta$ Dirac phase

$\alpha \beta$ Majorana phases
radiative neutrino masses
Specific particle content and/or symmetries can force the Weinberg operator to arise at \( L \geq 1 \) loop order.

\[
L_5 = \frac{1}{\Lambda} (\varphi^+ l)^T W (\varphi^+ l) + h.c.
\]

Topologies classified

- [1-loop: Bonnet, Hirsch, Ota, Winter 1204.5862]
- [2-loop: Sierra, Degee, Dorame, Hirsch 1411.7038]

At least 2 new multiplets required as intermediate states (or additional non-renormalizable operators)

[Geib, King, Merle, No, Panizzi 1512.04391]

Main motivation

Neutrino masses suppressed by loop factors: intermediate states can be light and probed at existing facilities (cfr. \( \nu_R \) in seesaw at the GUT scale)

Neutrino physics directly accessible at
- high-energy colliders (direct production of new particles \( 100 \text{ GeV} \leq m \leq 1 \text{ TeV} \))
- high-intensity facilities (LFV, \( \mu \rightarrow e\gamma \) at 1-loop, \( \mu \rightarrow 3e \) at tree-level,....)

Very interesting physics (it would deserve a dedicated review)
Also DM candidates among the new particles (with discrete symmetries)
price to pay

large variety of possible realizations

many independent topologies: $4 \text{ (1-loop)} + 20 \text{ (2-loop)} + \ldots$

for each topology, several different choices of intermediate states

large ($\infty$) number of models

uniqueness of tree-level seesaw lost

many new independent parameters

- tuning of parameters needed:
  - to cope with LFV
  - to reproduce $(m_i, \theta_{ij}, \ldots)$

flavour problem amplified

no insight about the origin of lepton mixing

connection to charged fermions/GUT weakened

embedding of intermediate states in GUT requires large representations

\[ k^{++} \subset 50 \text{ [SU(5)]} \subset \overline{126} \text{ [SO(10)]} \]

gauge coupling unification possible, but with ad-hoc particle content

[Hagedorn, Ohlsson, Riad, Schmidt 1605.03986]
lepton mixing from RGE flow
under appropriate conditions:
low-energy value $\alpha(Q) = \alpha^*$
independent from
the initial conditions $\alpha(\Lambda)$

more parameters ($\alpha, \beta, \ldots$)
running at the same time:

fixed point relation

$$f(\alpha^*, \beta^*, \ldots) = 0$$
starting point

\[ L_5 = \frac{1}{\Lambda} (\varphi^+ l)^T w(\varphi^+ l) + h.c. \]

at some large scale \( \Lambda \gg \text{e.w. scale} \)
e.g. from see-saw

what matters is the speed along the RGE trajectories. For \( U_{PMNS} \)

\[
\frac{m_i + m_j}{m_i - m_j} \times \frac{\eta}{16\pi^2} y_\tau^2 \times \log \frac{\Lambda}{Q} \approx \frac{m_i + m_j}{m_i - m_j} \times 10^{-5} \tan^2 \beta
\]

\[
(\Lambda = 10^{10} \, \text{GeV} \quad Q = 10^2 \, \text{GeV})
\]

sufficient speed requires strong degeneracy at \( Q = \Lambda \)

fixed point relations

\[
\sin^2 2\theta_{12} = s_{13}^2 \frac{\sin^2 2\theta_{23}}{s_{23}^2 + s_{13}^2 c_{13}^2}
\]

\[ \text{CP conserving case} \quad [\text{Chankowski, Krolikowki, Pokorski 2000}] \]

(0.75 + 0.92) [3\sigma]

(0.05 + 0.16) [3\sigma]

does not work

\[ \text{CP violating case} \quad [\text{Casas, Espinosa, Ibarra, Navarro 9910420}] \]

the only acceptable case is Inverted Hierarchy
with \( m_1 \approx m_2 \) at the scale \( \Lambda \)

\[ \text{Re}(U_{31}^{*} U_{32}) = 0 \]
fixed point relation reads

\[ s_{12}c_{12}(c_{23}s_{13} - s_{23})(c_{23}s_{13} + s_{23})\cos\alpha / 2 + \\
\; s_{13}s_{23}c_{23}(c_{12}^2 \cos(\alpha / 2 - \delta) - s_{12}^2 \cos(\alpha / 2 + \delta)) = 0 \]

phase convention for Majorana phases:

\[ |m_{ee}| = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3 \right|^2 \]

represents a constraint in (\delta, \alpha) plane

impact on 0\nu\beta\beta decay

predictions

- large scale \Lambda
- Inverted Hierarchy
- \pi \leq \delta \leq 2\pi correlated with \alpha \approx -150^0
- \Sigma_i m_i > 0.1 \text{ eV (degeneracy)}
- |m_{ee}| close to present bounds

but

- \( \vartheta \)\text{ }_{23} \text{ }_{13} \text{ do not appreciably run in this regime}
- no relation to quark sector
- can be altered by threshold corrections
GUTs: the open sea
(once called the big desert)
almost no symmetry

no striking hierarchies among lepton mixing angles or neutrino masses

Higgs multiplets

[Hall, Murayama, Weiner 1999, De Gouvea, Murayama 1204.1249
Brdar, König, Kopp 1511.0637]

Anarchy

$m_\nu = \begin{pmatrix}
\text{mixing angles and mass ratios from random } O(1) \\
\text{quantities: consistent with data}
\end{pmatrix}$

$|U_{PMNS}| \approx \begin{pmatrix}
0.8 & 0.5 & 0.2 \\
0.4 & 0.6 & 0.6 \\
0.4 & 0.6 & 0.8
\end{pmatrix}$

can we adopt this principle for quarks and charged leptons?

1. all Yukawa’s are described by anarchical matrices with $O(1)$ matrix elements

2. minimal $SU(5)$ field content: 3 copies of

fermion masses from

$10 = (q, u^c, e^c) \quad \bar{5} = (l, d^c)$  

$L_Y = 10 y_u 10 \varphi + \bar{5} y_d 10 \varphi + \frac{1}{M} \bar{5} w \bar{5} \varphi \varphi$  

Higgs multiplets
the observed hierarchies are generated by a rescaling

\[ F_{10} \rightarrow F_{\tilde{5}} \]
\[ \bar{5} \rightarrow F_{\tilde{5}} \bar{5} \]

\[ F_x = \begin{pmatrix} \varepsilon' \ 0 \ 0 \\ 0 \ \varepsilon_x \ 0 \\ 0 \ 0 \ 1 \end{pmatrix} = \begin{pmatrix} \end{pmatrix} \]

\[ 1 \geq \varepsilon_x \geq \varepsilon_x' \]

 hierarchy in up-quark sector is stronger than in the down-quark one:

\[ F_{\tilde{5}} = \begin{pmatrix} \end{pmatrix} \]

F\(_x\) can arise from:
- U(1)\(_{FN}\) symmetries,
- a 5\(^{th}\) Extra Dimension,
- Partial Compositness,
- conformal dynamics

m\(_u\) : m\(_c\) : m\(_t\) \approx m\(_d^2\) : m\(_s^2\) : m\(_b^2\) \approx m\(_e^2\) : m\(_\mu^2\) : m\(_\tau^2\) approximately true
\[ Y_e = F_{10} y_d^T F_5 = Y_d^T \]

\[ Y_d \text{ and } Y_e \text{ lopsided} \]

\[ F_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ m_v = \frac{\nu^2}{M} F_5 w F_5 \propto \]

\[ V_{ub} \approx V_{us} \times V_{cb} \]

[Hagiwara, Okamura '98; Berezhiani, Rossi '98; Altarelli, F. '98]

anarchy is just the special case

anarchy:
-- no preferred mass ordering
-- smallness of small parameters due to chance

worth to explore other possibilities beyond anarchy

\[ F_5 = \begin{pmatrix} \lambda^{Q_1} & 0 & 0 \\ 0 & \lambda^{Q_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ \begin{array}{|c|c|c|}
\hline
\text{sin}^2 \theta_{13} & \Delta m_{21}^2 / |\Delta m_{31}^2| \\
\hline
\text{NH} & 0.0214^{+0.0011}_{-0.0009} & 0.0295 \pm 0.0008 \\
\text{IH} & 0.0218^{+0.0009}_{-0.0012} & 0.0300 \pm 0.0009 \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|}
\hline
(Q_1, Q_2) & \lambda \\
\hline
A & (0,0) \\
A_{\mu\tau} & (1,0), 0.25 \\
PA_{\mu\tau} & (2,0), 0.35 \\
H & (2,1), 0.45 \\
\hline
\end{array} \]
Normal Hierarchy favored in non-anarchical examples

$\sin^2 \theta_{13} \approx \Delta m^2_{12} / \Delta m^2_{13}$

typical

[Buchmuller, Domcke, Schmitz, 1111.387;
Altarelli,F, Masina, Merlo 1207.0587;
Bergstrom, Meloni, Merlo, 1403.4528]

Impressive
-- minimal amount of symmetry required
-- reproduces qualitative features of both quark and lepton masses and mixings
-- explains why $U_{PMNS} \neq V_{CKM}$
-- compatible with GUT
-- compatible with leptogenesis
-- incorporates anarchy as a special case

Limits:
-- maximal $\theta_{23}$ unexplained
-- Dirac phase unpredicted
-- large number of independent $O(1)$ parameters: impossible to go beyond order-of-magnitude predictions

accurate existing data need precise predictions
flavour symmetries
can we do better?

perhaps some feature of lepton mixing is not accidental

\[ \theta_{23} \text{ nearly maximal?} \]

\[ \delta_{CP} = -\pi/2 ? \]

quark-lepton complementarity?

\[ \theta_{12} + \theta_{12}^q = \frac{\pi}{4} \leftrightarrow (1.023 \pm 0.023) \frac{\pi}{4} \]

[Smirnov; Raidal; Minakata and Smirnov 2004]

\[ U_{PMNS} \text{ close to TB, BM,...?} \]

\[ U_{PMNS} = U_{PMNS}^0 + \text{corrections} \]

\[ |U_{PMNS}|^2 = |U_{TB}|^2 = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} + \ldots \]

[Harrison, Perkins, Scott 0202074]

\[ |U_{PMNS}|^2 = |U_{BM}|^2 = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} + \ldots \]

[Marrone, Neutrino 2016]
discrete flavor symmetries showed very efficient to reproduce $U_{TB}$, $U_{BM}$,…

but also indirect: [King, Merle, Morisi, Shimizu Tanimoto 1402.4271]

3x3 matrix space

$U_ν$ diagonal matrices

$U_e (m_e^+ m_e)$

direct models 4 predictions

$\delta^0 \pmod{\pi}$

neutrino masses: fitted, not predicted

for simplest pattern such as $TB, BM,…$

required groups are small: $A_4, S_4$

but corrections are needed

corrections “by hand”

BM corrected from charged lepton sector

$U^0_{PMNS} = U_{BM} \rightarrow R_{12}(\alpha)U_{BM}$

sum rules

$\sin^2 \theta_{12} = \frac{1}{2} + \sin \theta_{13} \cos \delta_{CP} + O(\sin^2 \theta_{13})$

$\sin^2 \theta_{23} = \frac{1}{2} + O(\sin^2 \theta_{13})$

more freedom if also $R_{13}(\beta)$ is allowed [Altarelli, Machado, Meloni 1504.05514]
corrections by scanning discrete groups

complete classification of $|U_{PMNS}|$ for Majorana $\nu$ from any finite group available now!

[Fonseca, Grimus 1405.3678]

- patterns compatible with data: TriMaximal
- if mixing angles close to data then: $\delta_{CP}$ trivial
- large groups required, for example $\Delta(486)$

$U_{PMNS}^0 = U_{TB} \rightarrow U_{TB} R_{23}(\alpha)$

$U_{PMNS}^0 = U_{TB} \rightarrow U_{TB} R_{13}(\alpha)$

TM mixing leads to other sum rules

combining CP and discrete flavour symmetries

mixing angles and CP violating phases

$(\vartheta_{12}^0, \vartheta_{23}^0, \vartheta_{13}^0, \delta^0, \alpha^0, \beta^0)$

predicted in terms of a single real parameter $0 \leq \vartheta \leq \pi$

$G_f=\Delta(384)$

<table>
<thead>
<tr>
<th>$\sin^2 \theta_{13}$</th>
<th>$\sin^2 \theta_{12}$</th>
<th>$\sin^2 \theta_{23}$</th>
<th>$\sin \delta$</th>
<th>$\sin \alpha = \sin \beta$</th>
</tr>
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<tbody>
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Lam 1208.5527 and 1301.1736
Holthausen, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308
King, Neder 1403.1758
Ishimori, King, Okada, Tanimoto 1411.5845
Yao, Ding 1505.03798]

[F. F., C. Hagedorn and R. Ziegler 1211.5560, 1303.7178
Ding, King, Luhn, Stuart 1303.6180
Ding, King, Stuart 1307.4212
Fonseca, Grimus 1405.3678]

[Hagedorn, Meroni, Molinaro 1408.7118]
flavour symmetries are a useful tool in our quest of the origin of \((m_i, \theta_{ij})\) but no compelling and unique picture have emerged so far. Present data can be described within widely different frameworks [despite the constant, impressive progress on the experimental side]

simple schemes with a minimal amount of structure can well reproduce the main features of \((m_i, \theta_{ij})\) in both quark and lepton sectors also in a GUT framework
main drawbacks: -- no precise questions/no precision tests allowed [e.g. maximal \(\theta_{23}\) unexplained]
-- more structure needed to suppress FCNC and CPV if there is new physics at the TeV scale

some special features [\(\theta_{23}\) maximal, \(\delta_{CP} = -\pi/2\), \(U_{PMNS} \approx TB, BM,\ldots\)] can survive experimental refinements and guide us in the search of first principles ruling the flavour sector
back-up slides
available tools to predict masses and mixing angles

in Quantum Field Theory

assumed throughout this talk:
3 light active Majorana neutrinos [i.e. no sterile neutrinos, and $\Delta(B-L) = 2$]

$(m_i, \theta_{ij}, \ldots)$ are basic parameters of the theory that cannot be predicted unless they vanish in the Classical Lagrangian [with one exception $\rightarrow 3.$]

1. quantum corrections removing accidental zeros of the classical Lagrangian
[due to specific particle content and renormalizability]

$m_i = \frac{\mu}{16\pi^2} f_i(g) + \ldots$

radiative fermion masses

2. symmetries requiring relations among $(m_i, \theta_{ij}, \ldots)$

exact symmetries

example: $U(1)_{em} \rightarrow m_\gamma = 0$

do not apply to fermion masses and mixing angles. Largest (non-abelian) global symmetry of quark sector - $G_{\text{MFV}} = SU(3)^3$ - completely broken by $(m_i, \theta_{ij}, \ldots)$

no hint for exact symmetries in the lepton sector with Majorana neutrinos
approximate symmetries

\[ SU(2)_{\text{isospin}} \rightarrow m_n - m_p \approx 0 \]

\[ SU(2)_L \times U(1)_Y \rightarrow \begin{cases} m_W \propto g \langle v \rangle \\ m_Z \propto \sqrt{g^2 + g'^2} \langle v \rangle \end{cases} \]

huge number of models of \((m_i, \theta_{ij}, \ldots)\):
- abelian and non-abelian symmetries
- continuous and discrete symmetries
- explicitly or spontaneously broken

renormalization group flow driving \((m_i, \theta_{ij}, \ldots)\) to special low-energy values

under appropriate conditions:
low-energy value \(\alpha(Q) = \alpha^*\)
independent from the initial conditions \(\alpha(\Lambda)\)

(Infrared Stable) Fixed Point of RGE flow

more parameters \((\alpha, \beta, \ldots)\) running at the same time:

fixed point relation
\[ f(\alpha^*, \beta^*, \ldots) = 0 \]
flavour symmetries

two examples
Explanation of leptonic flavour mixing is a part of a more general problem, the "flavour puzzle"

Some open questions:

- Smallness of neutrino masses
- Lepton mixing angles versus quark mixing angles
- Hierarchy of charged fermion masses

ν sector very special: the only one where predictions are still possible

- Mass ordering
- $\Sigma_i m_i$ absolute mass scale
- $\delta$ Dirac phase
- $\alpha \beta$ Majorana phases
corrections by scanning discrete groups

complete classification of $|U_{PMNS}|$ for Majorana $\nu$ from any finite group available now!

[Fonseca, Grimus 1405.3678]

-- patterns compatible with data: only TriMaximal
-- if mixing angles close to data then: $\delta_{CP}$ trivial
-- large groups required, for example $\Delta(486)$
-- larger groups - e.g. $\Delta(1176)$ - can determine the Cabibbo angle as well, but not the other quark mixing angles.

combining $CP$ and discrete flavour symmetries

Next talk!

In summary
- discrete groups produce testable predictions but
- evidence for discrete symmetries in the quark sector is still very poor
- unified - lepton&quarks - descriptions typically badly broken in the quark sector
- approach too much centered on mixing/CP properties, masses take second place

F.F., C. Hagedorn, R. de A. Toroop  
Lam 1208.5527 and 1301.1736  
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Yao, Ding 1505.03798
Trimaximal [TM] mixing from TB

\[ U_{PMNS}^0 = U_{TB} \rightarrow U_{TB} R_{23}(\alpha) \]

leading to testable sum rules

\[
\sin^2 \theta_{12} = \frac{1}{3} - \frac{2}{3} \sin^2 \theta_{13} + O(\sin^4 \theta_{13})
\]

\[
\sin^2 \theta_{23} = \frac{1}{2} - \sqrt{2} \sin \theta_{13} \cos \delta_{CP} + O(\sin^2 \theta_{13})
\]

deviation from TB is linear in \( \alpha \) for \( \sin^2 \theta_{23} \), whereas is quadratic for \( \sin^2 \theta_{12} \), the best measured angle

sum rules can be tested by measuring \( \delta_{CP} \) and improving on \( \sin^2 \theta_{23} \)