Neutrinoless double-beta decay: Theory challenges

Fedor Šimkovic
I. Introduction
   Majorana, Pontecorvo, Weinberg

II. The $0\nu\beta\beta$-decay scenarios due neutrinos exchange
   (simpliest, sterile $\nu$, LR-symmetric model, interpolating formula)

III. DBD NMEs – Current status
   (deformed QRPA versus ISM, …)

IV. Is there a proportionality between $0\nu\beta\beta$- and $2\nu\beta\beta$-decay NMEs?
   (role of SU(4) symmetry …)

V. New modes of the double-beta decay with emission of a single electron from an atom

VI. Conclusion

Acknowledgements: A. Faesler (Tuebingen), P. Vogel (Caltech), S. Kovalenko (Valparaiso U.), M. Krivoruchenko (ITEP Moscow), D. Štefánik, R. Dvornický (Comenius U.), A. Babič, A. Smetana, J. Terasaki (IEAP CTU Prague), …
I. Introduction
Majorana fermion

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Sunto. - Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell’elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di Dirac ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di «antiparticelle» corrispondenti ai «vuoti» di energia negativa.

L’interpretazione dei cosiddetti «stati di energia negativa» proposta da Dirac (1) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici sopraeritti per darvi alla teoria una forma simmetrica che si accordi sia perché si, perché la sintesi procedimentale rimane difficile, devono concludere più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

ν ↔ ν̄ oscillation (neutrinos are Majorana particles)


It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν₁ and ν₂ of different combined parity.⁵
After 62 years we know

**Fundamental \( \nu \) properties**

- 3 families of light (V-A) neutrinos: \( \nu_e, \nu_\mu, \nu_\tau \)
- \( \nu \) are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

No answer yet

- Are \( \nu \) Dirac or Majorana?
- Is there a CP violation in \( \nu \) sector?
- Are neutrinos stable?
- What is the magnetic moment of \( \nu \)?
- Sterile neutrinos?
- Statistical properties of \( \nu \)? Fermionic or partly bosonic?

Currently main issue

*Nature, Mass hierarchy, CP-properties, sterile \( \nu \)*

The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties.
Beyond the Standard model physics

(EFT scenario)

The absence of the right-handed neutrino fields in the SM is the simplest, most economical possibility. In such a scenario Majorana mass term is the only possibility for neutrinos to be massive and mixed. This mass term is generated by the Lepton number violating Weinberg effective Lagrangian.

\[
\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)
\]
Weinberg, 1979: d=5

\[ \mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H) \]

Weinberg does not take credit for predicting neutrino masses, but he thinks it’s the right interpretation. What’s more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven’t been observed, such as violation of baryon-number conservations. “We don’t know anything about the details of those terms, but I’ll swear they are there.”
Amplitude for \((A,Z) \rightarrow (A,Z+2) + 2e^-\) can be divided into:

- Mass mechanism: \(d=5\)
- Long range: \(d=7\)
- Short range: \(d=9\) (\(d=11\))

\[
\mathcal{O}_W \propto \frac{c_{i,j}}{\Lambda} (L_i H)(L_j H)
\]

Weinberg, 1979

\[
\begin{align*}
\mathcal{O}_2 &\propto LLLe^c H \\
\mathcal{O}_3 &\propto LLQd^c H \\
\mathcal{O}_4 &\propto LL\bar{Q}u^c H \\
\mathcal{O}_5 &\propto LLQd^c HHH^\dagger \\
\mathcal{O}_6 &\propto LL\bar{Q}u^c HH^\dagger H \\
\mathcal{O}_7 &\propto LQ\bar{e}^c \bar{Q}HHH^\dagger \\
\mathcal{O}_8 &\propto L\bar{e}^c \bar{u}^c d^c H \\
\mathcal{O}_9 &\propto LLLe^c Le^c \\
\mathcal{O}_{10} &\propto LLLe^c Qd^c \\
\mathcal{O}_{11} &\propto LLQd^c Qd^c \\
\ldots \ldots
\end{align*}
\]

Babu, Leung: 2001

de Gouvea, Jenkins: 2007

Physics at LHC (Jose Valle talk)
II. Different $0\nu\beta\beta$-decay scenarios

If $0\nu\beta\beta$ is observed the $\nu$ is a Majorana particle

Can we say something about content of the black box?

Considering
i. Sterile $\nu$
ii. Different LNV scales
iii. Right-handed currents
iv. Non-standard $\nu$-interactions
The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

\[ \mathcal{L}_{5}^{\text{eff}} = -\frac{1}{\Lambda} \sum_{l_1 l_2} \left( \bar{\Psi}_{l_1 L} \tilde{\Phi} \right) \dot{Y}_{l_1 l_2} \left( \tilde{\Phi}^T \left( \Psi_{l_2 L}^\text{lep} \right)^c \right) \]

Heavy Majorana leptons $N_i$ ($N_i = N_{c_i}$) singlet of $SU(2)_L \times U(1)_Y$ group

Yukawa lepton number violating int.

$\Lambda \geq 10^{15}$ GeV

\[ m_i = \frac{v}{\Lambda} (y_i v), \quad i = 1, 2, 3 \]


The discovery of the $\beta\beta$-decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.
\[(A,Z) \to (A,Z+2) + e^- + e^- \]

\[
(T_{1/2}^{0\nu})^{-1} = \left( \frac{m_{\beta\beta}}{m_e} \right)^2 g_A^4 |M_\nu^{0\nu}|^2 G^{0\nu}
\]

| transition         | \(G^{01}(E_0, Z) \times 10^{14} \text{y} \) | \(Q_{\beta\beta} \) [MeV] | Abund. (%) | \(|M^{0\nu}_\nu|^2 \) |
|---------------------|---------------------------------------------|-------------------------|------------|-----------------|
| \(^{150}\text{Nd} \to ^{150}\text{Sm}\) | 26.9                                       | 3.667                   | 6          | ?               |
| \(^{48}\text{Ca} \to ^{48}\text{Ti}\)   | 8.04                                       | 4.271                   | 0.2        | ?               |
| \(^{96}\text{Zr} \to ^{96}\text{Mo}\)   | 7.37                                       | 3.350                   | 3          | ?               |
| \(^{116}\text{Cd} \to ^{116}\text{Sn}\) | 6.24                                       | 2.802                   | 7          | ?               |
| \(^{136}\text{Xe} \to ^{136}\text{Ba}\) | 5.92                                       | 2.479                   | 9          | ?               |
| \(^{100}\text{Mo} \to ^{100}\text{Ru}\) | 5.74                                       | 3.034                   | 10         | ?               |
| \(^{130}\text{Te} \to ^{130}\text{Xe}\) | 5.55                                       | 2.533                   | 34         | ?               |
| \(^{82}\text{Se} \to ^{82}\text{Kr}\)  | 3.53                                       | 2.995                   | 9          | ?               |
| \(^{76}\text{Ge} \to ^{76}\text{Se}\)  | 0.79                                       | 2.040                   | 8          | ?               |

The NMEs for 0\nu\beta\beta-decay must be evaluated using tools of nuclear theory
**Complementarity of $0\nu\beta\beta$-decay, $\beta$-decay and cosmology**

**$\beta$-decay** (Mainz, Troitsk)

\[
m^2_\beta = \sum_i |U^L_{ei}|^2 m^2_i \leq (2.2 \text{ eV})^2
\]

KATRIN: $(0.2 \text{ eV})^2$

**Cosmology (Planck)**

\[\Sigma < 110 \text{ meV}\]

\[m_0 > 26 \text{ meV (NS)}\]

87 meV (IS)

**Effective mass of Majorana neutrinos**

**GUT’s**

$m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$

(3 unknown parameters)
0νββ - half lives for NH and IH with included uncertainties in NMEe

**NH:**
\[ m_1 \ll m_2 \ll m_3 \quad m_3 \approx \sqrt{\Delta m^2} \]
\[ m_1 \ll \sqrt{\delta m^2} \quad m_2 \approx \sqrt{\delta m^2} \]
1.4 meV \leq m_{\beta\beta} \leq 3.6 \text{ meV}

**IH:**
\[ m_3 \ll m_1 < m_2 \quad m_1 \approx m_2 \approx \sqrt{\Delta m^2} \]
\[ m_3 \ll \sqrt{\Delta m^2} \]
20 meV \leq m_{\beta\beta} \leq 49 \text{ meV}

Lightest ν-mass equal to zero
Nuclear medium effect on the light neutrino mass exchange mechanism of the $0\nu\beta\beta$-decay


Low energy 4-fermion $\Delta L \neq 0$ Lagrangian

\[ L_{\text{eff}} = \frac{g^2}{m^2} \sum_A (\bar{q} O_A q)(\bar{\nu} O'_A \nu), \quad m_\chi > M_W. \]

Oscillation experiments, tritium $\beta$-decay, cosmology

$0\nu\beta\beta$-decay

$\sum_{\nu}^{\text{vac}} = -\chi$, 

$\sum_{\nu}^{\text{medium}} = -\chi +$
Mean field: \[ \bar{q}q \rightarrow \langle \bar{q}q \rangle \quad \text{and} \quad \langle \bar{q}q \rangle \approx 0.5 \langle q^\dagger q \rangle \approx 0.25 \text{ fm}^{-3} \]

The effect depends on \[ \langle \chi \rangle = -\frac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle \]

A comparison with \( G_F \): \[ \frac{g_\chi g_{ij}^a}{m_\chi^2} = \frac{G_F}{\sqrt{2}} \varepsilon_{ij}^a \]

Typical scale: \[ \langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25 \varepsilon_{ij}^a \text{ eV} \]

We expect: \[ 25 \varepsilon_{ij}^a < 1 \rightarrow m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1 \text{ TeV}^2 \]

Universal scalar interaction: \[ g_{ij}^a = \delta_{ij} g_a, \quad \varepsilon_{ij}^a = \delta_{ij} \varepsilon_a \]

In medium effective Majorana \( \nu \) mass:

\[
m_{\beta\beta} = \sum_{i=1}^{n} U_{ei}^2 \xi_i \sqrt{\left(m_i + \langle \chi \rangle g_1 \right)^2 + \left(\langle \chi \rangle g_2 \right)^2} \frac{\sqrt{\left(m_i + \langle \chi \rangle g_1 \right)^2 + \left(\langle \chi \rangle g_2 \right)^2}}{(1 - \langle \chi \rangle g_4)^2}.
\]
Complementarity between $\beta$-decay, $0\nu\beta\beta$ –decay and cosmological measurements might be spoiled.

Area $\langle \chi \rangle g_1$ [eV]

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II.b. *The sterile $\nu$ mechanism of the $0\nu\beta\beta$-decay*  
*$(D\text{-}M$ mass term, $V\text{-}A,SM$ int.)*  
*Interpolating formula*

**Dirac-Majorana mass term**

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha} \nu_\alpha$$

**Mixing of active-sterile neutrinos**

Small $\nu$ masses due to see-saw mechanism

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$

Light $\nu$ mass $\approx \left(\frac{m_D}{m_{LNV}}\right) m_D$

Heavy $\nu$ mass $\approx m_{LNV}$

**Neutrinos masses offer a great opportunity to jump beyond the EW framework via see-saw ...**

**Different motivations for the $LNV$ scale $\Lambda$**

Talk of Carlo Giunti

- **eV** light sterile $\nu$  
  $10^{-6}$ GeV

- **keV** hot DM  
  $10^{-6}$ GeV

- **Fermi** or **Si**  
  $10^{-6}$ GeV or $10^3$ GeV

- **TeV** LHC  
  $10^3$ GeV

- **GUT**  
  $10^{16}$ GeV

- **Planck**  
  $10^{19}$ GeV
Left-handed neutrinos: Majorana neutrino mass eigenstate $N$ with arbitrary mass $m_N$

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010

$$[T_{1/2}^{0\nu}]^{-1} = G_0^{\nu} g_A^4 \left| \sum_N \left( U_{eN}^2 m_N \right) m_p M^{0\nu}_N (m_N, g_A^{\text{eff}}) \right|^2$$

**General case**

$$M^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2 \pi^2 g_A^2} \sum_n \int d^3 x \, d^3 y \, d^3 p \, \langle 0_F^+ | J^{\mu \dagger}(x) | n \rangle \langle n | J^{\dagger}_\mu(y) | 0_I^+ \rangle \frac{\sqrt{p^2 + m_N^2 (\sqrt{p^2 + m_N^2} + E_n - E_i - E_f)}}{p^2}$$

**light $\nu$ exchange**

$$M^{0\nu}(m_N \to 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M^{0\nu}_N (g_A^{\text{eff}})$$

**heavy $\nu$ exchange**

$$M^{0\nu}(m_N \to \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M^{0\nu}_N (g_A^{\text{eff}})$$

**Particular cases**

$$[T_{1/2}^{0\nu}]^{-1} = G_0^{\nu} g_A^4 \times$$

$$\times \left\{ \begin{array}{ll} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M^{0\nu}_\nu (g_A^{\text{eff}}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M^{0\nu}_N (g_A^{\text{eff}}) \right|^2 & \text{for } m_N \gg p_F \end{array} \right.$$

$$\langle m_\nu \rangle = \sum_N U_{eN}^2 m_N$$

$$\langle \frac{1}{m_N} \rangle = \sum_N U_{eN}^2 m_N$$
\[ [T_{1/2}^{0\nu}]^{-1} = A \cdot \left| \sum_{N} U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2, \]

\[ \langle p^2 \rangle = m_p m_e \left| \frac{M_{N}^{0\nu}(g_{A}^{\text{eff}})}{M_{\nu}^{0\nu}(g_{A}^{\text{eff}})} \right| \approx 200 \text{ MeV} \]

Interpolating formula

Faessler, Gonzalez, Kovalenko, F. Š., PRD 90 (2014) 096010
The light and heavy neutrino exchange are basically degenerate with the NME scaling factor \((^{76}\text{Ge}, ^{130}\text{Te}, ^{136}\text{Xe})\)

\[ \text{Sqrt}(\langle p^2 \rangle_a) = 175(11) \text{ MeV (Arg. src)} \]
\[ 205(13) \text{ MeV (CDBonn src)} \]


E. Lisi, A. Rotunno, F. Š., PRD 92 (2014) 093004
Interpolating formula is justified by practically no dependence $<p^2>$ on $A$.

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2}$$

Exclusion plot in $|U_{eN}|^2 - m_N$ plane

Improvements: i) QRPA (constrained Hamiltonian by $2\nu\beta\beta$ half-life, self-consistent treatment of src, restoration of isospin symmetry ...), ii) More stringent limits on the $0\nu\beta\beta$ half-life

$T^{0\nu}_{1/2}(^{76}\text{Ge}) \geq 3.0 \times 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}\text{Xe}) \geq 3.4 \times 10^{25}$ yr
II.c. The $0\nu\beta\beta$-decay within L-R symmetric theories (interpolating formula) (D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)


$$[T_{1/2}^{0\nu}]^{-1} = \eta_{\nu N}^2 C_{\nu N}$$

$$C_{\nu N} = g_A^4 |M_\nu^{0\nu}|^2 G_{0\nu}$$

Mixing of light and heavy neutrinos

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$\nu_{eL} = \sum_{j=1}^{3} \left( U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left( T_{ej} (\nu_{jL})^C + V_{ej}^* N_{jR} \right)$$

Effective LNV parameter within LRS model

$$\eta_{\nu N}^2 = \left| \sum_{j=1}^{3} \left( U_{ej}^2 \frac{m_j}{m_e} + S_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

$$\left| \sum_{j=1}^{3} \left( T_{ej}^2 \frac{m_j}{m_e} + V_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

$$\langle p^2 \rangle = m_p m_e \frac{M_N^{0\nu}}{M_\nu^{0\nu}}$$
6x6 PMNS see-saw ν-mixing matrix (the most economical one)

\[ U = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \]

Basis

\[ (\nu_L, (N_R)^c)^T \]

6x6 neutrino mass matrix

\[ M = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix} \]

Assumptions:

i) the see-saw structure

ii) mixing between different generations is neglected

6x6 matrix: 15 angles, 10+5 CP phases

3x3 matrix: 3 angles, 1+2 CP phases

3x3 block matrices U, S, T, V are generalization of PMNS matrix

6x6 PMNS see-saw parameter

\[ \zeta = \frac{m_D}{m_{LNV}} \]

6x6 matrix: 3 angles, 1+2 CP phases, 1 see-saw par.
6x6 PMNS see-saw $\nu$-mixing matrix (the most economical one)

$$U_0 = U_{PMNS}$$

$$V_0 = U_{PMNS}^\dagger = \begin{pmatrix}
    c_{12} c_{13} e^{-i\alpha_1} & \left(-s_{12} c_{23} - c_{12} s_{13} s_{23} e^{-i\delta}\right) e^{-i\alpha_1} \\
    s_{12} \left(s_{23} c_{13} e^{-i\alpha_2} - c_{23} \right) e^{-i\alpha_2} & \left(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta}\right) e^{-i\alpha_1} \\
    s_{13} e^{i\delta} & \left(-c_{12} s_{23} - s_{12} s_{13} c_{23} e^{-i\delta}\right) e^{-i\alpha_2}
\end{pmatrix}$$

Assumption about heavy neutrino masses $M_i$ (by assuming see-saw)

Inverse proportional

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \sum_{j=1}^{3} (U_{0}^\dagger)_{e_j} m_j$$

Proportional

$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^R = \lambda \zeta^2 \sum_{j=1}^{3} (U_{0}^\dagger)_{e_j} \frac{\langle p^2 \rangle_a}{m_j}$$

Heavy Majorana mass $M_{\beta\beta}^R$ depends on the “Dirac” CP violating phase $\delta$

Unlike “Majorana” CP phases $\alpha_1$ and $\alpha_2$

Contribution from exchange of heavy neutrino to $0\nu\beta\beta$-decay rate might be large

**Inverse proportional**

\[
m_i M_i \approx m_D^2
\]

\[
M^R_{\beta\beta} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U^\dagger_0)_e^2 j m_j \right|
\]

\[
V_0 = U_{PMNS}^\dagger
\]

\[
M_i = \frac{m_D^2}{m_i} \quad m_D \approx 5 \text{ MeV}
\]

\[
\lambda = 7.7 \times 10^{-4}
\]

**Proportional**

\[
m_i \approx \zeta^2 M_i
\]

\[
M^R_{\beta\beta} = \lambda \zeta^2 \left| \sum_{j=1}^{3} (U^\dagger_0)_e^2 j \frac{\langle p^2 \rangle_a}{m_j} \right|
\]

\[
V_0 = U_{PMNS}^\dagger
\]

\[
\zeta = \frac{m_i}{M_i} \quad \zeta^2 \approx 5 \times 10^{-17}
\]

\[
\lambda = 7.7 \times 10^{-4}
\]
\[ \eta_{\nu N}^2 = \frac{1}{m_e^2} \left( m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2 \right) \]

\[ m_i M_i \approx m_D^2 \]

\[ M_{\beta\beta}^R = \lambda \frac{\langle P^2 \rangle_a}{m_D^2} \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 m_j \]

See-saw scenario

Normal spectrum

Inverted spectrum

---

II.d. The $0\nu\beta\beta$-decay within L-R symmetric theories

(D-M mass term, see-saw, V-A and V+A int., exchange of light neutrinos)

Mixing of light and heavy neutrinos

$$\nu_{eL} = \sum_{j=1}^{3} \left( U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left( T_{ej}^* (\nu_{jL})^C + V_{ej}^* N_{jR} \right),$$

Effective LNV parameter due to RHC

$$\langle \lambda \rangle = \lambda \left| \sum_{j=1}^{3} U_{ej} T_{ej}^* \right|$$

Ratio of masses of vector bosons

$$\lambda = \left( \frac{M_{W_1}}{M_{W_2}} \right)^2$$
\[ [T_{1/2}^{0\nu}]^{-1} = \left( \eta^2_\nu + \eta^2_\lambda f_{\lambda m} \right) C_{mm} \]
\[ \approx \left( \eta^2_\nu + \eta^2_\lambda f^G_{\lambda m} \right) g_A^4 M^2_\nu G_{01} \]

\[ f_{\lambda m} = \frac{C_{\lambda\lambda}}{C_{mm}} \]
\[ \approx f^G_{\lambda m} = \frac{G_{02}}{G_{01}} \]
$m_{\beta\beta} = 50$ meV ($^{136}$Xe), $g_A = 1.269$, QRPA NMEs
III. $0\nu\beta\beta$ decay NMEs
0νββ-decay NME (light ν mass) – status 2017

J. Engel, J. Menendez, Rept. Prog. Phys. 80, 046301 (2017)

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unquenched $g_A$
Heavy $\nu$: $0\nu\beta\beta$ NMEs - status 2017

$M_{0\nu}^N$
Suppression of the 0νββ-decay NMEs due to different deformation of initial and final nuclei

The suppression of the NME depends on the relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler, NPA 733 (2004) 321

Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (light neutrino exchange)

D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)
$0\nu\beta\beta$-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (heavy neutrino exchange, Argonne src)

D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)
Ab Initio Nuclear Structure
(Often starts with chiral effective-field theory)

Nucleons, pions sufficient below chiral symmetry breaking scale. Expansion of operators in power of $Q/\Lambda_\chi$. $Q=m_\pi$ or typical nucleon momentum.

Calculation for the hypothetical $0\nu\beta\beta$ decay of $^{10}$He:
$^{10}$He $\rightarrow ^{10}$Be + e$^- + e^-$

masses, spectra
Supporting nuclear physics experiments
(2νββ-decay ChER, pion and heavy ion DCX, nucleon transfer reactions etc)

Heavy ion DCX:
NUMEN (LNC-INFN),
HIDCX (RCNP/RIKEN)

\[
^{18}O + ^{40}Ca \rightarrow ^{18}F + ^{40}K \rightarrow ^{18}Ne + ^{40}Ar.
\]

H. Lenske group
Theory of heavy ion DCX and Connection to DBD NMEs

Double GT Giant resonances
(exhausts a major part of sum-rule strength)
V. *Is there a proportionality between 0νββ- and 2νββ-decay NMEs?*
Understanding of the $2\nu\beta\beta$-decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$-decay NMEs.

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons.

Explaining $2\nu\beta\beta$-decay is necessary but not sufficient.

There is no reliable calculation of the $2\nu\beta\beta$-decay NMEs.

Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.) ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF
Is there a proportionality between $0\nu\beta\beta$- and $2\nu\beta\beta$-decay NMEs?

Known krom measured $2\nu\beta\beta$-decay half-life

Calc. within nuclear model

$M^{0\nu}_\beta/(m_e M^{2\nu}_{\text{exp}})$
$M^{0\nu} \propto M^{2\nu}_{\text{GT-cl}}$: ISM, EDF

$M^{\text{DGT}} = M^{2\nu}_{\text{GT}}$

<table>
<thead>
<tr>
<th>Element</th>
<th>SSD</th>
<th>ChER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>$^{76}\text{Ge}$</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>$^{116}\text{Cd}$</td>
<td>0.35</td>
<td>0.30</td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>0.41</td>
<td></td>
</tr>
</tbody>
</table>

EDF: $0.6 \to 1.2$
ISM: $0.1 \to 0.7$
IBM: $1.6 \to 4.4$
QRPA: $|0.1| \to |0.7|$

QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$M^{\text{DGT}}$ – only $1^+$
$M^{0\nu}$ - contribution from many $J^\pi$ (!)


Fedor Simkovic
QRPA: There is no proportionality between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay NMEs

F. Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

Region of GT resonance

ISM model space
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$M_{2\nu,GT-cl}^0 = \int_0^\infty C_{GT-cl}^{2\nu}(r) \, dr$

$M_{\nu,N-I}^{0\nu} = \int_0^\infty P_{I-src}^{\nu,N}(r) \, C_{I-cl}^{2\nu}(r) \, dr$

$I = F, GT$ and $T$

$r$-relative distance of two decaying nucleons

Neutrino potential prefers short distances

$\nu$-and $N$- exchange potential ($x f_{src}^2(r)$)
The only non-zero contribution from $J^\pi=1^+$

$$M_{GT-cl}^{2\nu} = \sum_{J^\pi} \int_0^\infty C_{GT-J^\pi}^{2\nu}(r) \, dr$$

$$\sum_{J^\pi, m} \langle 0_f^+ | \tau^+ \bar{\sigma} | J^\pi, m \rangle \cdot \langle J^\pi, m | \tau^+ \bar{\sigma} | 0_i^+ \rangle$$

$$\sum_{m} \langle 0_f^+ | \tau^+ \bar{\sigma} | 1^+, m \rangle \cdot \langle 1^+, m | \tau^+ \bar{\sigma} | 0_i^+ \rangle$$

Many multipole contributions not included within the ISM due to truncation of the model space.
ISM: \[ \text{Tail} \approx 0 \ (?! \) \Rightarrow M^{2\nu}_{\text{cl}} \gg 0 \]

QRPA: \[ \text{Bump} \approx -\text{Tail} \Rightarrow M^{2\nu}_{\text{cl}} \approx 0 \]

Close to restoration of the SU(4) symmetry of residual Hamiltonian

N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)
What is the origin of this peak?

2νββ–decay within the QRPA
(restoration of the SU(4) symmetry – $M_{2n}^{\text{cl}} = 0$)

$$g_A^{\text{eff}} = q \times g_A^{\text{free}} = 0.901$$
$$g_A^{\text{free}} = 1.269, \quad q = 0.710$$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$d_{pp}^i$</th>
<th>$d_{pp}^f$</th>
<th>$d_{nn}^i$</th>
<th>$d_{nn}^f$</th>
<th>$g_{pp}^{T=1}$</th>
<th>$g_{pp}^{T=0}$</th>
<th>$M_F^{2\nu}$ [MeV$^{-1}$]</th>
<th>$M_{GT}^{2\nu} \times q^2$ [MeV$^{-1}$]</th>
<th>$M_{exp}^{2\nu}$ [MeV$^{-1}$]</th>
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<tbody>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>-</td>
<td>1.069</td>
<td>-</td>
<td>0.982</td>
<td>1.028</td>
<td>0.745</td>
<td>-0.003</td>
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<tr>
<td>$^{76}\text{Ge}$</td>
<td>0.922</td>
<td>0.960</td>
<td>1.053</td>
<td>1.085</td>
<td>1.021</td>
<td>0.733</td>
<td>0.003</td>
<td>0.076</td>
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<tr>
<td>$^{82}\text{Se}$</td>
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<td>0.921</td>
<td>1.063</td>
<td>1.108</td>
<td>1.016</td>
<td>0.737</td>
<td>0.001</td>
<td>0.070</td>
<td>0.100</td>
</tr>
<tr>
<td>$^{96}\text{Zr}$</td>
<td>0.910</td>
<td>0.984</td>
<td>0.752</td>
<td>0.938</td>
<td>0.961</td>
<td>0.739</td>
<td>0.001</td>
<td>0.161</td>
<td>0.097</td>
</tr>
<tr>
<td>$^{100}\text{Mo}$</td>
<td>1.000</td>
<td>1.021</td>
<td>0.926</td>
<td>0.953</td>
<td>0.985</td>
<td>0.799</td>
<td>-0.001</td>
<td>0.304</td>
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<tr>
<td>$^{116}\text{Cd}$</td>
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<td>-</td>
<td>0.934</td>
<td>0.890</td>
<td>0.892</td>
<td>0.877</td>
<td>-0.000</td>
<td>0.059</td>
<td>0.136</td>
</tr>
<tr>
<td>$^{128}\text{Te}$</td>
<td>0.816</td>
<td>0.857</td>
<td>0.889</td>
<td>0.918</td>
<td>0.965</td>
<td>0.741</td>
<td>0.017</td>
<td>0.075</td>
<td>0.052</td>
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<tr>
<td>$^{130}\text{Te}$</td>
<td>0.847</td>
<td>0.922</td>
<td>0.971</td>
<td>1.011</td>
<td>0.963</td>
<td>0.737</td>
<td>0.016</td>
<td>0.064</td>
<td>0.037</td>
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<tr>
<td>$^{136}\text{Xe}$</td>
<td>0.782</td>
<td>0.885</td>
<td>-</td>
<td>0.926</td>
<td>0.910</td>
<td>0.685</td>
<td>0.014</td>
<td>0.039</td>
<td>0.022</td>
</tr>
</tbody>
</table>
Fermi, Gamow-Teller and tensor

Role of the change of the isospin

$S=0$ and $S=1$ contributions
V. Quenching of $g_A$
Quenching in nuclear matter: $g_{A}^{\text{eff}} = q g_{A}^{\text{free}}$

(from theory: $T_{1/2}^{0n}$ up 50 x larger)

$\mathcal{L} = -\frac{G_{\beta}}{\sqrt{2}} \left[ \bar{u} \gamma^{\alpha} (1 - \gamma^{5}) d \right] \left[ \bar{e} \gamma^{\alpha} (1 - \gamma^{5}) \nu_{e} \right]$

$\mathcal{L} = -\frac{G_{\beta}}{\sqrt{2}} \left[ \bar{p} \gamma^{\alpha} \left( g_{V} - g_{A} \gamma^{5} \right) n \right] \left[ \bar{e} \gamma^{\alpha} (1 - \gamma^{5}) \nu_{e} \right]$

CVC hypothesis

$g_{V} = 1$ at the quark level

$g_{V} = 1$ at the nucleon level

$g_{V} = 1$ inside nuclei

Quenching of $g_{A}$

$g_{A} = 1$ at the quark level

$g_{A}^{\text{free}} = 1.27$ at the nucleon level

$g_{A}^{\text{eff}} = ?$ inside nuclei

ISM: $(g_{A}^{\text{eff}})^{4} \approx 0.66 \ (^{48}\text{Ca}), 0.66 \ (^{76}\text{Ge}), 0.30 \ (^{76}\text{Se}), 0.20 \ (^{130}\text{Te})$ and $0.11 \ (^{136}\text{Xe})$

IBM: $(g_{A}^{\text{eff}})^{4} \approx (1.269 A^{-0.18})^{4} = 0.063$

QRPA: $(g_{A}^{\text{eff}})^{4} = 0.30$ and $0.50$ for $^{100}\text{Mo}$ and $^{116}\text{Cd}$
\[ g_A^4 = (1.269)^4 = 2.6 \]

**Quenching of** \( g_A \) **(from exp.:** \( T_{1/2}^{0n} \) **up 2.5 x larger)**

\[ (g_{\text{eff}}_A)^4 = 1.0 \]

**Strength of GT trans. (approx. given by Ikeda sum rule =3(\( N-Z \))) has to be quenched to reproduce experiment**

\[ 76^{32}\text{Ge} \rightarrow 76^{76}\text{Ge} \quad S_\beta^- - S_\beta^+ = 3(N-Z) = 36 \]

**Cross-section for charge exchange reaction:**

\[
\left[ \frac{d\sigma}{d\Omega} \right] = \left[ \frac{\mu}{\pi \hbar} \right]^2 \frac{k_f}{k_i} Nd \left| V_{\sigma\tau} \right|^2 \left| \langle f \mid \sigma \tau \mid i \rangle \right|^2
\]

\( q = 0!! \)

largest at 100 - 200 MeV/A
\((g_{\text{eff}}^A)^4 = 0.30\) and \(0.50\) for \(^{100}\text{Mo}\) and \(^{116}\text{Cd}\), respectively (The QRPA prediction). \(g_{\text{eff}}^A\) was treated as a completely free parameter alongside \(g_{\text{pp}}\) (used to renormalize particl-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of \(g_{\text{eff}}^A\) and \(g_{\text{pp}}\), where possible, to the \(\beta-\)decay rate and \(\beta^+/EC\) rate of the \(J = 1^+\) ground state in the intermediate nuclei involved in double-beta decay in addition to the \(2\nu\beta\beta\) rates of the initial nuclei, leads to an effective \(g_{\text{eff}}^A\) of about 0.7 or 0.8.

![Diagram showing \((g_{\text{pp}}, g_A)\) allowed regions for \(^{100}\text{Mo}\) and \(^{116}\text{Cd}\).](image)

Extended calculation also for neighbor isotopes performed by F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

Dependence of \(g_{\text{eff}}^A\) on \(A\) was not established.
Quenching of $g_A$ -IBM ($T_{1/2}^{0+n}$ suppressed up to factor 50)

$(g_{A,\text{eff}}^4 \approx (1.269 A^{-0.18})^4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the $2\nu\beta\beta$-decay half-lives, which were based on within closure approximation calculated Corresponding NMEs, with the measured half-lives.

Improved description of the 0νββ–decay rate (and novel approach of fixing $g_A^{\text{eff}}$)

Let perform Taylor expansion

\[
M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{(E_n - (E_i + E_f)/2)^2 - \xi_{K,L}^2}
\]

\[
\xi_{K,L} = \frac{E_e^2 + E_{\nu_2} - E_{e_1} - E_{\nu_1}}{2}
\]

We get

\[
\left[ T_{1/2}^{2\nu\beta\beta} \right]^{-1} \approx \left( g_A^{\text{eff}} \right)^4 \left| M_{GT-3}^{2\nu} \right|^2 \frac{1}{\xi_{13}^{2\nu}} \left( G_{0}^{2\nu} + \xi_{13}^{2\nu} G_{2}^{2\nu} \right)
\]

\[
M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)}
\]

\[
M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3}
\]

\[
\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}
\]

The $g_A^{\text{eff}}$ can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)
The running sum of the $2\nu\beta\beta$–decay NMEs (QRPA)

$$M^2_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)}$$

$$M^2_{GT-3} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3}$$
Normalized to unity
different partial energy distributions

\[
\left[ T_{1/2}^{2\nu\beta\beta} \right]^{-1} \equiv \frac{\Gamma_{2\nu}}{\ln(2)} \approx \frac{\Gamma_{0}^{2\nu} + \Gamma_{2}^{2\nu} + \Gamma_{4}^{2\nu}}{\ln(2)}
\]
\( \xi_{13} \) tell us about importance of higher lying states of int. nucl.

\( \xi_{13} \) can be determined phenomenologically from the shape of energy distributions of emitted electrons.

\( \xi_{13} = 0 \) for HSD mechanism.
Solution: measurement of $\xi$ and calculation of $M_{GT-3}$ have to be calculated by nuclear theory - ISM

\[
\left( g_A^{\text{eff}} \right)^2 = \frac{1}{M_{GT-3}^{2\nu}} \left| \xi_{31}^{2\nu} \right| \sqrt{T_{1/2}^{2\nu-\text{exp}}} \left( G_0^{2\nu} + \xi_{31}^{2\nu} G_2^{2\nu} \right)
\]

Preliminary

excluded by the KamLAND-Zen exp.
New modes of the double beta decay
Double Beta Decay with emission of a single electron


[Jung et al. (GSI), 1992] observed beta decay of $^{163}_{66}$Dy$^{66+}$ ions with Electron Production (EP) in K or L shells: $T^{EP}_{1/2} = 47$ d

Bound-state double-beta decay $0\nu EP\beta^-$ ($2\nu EP\beta^-$) with EP in available $s_{1/2}$ or $p_{1/2}$ subshell of daughter $2^+$ ion:

$$^{A}ZX \rightarrow ^{Z+2}A Y + e^- + e^- + (\bar{\nu}_e + \bar{\nu}_e)$$

Search for possible manifestation in single-electron spectra...
Phase space factors

0νEPβ⁻ and 2νEPβ⁻ phase-space factors:

\[
G^{0\nu\text{EP}\beta^-}(Z, Q) = \frac{G_\beta^4 m_e^2}{32\pi^4 R^2 \ln 2} \sum_{n=n_{\text{min}}}^{\infty} B_n(Z, A) F(Z + 2, E) E p
\]

\[
G^{2\nu\text{EP}\beta^-}(Z, Q) = \frac{G_\beta^4}{8\pi^6 m_e^2 \ln 2} \sum_{n=n_{\text{min}}}^{\infty} B_n(Z, A) \int_{m_e}^{m_e+Q} dE \int_{0}^{m_e+Q-E} d\omega_1 \omega_1^2 \omega_2^2 F(Z + 2, E) E p
\]

Single-electron spectra for \( ^{82}\text{Se} \) (\( Q = 2.998 \text{ MeV} \)):

Bound- and free-electron Fermi functions:

\[
B_n(Z, A) = f_{n,-1}^2(R) + g_{n,+1}^2(R)
\]

\[
F(Z, E) = f_{-1}^2(R, E) + g_{+1}^2(R, E)
\]

Relativistic electron wave functions in central field:

\[
\psi_{\kappa\mu}(\vec{r}) = \begin{pmatrix} f_\kappa(r) \Omega_{\kappa\mu}(\hat{r}) \\ i g_\kappa(r) \Omega_{-\kappa\mu}(\hat{r}) \end{pmatrix}
\]

\[
\kappa = (l - j)(2j + 1) = \pm 1, \pm 2, \ldots
\]

\[
j = |l \pm 1/2|
\]

\[
\mu = -j, \ldots, +j
\]
Stationary $N$-particle Dirac eq. with separable central atomic Hamiltonian [a.u.]:

$$\sum_{i=1}^{N} -i \nabla_i \cdot \tilde{\alpha} c + \beta c^2 - \frac{Z}{r_i} + V(r_i) \right] \Psi = E \Psi$$

$$\Psi = \frac{1}{\sqrt{N!}} \begin{bmatrix} \psi_1(\tilde{r}_1) & \cdots & \psi_1(\tilde{r}_N) \\ \vdots & \ddots & \vdots \\ \psi_N(\tilde{r}_1) & \cdots & \psi_N(\tilde{r}_N) \end{bmatrix}$$

Multiconfiguration Dirac–Hartree–Fock package GRASP2K:

- Fit of non-convergent orbitals: $f_{n,-1}^2, g_{n,+1}^2(R) \approx aZ^b$
- Fit of orbitals beyond $n = 9$: $f_{n,-1}^2, g_{n,+1}^2(R) \approx cn^d$

$$\sum_{n} B_n(Z, A) \text{ [a. u.]}$$

Diagram illustrating the behavior of $B_n(Z, A)$ for different values of $Z$.
$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ single-electron spectra $1/\Gamma^{0\nu\beta\beta} \, d\Gamma/dE$ vs. electron kinetic energy $E - m_e$ for $^{82}\text{Se}$ ($Q = 2.996$ MeV)
$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ half-lives $T^{0\nu\beta\beta}_{1/2}$ and $T^{0\nu\text{EP}\beta}_{1/2}$ estimated for $\beta^-\beta^-$ isotopes with known NME $|M^{0\nu\beta\beta}|$, assuming unquenched $g_A = 1.269$ and $|m_{\beta\beta}| = 50$ meV

Suppressed by about $10^{-4}$
$2\nu\beta^-\beta^-$ and $2\nu{\text{EP}}\beta^-$ single-electron spectra $1/\Gamma \, d\Gamma/dE$ vs. electron kinetic energy $E - m_e$ for $^{82}\text{Se}$ ($Q = 2.996$ MeV)
$2\nu\beta^-\beta^-$ and $2\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{2\nu\beta\beta}$ and $T_{1/2}^{2\nu\text{EP}\beta}$ calculated for $\beta^-\beta^-$ isotopes observed experimentally, assuming unquenched $g_A = 1.269$. 

Suppressed by about $10^{-3}$
**DBD theoretical challenges**

*Particle physics:*

1. Understanding of the effective Majorana mass
2. What is the dominant mechanism of the $0\nu\beta\beta$-decay
3. Connection to laboratory $\nu$-mass measurement, cosmology, LHC physics, etc

*Nuclear physics:*

1. Progress in nuclear structure theory
   - reliable description of the $\beta$-, EC-, $2\nu\beta\beta$-decay, ChER, DCX etc
   - role of the isospin and spin-isospin symmetry
   - understanding of uncertainty in calculated NMEs
2. Understanding of quenching of $g_A$
We are at the beginning of the Beyond Standard Model Road…

The future of neutrino physics is bright

Instead of Conclusions

\[ \frac{1}{\Lambda^2} \sum_i c_i^{(6)} O_i^{(6)} + \mathcal{O}(\frac{1}{\Lambda^3}) \]

Neutrino physics

\[ \frac{1}{\Lambda} \sum_i c_i^{(5)} O_i^{(5)} \]

\( \theta_{\nu\beta\beta} \)

LHC physics

Progress in nuclear structure calculations is highly required