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Measurement-theoretical analysis of neutron interference at low transmission probability

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Recent experimental results of neutron interferometry at low transmission probability are analyzed when an absorber is present in one of the two routes of the interferometer. The process is regarded as a partial dephasing characterized by the *decoherence* parameter, along the same line of thought as in the many-Hilbert-space approach to the quantum measurement problem. It is shown that the density fluctuations of the elementary constituents of the absorber provoke a reduction of the visibility of the interference pattern. The effect is evaluated analytically in the case of Gaussian fluctuations, and found to be in agreement with the experimental data.

A few years ago, a very interesting neutron interferometry experiment was performed by the Vienna group [1], showing a big discrepancy of the experimental data from the theoretical prediction for the visibility: The experimental points lay remarkably below the quantum-mechanical curve, which is proportional to $\sqrt{t}$ in the low t region, t being the transmission probability. In a previous paper [2], two of us discussed a possible reduction of the interference term in the above-mentioned experiment, due to statistical fluctuations of the absorber constituents. The process was characterized by means of the *decoherence* parameter, which was used later as a sort of "order parameter" to measure the wave function collapse within the framework of the many-Hilbert-space (MHS) approach to the quantum measurement problem [3,4]. However, only a qualitative analysis based on a perturbative calculation was proposed, and not a quantitative one liable to comparison with experiment.

The aim of this note is to improve our previous analysis [2], by showing nonperturbatively that the reduction of the visibility in a neutron-interferometry experiment is a physical consequence of the presence of statistical fluctuations in the absorber. Incidentally, this will show that the MHS approach [3,4] can be discriminated experimentally from von Neumann's theory [5]. We shall start by analyzing a typical double-slit experiment.

Let the incident neutron wave packet be split into two branch waves ψ_1 and ψ_2, corresponding to two different routes in the apparatus, and assume that ψ_2 interacts first with a phase shifter and then with an absorber. The first contributes a phase factor $e^{i\delta}$ while the second is assumed to simply multiply the wave function by a transmission coefficient T, so that

$$\psi_2 \to e^{i\delta} T \psi_2 . \tag{1}$$

If ψ_1 and ψ_2 are in phase and $|\psi_1| = |\psi_2| = 1$, the intensity after recombination of the two branch waves is

$$I \propto |\psi_1 + e^{i\delta} T \psi_2|^2 = 1 + |T|^2 + 2\,\mathrm{Re}(T e^{i\delta})$$
$$= 1 + t + 2\sqrt{t}\cos(\alpha+\delta)\,, \qquad (2)$$

where we have written $T = |T|e^{i\alpha}$, and have defined the transmission probability $t = |T|^2$. In this way, the visibility of the interference pattern is

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{t}}{1+t}\,. \qquad (3)$$

These are standard quantum mechanical formulae: Notice that the behaviour of the macroscopic apparatuses has been ignored, and their effect on the wave function has been "summarized" by introducing two "constants" (δ and T in eq. (1)). On the other hand, according to the MHS theory, the internal structure of macroscopic apparatuses cannot be neglected, and must be taken into account. In particular, we have to consider the density fluctuations of the elementary constituents of the absorber. Let us focus our attention on the Vienna experiments [1], in which the absorber is a solution of Gd in H_2O. Our point is the following: In an interference experiment one must accumulate a huge number of experimental results, produced by neutrons sent through a very weak and steady beam, in order to obtain an interference pattern. Even though, during an experimental run, the *macroscopic* state of the absorber is always the same, each neutron will interact with a slightly different *microscopic* state of it: Indeed, the Gd atoms (which are responsible for neutron absorption) are subject to their own internal motion, and their positions change all the time; moreover, different neutrons will go through (and interact with) different parts of the absorber, due to the finite lateral size of the beam. Roughly speaking we can say that, *as far as absorption and transmission probabilities are concerned*, each neutron interacts with a small cylinder of the Gd–H_2O solution: This cylinder has a height roughly equal to the length of the absorber and a base roughly equal to the neutron–Gd total cross section (see fig. 1). Notice that we are neglecting the role of water in the process. This assumption is sound, because water does not strongly absorb neutrons, and, as we shall see, the density fluctuations of the water molecules are completely negligible when compared to Gd.

In the light of the previous discussion, we shall label different incoming neutrons with j ($j=1, ..., N_p$, where N_p is the total number of neutrons in an experimental run), and write T_j for the transmission coefficient of the jth neutron. The only experimentally meaningful quantities will then be

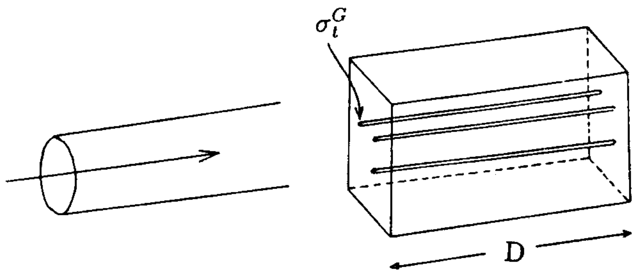

Fig. 1. Different paths corresponding to different neutrons.

$$\bar{T} = \frac{1}{N_p}\sum_{j=1}^{N_p} T_j\,, \qquad (4)$$

$$\overline{|T|^2} = \frac{1}{N_p}\sum_{j=1}^{N_p} |T_j|^2\,. \qquad (5)$$

We identify $\overline{|T|^2}$ with t, the *experimentally measured* value of the transmission probability. It is then very simple to show that [2,4]

$$|\bar{T}|^2 \leqslant \overline{|T|^2} \equiv t\,, \qquad (6)$$

and in particular,

$$|\bar{T}|^2 = t(1-\epsilon)\,, \qquad (7)$$

where ϵ ($0 \leqslant \epsilon \leqslant 1$) has been named *decoherence parameter* [2,4]. Its definition is therefore

$$\epsilon = 1 - \frac{|\bar{T}|^2}{t}\,. \qquad (8)$$

The intensity and visibility (eqs. (2) and (3)) are now given by

$$I' \propto 1 + \overline{|T|^2} + 2\,\mathrm{Re}(\bar{T} e^{i\delta})$$
$$= 1 + t + 2\sqrt{t}\sqrt{1-\epsilon}\cos(\beta+\delta)\,,$$
$$V' = \frac{2\sqrt{t(1-\epsilon)}}{1+t} = V\sqrt{1-\epsilon}\,, \qquad (9)$$

where we have written $\bar{T} = |\bar{T}|e^{i\beta}$. One sees clearly that for $\epsilon = 1$ interference disappears: This represents the case of *total loss of coherence* between the two branch waves.

In ref. [2] the decoherence parameter ϵ was evaluated in the first order approximation by following a perturbative approach in terms of the interaction Hamiltonian for the neutron–absorber system. On the other hand, some recent data obtained by the Vienna group [6] seem to indicate that the value of the visibility is strongly reduced in the region of very low transmission probability. We are therefore compelled to search for a nonperturbative estimate of ϵ.

Generalizing the well-known Goldberger formula for the refraction index in neutron optics [7], let us start by writing

$$T=\exp\left(ikF_G\int_0^D \rho_G(x)\,dx\right)\exp(ikF_W\rho_W D) \tag{10}$$

for the transmission coefficient of a neutron with wave number k, passing through an absorber of thickness D, containing a solution of Gd in water, where

$$F_G=-\frac{2\pi b_R^G}{k^2}+i\frac{\sigma_a^G}{2k}, \tag{11}$$

$$F_W=-\frac{2\pi b_R^W}{k^2}+i\frac{\sigma_a^W}{2k}, \tag{12}$$

and we have denoted the real part of the scattering length of the elastic neutron–Gd (neutron–water) collision by b_R^G (b_R^W), and the absorption cross section for the neutron–Gd (neutron–water) collision by σ_a^G (σ_a^W). We have neglected small reflection-deflection effects at every point along the incident direction, conventionally set as x-axis. The quantities ρ_G and ρ_W are the Gd and water densities, respectively, expressed in the number of particles per unit volume. For simplicity, we assumed ρ_W constant, and took only the spatial variation of $\rho_G(x)$ into account. Moreover, we used the following averaged density,

$$\rho_G(x)=\frac{1}{\sigma_t^G}\iint_{\sigma_t^G}\rho_G(\boldsymbol{r})\,dy\,dz, \tag{13}$$

σ_t^G and $\rho_G(\boldsymbol{r})$ being the total cross section of the neutron–Gd collision and the density of Gd atoms, respectively.

We decompose the Gd density function into its average and fluctuation part as follows,

$$\rho_G(\boldsymbol{r})=\langle\rho_G\rangle+\delta\rho_G(\boldsymbol{r}), \tag{14}$$

and set

$$\langle\delta\rho_G(\boldsymbol{r})\delta\rho_G(\boldsymbol{r}')\rangle=\langle\rho_G\rangle G(\boldsymbol{r}-\boldsymbol{r}'), \tag{15}$$

in which $G(\boldsymbol{r}-\boldsymbol{r}')$ represents the density correlation function of Gd atoms in the H_2O–Gd solution (note that $G(\boldsymbol{r})=G(-\boldsymbol{r})$). In general, $G(\boldsymbol{r}-\boldsymbol{r}')$ contains not only an autocorrelation part (approximately) proportional to $\delta(\boldsymbol{r}-\boldsymbol{r}')$, but also a long-range pair-correlation part peculiar to liquids. A theoretical analysis of $G(\boldsymbol{r}-\boldsymbol{r}')$ is not so easy, because we should solve exactly the complicated molecular theory of the two-constituent liquid. However, even though we know that the long-range pair-correlation part should have an important effect on the spectral distribution of the fluctuations, in the present approximation, based on eq. (10), it appears only in the integrated strength of the correlation, as will also be seen below.

By writing $\rho_G(x)$, defined by eq. (13), as the sum of the mean value $\langle\rho_G\rangle$ and the fluctuating component $\delta\rho_G(x)$, we easily obtain

$$\langle\delta\rho_G(x)\delta\rho_G(x')\rangle=\langle\rho_G\rangle\bar{G}(x-x'), \tag{16}$$

where

$$\bar{G}(x-x')\equiv\frac{1}{(\sigma_t^G)^2}\times\int\ldots\int G(x-x',y-y',z-z')\,dy\,dz\,dy'\,dz'. \tag{17}$$

Note that $\bar{G}(x)$ is an even function of x. Equation (16) yields

$$\int_0^D\int_0^D\langle\delta\rho_G(x)\delta\rho_G(x')\rangle\,dx\,dx'=\frac{g\langle\rho_G\rangle D}{\sigma_t^G}, \tag{18}$$

where we have defined the dimensionless c-number

$$g\equiv 2\sigma_t^G\int_0^D\bar{G}(x)\,dx, \tag{19}$$

representing the strength of the correlation. We can use $\bar{G}(x-x')\simeq G(x-x',0,0)$ if $\sqrt{\sigma_t^G}$ is much smaller than the correlation length in the liquid.

Let us briefly discuss the role of g. If we had just an ideal Gd gas in the absorber, we could set $G(\boldsymbol{r}-\boldsymbol{r}')=\delta(\boldsymbol{r}-\boldsymbol{r}')$ (dilute-solution limit [8]), and obtain $g=1$. In this case we would overestimate the fluctuations, in comparison with a realistic absorber.

For this reason we can generally consider g as a reduction factor smaller than unity.

Notice that, by eq. (10), fluctuations of T stem only from the density function $\rho_G(x)$, for fixed k and D, because all other quantities are given in terms of constants of the elementary neutron–Gd and neutron–water collisions. Thus we infer that every averaged quantity can be written in terms of g.

In the above equations, $\langle ... \rangle$ is a statistical ensemble average over the absorber microstates. We shall make the following ergodic hypothesis,

$$\overline{...} = \langle ... \rangle \,, \tag{20}$$

where $\overline{...}$ is the average over many particles, introduced in eqs. (4) and (5). From eqs. (10), (18) and the Gaussian reduction formula, we immediately obtain

$$\begin{aligned}\bar{T} = \langle T \rangle &= \exp(ikF_G \langle \rho_G \rangle D) \exp(ikF_W \rho_W D) \\ &\times \exp[-\tfrac{1}{2}k^2 F_G^2 (g\langle \rho_G \rangle / \sigma_t^G) D] \\ &= T_0 \exp[-\tfrac{1}{2}k^2 F_G^2 (g\langle \rho_G \rangle / \sigma_t^G) D] \,,\end{aligned}$$

$$T_0 \equiv \exp(ikF_G \langle \rho_G \rangle D) \exp(ikF_W \rho_W D) \,, \tag{21}$$

which yields

$$|\bar{T}|^2 = t_0 \exp[-k^2 \mathrm{Re}(F_G^2)(g\langle \rho_G \rangle / \sigma_t^G) D] \,, \tag{22}$$

$$t_0 \equiv |T_0|^2 = \exp(-\sigma_a^G \langle \rho_G \rangle D) \exp(-\sigma_a^W \rho_W D) \,, \tag{23}$$

where t_0 is the transmission probability in the absence of fluctuations. Similarly, we obtain

$$\begin{aligned}\overline{|T|^2} &= \langle |T|^2 \rangle \\ &= t_0 \exp[2k^2 (\mathrm{Im}\, F_G)^2 (g\langle \rho_G \rangle / \sigma_t^G) D] \,.\end{aligned} \tag{24}$$

As already pointed out after eq. (5), this is the quantity we have to identify with the experimentally measured transmission probability

$$t \equiv \overline{|T|^2} = \langle |T|^2 \rangle \,. \tag{25}$$

We obtain therefore the following expression for the decoherence parameter,

$$\epsilon = 1 - \exp\left(-\frac{\pi}{k^2} \frac{\sigma_s^G}{\sigma_t^G} g \langle \rho_G \rangle D \right) . \tag{26}$$

Eliminating D from eqs. (22), (24) and (25), we obtain

$$|\bar{T}|^2 = t^{1+\gamma} \,, \tag{27}$$

where

$$\gamma = \frac{g(\pi/k^2)\sigma_s^G / \sigma_t^G \sigma_a^G}{1 - \tfrac{1}{2} g \sigma_a^G / \sigma_t^G + \sigma_a^W \rho_W / \sigma_a^G \langle \rho_G \rangle} \,. \tag{28}$$

Here σ_s^G stands for the cross section of the elastic neutron–Gd collision,

$$\sigma_s^G = 4\pi [(b_R^G)^2 + (k\sigma_a^G / 4\pi)^2] \,. \tag{29}$$

The decoherence parameter can be rewritten in terms of γ as

$$\epsilon = 1 - t^\gamma \,. \tag{30}$$

We do not know the actual values of $\langle \rho_G \rangle$ and g. The Vienna group derived the value of $\langle \rho_G \rangle$ from the experimental value of t_0 by means of eq. (23), neglecting all small contributions from the water component. The present approach suggests, however, that we have to use eq. (25), with (24), instead of (23). Nevertheless, for the sake of simplicity, let us provisionally use the Vienna value

$$\langle \rho_G \rangle = 5 \times 10^{26} \; [\mathrm{m}^{-3}] \,, \tag{31}$$

then we can determine γ, $|\bar{T}|^2$ and the visibility by leaving g as an unknown free parameter. Figure 2 shows the numerical results in the low t region for the parameter values $g=0$, 0.3, 0.5, 0.7 and 1. The experimental points in fig. 2 were obtained by the Vienna group [6].

It is worth searching for some possible improvements of the calculation here presented. First, we should remark that $\bar{T}$ and $t = \overline{|T|^2}$ depend on the two independent quantities D and $\langle \rho_G \rangle$ (notice that g is also a function of them). In order to obtain a formula for the visibility capable of fitting well the experimental data over the whole range of t, we should investigate the t-dependence of $\langle \rho_G \rangle$ in a molecular theoretical way, with the help of statistical mechanics, by making use of detailed properties of the Gd–water solution. This is a delicate problem to be tackled in the future. At the present stage we can only draw qualitative conclusions about this dependence: $\langle \rho_G \rangle \to 0$ as t goes to 1. This means that fluctuation effects rapidly decrease, and V' rapidly approaches V, as $t \to 1$.

Second, we should remark that eq. (27) must be handled very carefully, due to the $\langle \rho_G \rangle$-dependence

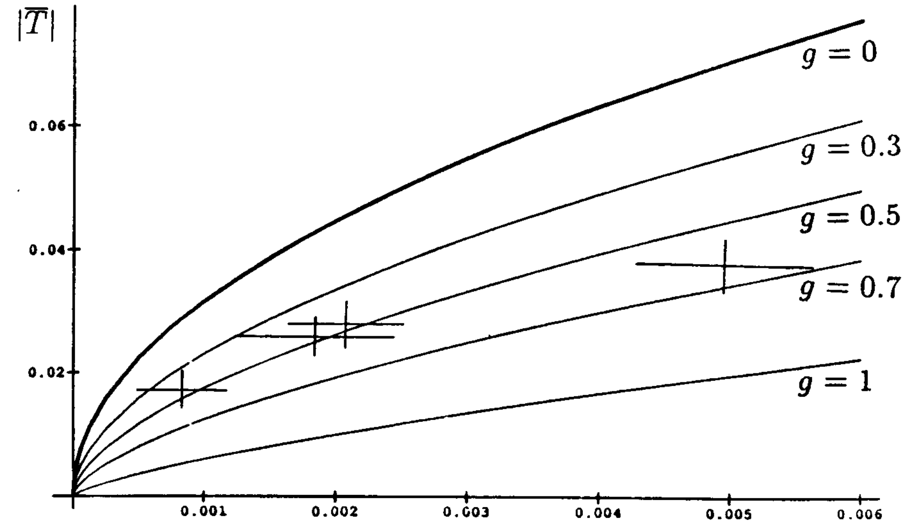

Fig. 2. Comparison of the experimental data with the theoretical predictions for different values of the parameter g. The cases $g=0$, 1 correspond to the absence of fluctuations and the ideal-gas correlation function, respectively.

of γ, in eq. (28). For instance, when $t\simeq 1$ we have $\langle\rho_G\rangle\simeq 0$, and therefore $\gamma\to 0$. In this case, eq. (27) yields

$$|\bar{T}|^2\simeq t\simeq 1\,, \qquad (32)$$

so that fluctuation effects become negligible, and the usual relation between transmission coefficient and probability is recovered.

Finally, we did not consider any effects deriving from inhomogeneities of the sample thickness D, and from dispersion in momentum k. As suggested by Rauch [6], these factors should also bear consequences on the observed reduction of the visibility of the interference pattern.

In this paper we have shown in a nonperturbative way that the experimentally observed reduction of the neutron interference at low transmission probability can be explained by taking into account the density fluctuations in the absorber, on the basis of Goldberger's formula, eq. (10). However, we should remark that eq. (10) is an approximate formula, by which we cannot analyze the details of the space–time relaxation or the spectral distribution of the fluctuations. In order to do this, we have to develop a mathematical technique similar to the one in our previous paper [2]. Let us give an outline of a possible theoretical approach to this problem, in a perturbative way, by improving a few points of our previous analysis: First of all, note that the transmission coefficient stems from the S-matrix element $\langle\phi_{k'}|S|\phi_k\rangle$ between two plane waves $\phi_{k'}(\boldsymbol{r})=(1/\sqrt{\Omega})\times\exp(\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{r})$ and $\phi_k(\boldsymbol{r})=(1/\sqrt{\Omega})\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$, Ω being the normalization volume. After the decomposition procedure of the Hamiltonian outlined in our previous paper, we expand $\langle\phi_{k'}|U_I(T)|\phi_k\rangle$ in the Dyson series of the interaction Hamiltonian, whose first-order term is given by

$$\Delta_1=-\frac{\mathrm{i}}{\hbar}\frac{V_0\delta\Omega}{\Omega}\int_0^T\mathrm{d}t\int\mathrm{d}^3r\,\delta\rho_G(\boldsymbol{r},t)$$

$$\times\exp[\mathrm{i}(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}-\mathrm{i}(\omega_k-\omega_{k'})(t-t')] \qquad (33)$$

for the potential $V(\boldsymbol{r})=V_0\delta\Omega\delta(\boldsymbol{r})$, in which $\omega_k=\hbar k^2/2m$.

By introducing the statistical rule

$$\langle\delta\rho_G(\boldsymbol{r},t)\rangle=0\,, \qquad (34)$$

$$\langle\delta\rho_G(\boldsymbol{r},t)\delta\rho_G(\boldsymbol{r}',t')\rangle$$

$$=\langle\rho_G\rangle G(\boldsymbol{r}-\boldsymbol{r}',t-t')\,, \qquad (35)$$

for the density fluctuations, the average absolute value of the first order term becomes

$$\overline{|\Delta_1|^2}=\frac{\langle\rho_G\rangle|V_0|^2(\delta\Omega)^2}{\hbar^2\Omega^2}$$

$$\times T\Omega\tilde{G}(\boldsymbol{k}'-\boldsymbol{k},\omega_{k'}-\omega_k)\,, \qquad (36)$$

where $\tilde{G}(\boldsymbol{k}, \omega)$ is the Fourier transform of $G(\boldsymbol{r}, t)$. In order to obtain a theoretical formula useful for comparison with the experiments, we have to sum this quantity over $\boldsymbol{k}'$ around $\boldsymbol{k}$, that is, over $\boldsymbol{K}=\boldsymbol{k}'-\boldsymbol{k}$ around 0, with a weight $w(\boldsymbol{K})$ given by the $\boldsymbol{k}$ and $\boldsymbol{k}'$ distributions. We finally obtain the following first-order estimate for the decoherence parameter,

$$\epsilon \simeq \sum_{\boldsymbol{k}' \text{ around } \boldsymbol{k}} \overline{|\Delta_1|^2} \to \frac{\Omega}{(2\pi)^3} \int d^3\boldsymbol{K}\, w(\boldsymbol{K}) \overline{|\Delta_1|^2}$$

$$= \frac{|V_0|^2 (\delta\Omega)^2}{\hbar^2} T\langle\rho_G\rangle \int \frac{d^3\boldsymbol{K}}{(2\pi)^3} w(\boldsymbol{K}) \tilde{G}(\boldsymbol{K}, \boldsymbol{K}\cdot\boldsymbol{v}_0)$$

$$= \tfrac{1}{4}(\ln a_0)^2 [1 + (V_{0R}/V_{0I})^2] f, \qquad (37)$$

where a_0 is given by eq. (32) of ref. [2] and we set $\omega_{\boldsymbol{k}'} - \omega_{\boldsymbol{k}} \simeq \boldsymbol{K}\cdot\boldsymbol{v}_0$, $\boldsymbol{v}_0$ being the particle velocity. The quantity f is a dimensionless constant given by

$$f = \frac{1}{\langle\rho_G\rangle T} \int \frac{d^3\boldsymbol{K}}{(2\pi)^3} w(\boldsymbol{K}) \tilde{G}(\boldsymbol{K}, \boldsymbol{K}\cdot\boldsymbol{v}_0), \qquad (38)$$

and is written in terms of the spectral distribution function $\tilde{G}$, which is the Fourier transform of the space–time correlation function.

We conclude by observing that the effect we are pointing out, and in general any "dephasing" effects describable by the decoherence parameter ϵ, are extraneous to von Neumann's theory and to the standard Copenhagen interpretation. It is somewhat gratifying that a traditionally academic issue like the problem of measurement in quantum mechanics may contribute actively to physics, by proposing new experimental tests and by analyzing the most recent experimental data. To the authors' knowledge, it is the first time that a theory can be discriminated *experimentally* from von Neumann's, after many years of philosophical and academic discussions.

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